

GCE Examinations
Advanced Subsidiary

Core Mathematics C3

Paper K

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks could be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.



Written by Shaun Armstrong

© *Solomon Press*

These sheets may be copied for use solely by the purchaser's institute.

C3 Paper K – Marking Guide

1. (a) $\arctan(x-2) = -\frac{\pi}{3}$ M1
 $x-2 = \tan(-\frac{\pi}{3}) = -\sqrt{3}$ M1
 $x = 2 - \sqrt{3}$ A1
- (b) $1 - 2\sin^2\theta - \sin\theta - 1 = 0$ M1
 $2\sin^2\theta + \sin\theta = 0$
 $\sin\theta(2\sin\theta + 1) = 0$ M1
 $\sin\theta = 0$ or $-\frac{1}{2}$ A1
 $\theta = 0$ or $-\frac{\pi}{6}, -\pi + \frac{\pi}{6}$
 $\theta = -\frac{5\pi}{6}, -\frac{\pi}{6}, 0$ A2 (8)
-

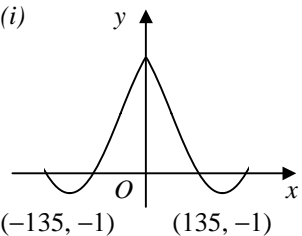
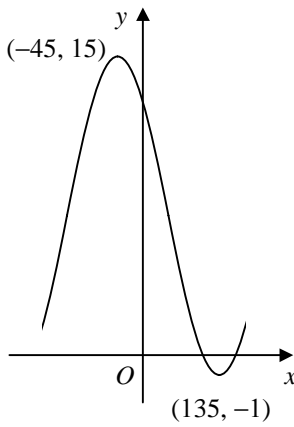
2. (a) $= \frac{4x}{(x+3)(x-3)} - \frac{2}{x+3}$ M1
 $= \frac{4x-2(x-3)}{(x+3)(x-3)}$ M1
 $= \frac{2x+6}{(x+3)(x-3)} = \frac{2(x+3)}{(x+3)(x-3)}$ M1
 $= \frac{2}{x-3}$ A1
- (b) $2^3 - 8 = 0 \therefore (x-2)$ is a factor of $(x^3 - 8)$ B1
- $$\begin{array}{r}
 x^2 + 2x + 4 \\
 x-2 \overline{) x^3 + 0x^2 + 0x - 8} \\
 \underline{x^3 - 2x^2} \\
 2x^2 + 0x \\
 \underline{2x^2 - 4x} \\
 4x - 8 \\
 \underline{4x - 8} \\
 0
 \end{array}$$
- $\therefore x^3 - 8 = (x-2)(x^2 + 2x + 4)$ M1 A1
 $\therefore \frac{x^3 - 8}{3x^2 - 8x + 4} = \frac{(x-2)(x^2 + 2x + 4)}{(3x-2)(x-2)} = \frac{x^2 + 2x + 4}{3x-2}$ M1 A1 (9)
-

3. (a) $= -\operatorname{cosec}^2 x^2 \times 2x = -2x \operatorname{cosec}^2 x^2$ M1 A1
- (b) $= 2x \times e^{-x} + x^2 \times (-e^{-x}) = xe^{-x}(2-x)$ M1 A2
- (c) $= \frac{\cos x \times (3+2\cos x) - \sin x \times (-2\sin x)}{(3+2\cos x)^2}$ M1 A1
 $= \frac{3\cos x + 2\cos^2 x + 2\sin^2 x}{(3+2\cos x)^2} = \frac{3\cos x + 2}{(3+2\cos x)^2}$ M1 A1 (9)
-

4. (a) $(e^x - 3)(e^x - 5) = 0$ M1 A1
 $e^x = 3, 5$
 $x = \ln 3, \ln 5$ M1 A1
- (b) assume $\log_2 3$ is rational B1
 $\therefore \log_2 3 = \frac{p}{q}$ where p and q are integers and $q \neq 0$ M1
 $\Rightarrow 2^{\frac{p}{q}} = 3$ M1
 $\Rightarrow 2^p = 3^q$ A1
2 and 3 are co-prime \therefore only solution is $p = q = 0$ M1
but $q \neq 0 \therefore$ contradiction $\therefore \log_2 3$ is irrational A1 (10)
-

5. (a) $f(x) > 0$ B1
 (b) $y = 3e^{x-1}, \quad x - 1 = \ln \frac{y}{3}$ M1
 $x = 1 + \ln \frac{y}{3}$
 $f^{-1}(x) = 1 + \ln \frac{x}{3}, \quad x \in \mathbb{R}, \quad x > 0$ M1 A2
 (c) $f(\ln 2) = 3e^{\ln 2 - 1} = 3e^{-1}e^{\ln 2} = 6e^{-1}$ M1 A1
 $gf(\ln 2) = g(6e^{-1}) = 30e^{-1} - 2$ A1
 (d) $f^{-1}g(x) = f^{-1}(5x - 2) = 1 + \ln \frac{5x-2}{3}$ M1 A1
 $\therefore 1 + \ln \frac{5x-2}{3} = 4, \quad \frac{5x-2}{3} = e^3$ M1
 $x = \frac{1}{5}(3e^3 + 2)$ A1 (12)

6. (a) $2x^2 + 3 \ln(2-x) = 0 \Rightarrow 3 \ln(2-x) = -2x^2$
 $\ln(2-x) = -\frac{2}{3}x^2$ M1
 $2-x = e^{-\frac{2}{3}x^2}$ M1
 $x = 2 - e^{-\frac{2}{3}x^2} \quad [k = -\frac{2}{3}]$ A1
 (b) $x_1 = 1.90988, x_2 = 1.91212, x_3 = 1.91262, x_4 = 1.91273$ M1 A1
 $\therefore \alpha = 1.913$ (3dp) A1
 $f(1.9125) = 0.0070, f(1.9135) = -0.020$ M1
 sign change, $f(x)$ continuous \therefore root A1
 (c) $f'(x) = 4x + \frac{3}{2-x} \times (-1) = 4x - \frac{3}{2-x}$ M1 A1
 $\therefore 4x - \frac{3}{2-x} = 0, \quad 4x = \frac{3}{2-x}, \quad 4x(2-x) = 3$ M1
 $4x^2 - 8x + 3 = 0, \quad (2x-3)(2x-1) = 0$ M1
 $x = \frac{1}{2}, \frac{3}{2}$ A1 (13)

7. (a) (i)  (ii)  B3
 M1 A2
 (b) $2\sqrt{2} \cos x - 2\sqrt{2} \sin x = R \cos x \cos \alpha - R \sin x \sin \alpha$
 $R \cos \alpha = 2\sqrt{2}, \quad R \sin \alpha = 2\sqrt{2}, \quad \therefore R = \sqrt{8+8} = 4$ M1 A1
 $\tan \alpha = 1, \quad \alpha = 45$ B1
 $\therefore f(x) = A + 4 \cos(x + 45)^\circ$
 (c) 3 B1
 (d) $3 + 4 \cos(x + 45) = 0, \quad \cos(x + 45) = -\frac{3}{4}$ M1
 $x + 45 = 180 - 41.4, 180 + 41.4 = 138.6, 221.4$ M1
 $x = 93.6, 176.4$ (1dp) A2 (14)

Total (75)

