

GCE Examinations
Advanced Subsidiary

Core Mathematics C3

Paper J

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks could be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.



Written by Shaun Armstrong

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C3 Paper J – Marking Guide

1. (a) $\cos^2 x = (\sqrt{3} - 1)^2 = 3 - 2\sqrt{3} + 1 = 4 - 2\sqrt{3}$ M1
 $\cos 2x = 2 \cos^2 x - 1 = 2(4 - 2\sqrt{3}) - 1 = 7 - 4\sqrt{3}$ M1 A1
- (b) $2(\cos y \cos 30 - \sin y \sin 30) = \sqrt{3}(\sin y \cos 30 - \cos y \sin 30)$ M1 A1
 $\sqrt{3} \cos y - \sin y = \frac{3}{2} \sin y - \frac{1}{2} \sqrt{3} \cos y$ B1
 $\frac{3}{2} \sqrt{3} \cos y = \frac{5}{2} \sin y$
 $\tan y = \frac{3}{2} \sqrt{3} \div \frac{5}{2} = \frac{3}{5} \sqrt{3}$ M1 A1 (8)
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2. (a) $f(x) = (x - \frac{3}{2})^2 - \frac{9}{4} + 7 = (x - \frac{3}{2})^2 + \frac{19}{4}$ M1 A1
 $\therefore f(x) \geq \frac{19}{4}$ A1
- (b) $= g(11) = 21$ M1 A1
- (c) $fg(x) = f(2x - 1) = (2x - 1)^2 - 3(2x - 1) + 7$ M1
 $\therefore 4x^2 - 4x + 1 - 6x + 3 + 7 = 17$
 $2x^2 - 5x - 3 = 0$ A1
 $(2x + 1)(x - 3) = 0$ M1
 $x = -\frac{1}{2}, 3$ A1 (9)
-
3. (a)
$$\begin{array}{r} x^2 + 4x - 4 \\ x^2 - 3x + 3 \overline{) x^4 + x^3 - 13x^2 + 26x - 17} \\ \underline{x^4 - 3x^3 + 3x^2} \\ 4x^3 - 16x^2 + 26x \\ \underline{4x^3 - 12x^2 + 12x} \\ -4x^2 + 14x - 17 \\ \underline{-4x^2 + 12x - 12} \\ 2x - 5 \end{array}$$
 M1
- $\therefore f(x) = x^2 + 4x - 4 + \frac{2x-5}{x^2-3x+3}, A = 4, B = -4, C = 2, D = -5$ A3
- (b) $f'(x) = 2x + 4 + \frac{2 \times (x^2 - 3x + 3) - (2x - 5) \times (2x - 3)}{(x^2 - 3x + 3)^2}$ M1 A2
 $x = 1 \Rightarrow y = -2, \text{ grad} = 5$
 $\therefore \text{grad of normal} = -\frac{1}{5}$ M1
 $\therefore y + 2 = -\frac{1}{5}(x - 1)$ M1
 $5y + 10 = -x + 1$
 $x + 5y + 9 = 0$ A1 (10)
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4. (a) $\frac{dx}{dy} = \frac{1}{2} \sec \frac{y}{2} \tan \frac{y}{2}$ M1
- $0 \leq y < \pi \therefore \tan \frac{y}{2} \geq 0 \therefore \frac{dx}{dy} = \frac{1}{2} \sec \frac{y}{2} \sqrt{\sec^2 \frac{y}{2} - 1} = \frac{1}{2} x \sqrt{x^2 - 1}$ M1 A1
- $\frac{dy}{dx} = 1 \div \frac{dx}{dy} = \frac{2}{x\sqrt{x^2-1}}$ M1 A1
- (b) $\frac{dy}{dx} = \frac{1}{2}(3 + 2 \cos x)^{-\frac{1}{2}} \times (-2 \sin x) = -\frac{\sin x}{\sqrt{3 + 2 \cos x}}$ M1 A1
 $x = \frac{\pi}{3}, y = 2, \text{ grad} = -\frac{1}{4} \sqrt{3}$ M1 A1
 $\therefore y - 2 = -\frac{1}{4} \sqrt{3} (x - \frac{\pi}{3}) \quad [3\sqrt{3}x + 12y - 24 - \pi\sqrt{3} = 0]$ M1 A1 (11)
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5.	(a)	$f(x) > 5$	B1
	(b)	$y = 5 + e^{2x-3}$ $2x - 3 = \ln(y - 5)$ $x = \frac{1}{2}[3 + \ln(y - 5)]$ $\therefore f^{-1}(x) = \frac{1}{2}[3 + \ln(x - 5)], x \in \mathbb{R}, x > 5$	M1 M1 A2
	(c)	$x = f^{-1}(7) = \frac{1}{2}(3 + \ln 2)$	M1 A1
	(d)	$f'(x) = 2e^{2x-3}$ grad = 4 $\therefore y - 7 = 4[x - \frac{1}{2}(3 + \ln 2)]$	M1 A1 M1 A1
		[$y = 4x + 1 - 2 \ln 2$]	(11)

6.	(a)	LHS $\equiv \frac{2\cos 2x}{\sin 2x} + \frac{\sin x}{\cos x}$ $\equiv \frac{\cos 2x}{\sin x \cos x} + \frac{\sin x}{\cos x}$ $\equiv \frac{\cos 2x + \sin^2 x}{\sin x \cos x}$ $\equiv \frac{(\cos^2 x - \sin^2 x) + \sin^2 x}{\sin x \cos x}$ $\equiv \frac{\cos^2 x}{\sin x \cos x}$ $\equiv \frac{\cos x}{\sin x}$ $\equiv \cot x \equiv \text{RHS}$	M1 M1 A1 M1 A1
	(b)	$\cot x = \operatorname{cosec}^2 x - 7$ $\cot x = 1 + \cot^2 x - 7$ $\cot^2 x - \cot x - 6 = 0$ $(\cot x + 2)(\cot x - 3) = 0$ $\cot x = -2$ or 3 $\tan x = -\frac{1}{2}$ or $\frac{1}{3}$ $x = \pi - 0.4636$ or 0.32 $x = 0.32, 2.68$ (2dp)	M1 M1 M1 A1 M1 A2
			(11)

7.	(a)	$f(x) \geq 0$	B1
	(b)	$= f(0) = 5$	M1 A1
	(c)	$fg(x) = f[\ln(x + 3)] = 2 \ln(x + 3) - 5 $ $\therefore 2 \ln(x + 3) - 5 = 3$ $2 \ln(x + 3) = 2, 8$ $\ln(x + 3) = 1, 4$ $x = e - 3, e^4 - 3$	M1 M1 A1 M1 A1
	(d)	let $h(x) = f(x) - g(x)$ $h(3) = -0.79, f(4) = 1.1$ sign change, $h(x)$ continuous \therefore root	M1 A1
	(e)	$x_1 = 3.396, x_2 = 3.428, x_3 = 3.430, x_4 = 3.431$	M1 A2
	(f)	$h(3.4305) = -0.000052, f(3.4315) = 0.0018$ sign change, $h(x)$ continuous \therefore root $\therefore \alpha = x_4$ to 4sf	M1 A1
			(15)

Total (75)

