

GCE Examinations  
Advanced Subsidiary

## Core Mathematics C3

Paper K

Time: 1 hour 30 minutes

### *Instructions and Information*

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Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration.

Full marks may be obtained for answers to ALL questions.

Mathematical formulae and statistical tables are available.

This paper has seven questions.

### *Advice to Candidates*

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You must show sufficient working to make your methods clear to an examiner.  
Answers without working may gain no credit.



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1. (a) Find the exact value of  $x$  such that

$$3 \arctan (x - 2) + \pi = 0. \quad (3)$$

- (b) Solve, for  $-\pi < \theta < \pi$ , the equation

$$\cos 2\theta - \sin \theta - 1 = 0,$$

giving your answers in terms of  $\pi$ . (5)

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2. (a) Express

$$\frac{4x}{x^2 - 9} - \frac{2}{x + 3}$$

as a single fraction in its simplest form. (4)

- (b) Simplify

$$\frac{x^3 - 8}{3x^2 - 8x + 4}. \quad (5)$$

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3. Differentiate each of the following with respect to  $x$  and simplify your answers.

(a)  $\cot x^2$  (2)

(b)  $x^2 e^{-x}$  (3)

(c)  $\frac{\sin x}{3 + 2 \cos x}$  (4)

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4. (a) Find, as natural logarithms, the solutions of the equation

$$e^{2x} - 8e^x + 15 = 0. \quad (4)$$

- (b) Use proof by contradiction to prove that  $\log_2 3$  is irrational. (6)
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5. The function  $f$  is defined by

$$f : x \rightarrow 3e^{x-1}, \quad x \in \mathbb{R}.$$

(a) State the range of  $f$ . (1)

(b) Find an expression for  $f^{-1}(x)$  and state its domain. (4)

The function  $g$  is defined by

$$g : x \rightarrow 5x - 2, \quad x \in \mathbb{R}.$$

Find, in terms of  $e$ ,

(c) the value of  $gf(\ln 2)$ , (3)

(d) the solution of the equation

$$f^{-1}g(x) = 4. \quad (4)$$

6.  $f(x) = 2x^2 + 3 \ln(2 - x), \quad x \in \mathbb{R}, \quad x < 2.$

(a) Show that the equation  $f(x) = 0$  can be written in the form

$$x = 2 - e^{kx^2},$$

where  $k$  is a constant to be found. (3)

The root,  $\alpha$ , of the equation  $f(x) = 0$  is 1.9 correct to 1 decimal place.

(b) Use the iteration formula

$$x_{n+1} = 2 - e^{kx_n^2},$$

with  $x_0 = 1.9$  and your value of  $k$ , to find  $\alpha$  to 3 decimal places and justify the accuracy of your answer. (5)

(c) Solve the equation  $f'(x) = 0$ . (5)

**Turn over**

7.

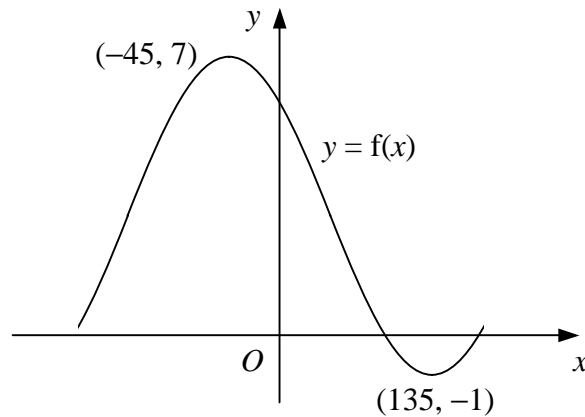
**Figure 1**

Figure 1 shows the curve  $y = f(x)$  which has a maximum point at  $(-45, 7)$  and a minimum point at  $(135, -1)$ .

(a) Showing the coordinates of any stationary points, sketch on separate diagrams the graphs of

(i)  $y = f(|x|)$ ,

(ii)  $y = 1 + 2f(x)$ .

**(6)**

Given that

$$f(x) = A + 2\sqrt{2} \cos x^\circ - 2\sqrt{2} \sin x^\circ, \quad x \in \mathbb{R}, \quad -180 \leq x \leq 180,$$

where  $A$  is a constant,

(b) show that  $f(x)$  can be expressed in the form

$$f(x) = A + R \cos(x + \alpha)^\circ,$$

where  $R > 0$  and  $0 < \alpha < 90$ ,

**(3)**

(c) state the value of  $A$ ,

**(1)**

(d) find, to 1 decimal place, the  $x$ -coordinates of the points where the curve  $y = f(x)$  crosses the  $x$ -axis.

**(4)****END**