FOR EDEXCEL

GCE Examinations
Advanced Subsidiary

Core Mathematics C3

Paper I
Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and / or integration.

Full marks may be obtained for answers to ALL questions.

Mathematical formulae and statistical tables are available.

This paper has eight questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working may gain no credit.

Written by Shaun Armstrong

1. Express

$$\frac{2x}{2x^2+3x-5} \div \frac{x^3}{x^2-x}$$

as a single fraction in its simplest form. **(5)**

2.

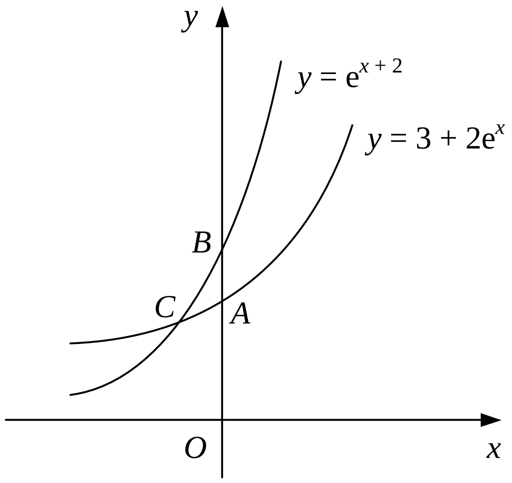

Figure 1

Figure 1 shows the curves $y = 3 + 2e^x$ and $y = e^{x+2}$ which cross the y-axis at the points A and B respectively.

(a) Find the exact length AB. **(3)**

The two curves intersect at the point C.

(b) Find an expression for the x-coordinate of C and show that the y-coordinate of C is $\frac{3e^2}{e^2-2}$. **(5)**

3.

$$f(x) = \frac{x^2+3}{4x+1}, \quad x \in \mathbb{R}, \quad x \neq -\tfrac{1}{4}.$$

(a) Find and simplify an expression for $f'(x)$. **(3)**

(b) Find the set of values of x for which $f(x)$ is increasing. **(5)**

---

4. The curve C has the equation $y = x^2 - 5x + 2\ln\frac{x}{3}$, $x > 0$.

(a) Show that the normal to C at the point where $x = 3$ has the equation

$$3x + 5y + 21 = 0.$$ **(5)**

(b) Find the x-coordinates of the stationary points of C. **(3)**

---

5. The functions f and g are defined by

$$\text{f}(x) \equiv 6x - 1, \quad x \in \mathbb{R},$$

$$\text{g}(x) \equiv \log_2 (3x + 1), \quad x \in \mathbb{R}, \quad x > -\tfrac{1}{3}.$$

(a) Evaluate gf(1). **(2)**

(b) Find an expression for $\text{g}^{-1}(x)$. **(3)**

(c) Find, in terms of natural logarithms, the solution of the equation

$$\text{fg}^{-1}(x) = 2.$$ **(4)**

---

6. *(a)* Use the identities for $\cos (A + B)$ and $\cos (A - B)$ to prove that

$$\cos P - \cos Q \equiv -2\sin\frac{P+Q}{2}\sin\frac{P-Q}{2}.$$ **(4)**

(b) Hence find all solutions in the interval $0 \le x < 180$ to the equation

$$\cos 5x° + \sin 3x° - \cos x° = 0.$$ **(7)**

Turn over

7. The function f is defined by

$$f(x) \equiv x^2 - 2ax, \quad x \in \mathbb{R},$$

where a is a positive constant.

(a) Showing the coordinates of any points where each graph meets the axes, sketch on separate diagrams the graphs of

(i) $y = |f(x)|$,

(ii) $y = f(|x|)$. **(6)**

The function g is defined by

$$g(x) \equiv 3ax, \quad x \in \mathbb{R}.$$

(b) Find $fg(a)$ in terms of a. **(2)**

(c) Solve the equation

$$gf(x) = 9a^3.$$ **(4)**

---

8. $$f(x) = 2x + \sin x - 3\cos x.$$

(a) Show that the equation $f(x) = 0$ has a root in the interval $[0.7, 0.8]$. **(2)**

(b) Find an equation for the tangent to the curve $y = f(x)$ at the point where it crosses the y-axis. **(4)**

(c) Find the values of the constants a, b and c, where $b > 0$ and $0 < c < \frac{\pi}{2}$, such that

$$f'(x) = a + b\cos(x - c).$$ **(4)**

(d) Hence find the x-coordinates of the stationary points of the curve $y = f(x)$ in the interval $0 \le x \le 2\pi$, giving your answers to 2 decimal places. **(4)**

END