

GCE Examinations  
Advanced Subsidiary

# Core Mathematics C3

Paper I

## MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks could be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.



*Written by Shaun Armstrong*

© *Solomon Press*

*These sheets may be copied for use solely by the purchaser's institute.*

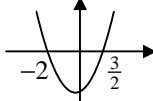
## C3 Paper I – Marking Guide

$$\begin{aligned}
 1. \quad &= \frac{2x}{2x^2+3x-5} \times \frac{x^2-x}{x^3} && \text{M1} \\
 &= \frac{2x}{(2x+5)(x-1)} \times \frac{x(x-1)}{x^3} && \text{M1 A1} \\
 &= \frac{2}{x(2x+5)} && \text{M1 A1} \quad (5)
 \end{aligned}$$


---

$$\begin{aligned}
 2. \quad (a) \quad &A(0, 5), B(0, e^2) && \text{B2} \\
 &\therefore AB = e^2 - 5 && \text{B1} \\
 (b) \quad &3 + 2e^x = e^{x+2} = e^2 e^x && \text{M1} \\
 &3 = e^x(e^2 - 2) && \\
 &e^x = \frac{3}{e^2 - 2}, \quad x = \ln \frac{3}{e^2 - 2} && \text{M1 A1} \\
 &\therefore y = e^2 e^x = e^2 \times \frac{3}{e^2 - 2} = \frac{3e^2}{e^2 - 2} && \text{M1 A1} \quad (8)
 \end{aligned}$$


---

$$\begin{aligned}
 3. \quad (a) \quad &f'(x) = \frac{2x \times (4x+1) - (x^2+3) \times 4}{(4x+1)^2} = \frac{4x^2+2x-12}{(4x+1)^2} && \text{M1 A2} \\
 (b) \quad &\frac{4x^2+2x-12}{(4x+1)^2} \geq 0 && \text{M1} \\
 &\text{for } x \neq -\frac{1}{4}, (4x+1)^2 > 0 \quad \therefore 4x^2+2x-12 \geq 0 && \text{M1 A1} \\
 &2(2x-3)(x+2) \geq 0 && \text{M1} \\
 &x \leq -2 \text{ or } x \geq \frac{3}{2} && \text{A1} \quad (8)
 \end{aligned}$$



---

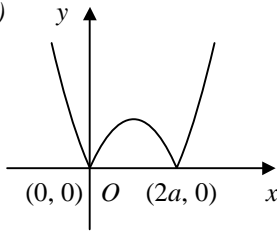
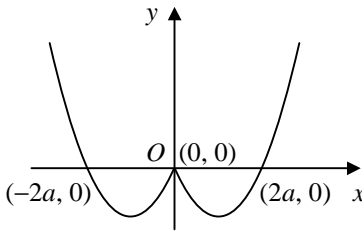
$$\begin{aligned}
 4. \quad (a) \quad &\frac{dy}{dx} = 2x - 5 + \frac{2}{x} && \text{M1} \\
 &x = 3, y = -6, \text{ grad} = \frac{5}{3} && \text{A1} \\
 &\text{grad of normal} = -\frac{3}{5} && \text{M1} \\
 &\therefore y + 6 = -\frac{3}{5}(x - 3) && \text{M1} \\
 &5y + 30 = -3x + 9 && \\
 &3x + 5y + 21 = 0 && \text{A1} \\
 (b) \quad \text{SP:} \quad &2x - 5 + \frac{2}{x} = 0 && \\
 &2x^2 - 5x + 2 = 0 && \text{M1} \\
 &(2x - 1)(x - 2) = 0 && \text{M1} \\
 &x = \frac{1}{2}, 2 && \text{A1} \quad (8)
 \end{aligned}$$


---

$$\begin{aligned}
 5. \quad (a) \quad &= g(5) = \log_2 16 = 4 && \text{M1 A1} \\
 (b) \quad &y = \log_2(3x + 1) && \\
 &3x + 1 = 2^y && \text{M1} \\
 &x = \frac{1}{3}(2^y - 1) && \\
 &g^{-1}(x) = \frac{1}{3}(2^x - 1) && \text{M1 A1} \\
 (c) \quad &fg^{-1}(x) = f\left[\frac{1}{3}(2^x - 1)\right] = 2(2^x - 1) - 1 = 2(2^x) - 3 && \text{M1} \\
 &\therefore 2(2^x) - 3 = 2 && \\
 &2^x = \frac{5}{2} && \text{A1} \\
 &x = \frac{\ln \frac{5}{2}}{\ln 2} \text{ or } \frac{\ln 5 - \ln 2}{\ln 2} && \text{M1 A1} \quad (9)
 \end{aligned}$$


---

6. (a)  $\cos(A+B) \equiv \cos A \cos B - \sin A \sin B$   
 $\cos(A-B) \equiv \cos A \cos B + \sin A \sin B$   
 subtracting,  $\cos(A+B) - \cos(A-B) \equiv -2 \sin A \sin B$  M1 A1  
 let  $P = A+B$ ,  $Q = A-B$   
 adding,  $P+Q = 2A \Rightarrow A = \frac{P+Q}{2}$  M1  
 subtracting,  $P-Q = 2B \Rightarrow B = \frac{P-Q}{2}$   
 $\therefore \cos P - \cos Q \equiv -2 \sin \frac{P+Q}{2} \sin \frac{P-Q}{2}$  A1
- (b)  $(\cos 5x - \cos x) + \sin 3x = 0$   
 $-2 \sin 3x \sin 2x + \sin 3x = 0$  M1  
 $\sin 3x(1 - 2 \sin 2x) = 0$  M1  
 $\sin 3x = 0$  or  $\sin 2x = \frac{1}{2}$  A1  
 $3x = 0, 180, 360$  or  $2x = 30, 150$  B1  
 $x = 0, 15, 60, 75, 120$  M1 A2 (11)

7. (a) (i)  (ii) 
- (b)  $= f(3a^2) = 9a^4 - 6a^3$  M1 A1
- (c)  $gf(x) = 3a(x^2 - 2ax)$  M1  
 $\therefore 3a(x^2 - 2ax) = 9a^3$   
 $x^2 - 2ax - 3a^2 = 0$  A1  
 $(x+a)(x-3a) = 0$  M1  
 $x = -a, 3a$  A1 (12)

8. (a)  $f(0.7) = -0.25$ ,  $f(0.8) = 0.23$  M1  
 sign change,  $f(x)$  continuous  $\therefore$  root A1
- (b)  $f'(x) = 2 + \cos x + 3 \sin x$  M1  
 $x = 0$ ,  $y = -3$ ,  $\text{grad} = 3$  A1  
 $\therefore y = 3x - 3$  M1 A1
- (c)  $\cos x + 3 \sin x = b \cos x \cos c + b \sin x \sin c$   
 $b \cos c = 1$ ,  $b \sin c = 3$   
 $\therefore b = \sqrt{1^2 + 3^2} = \sqrt{10}$  M1  
 $\tan c = 3$ ,  $c = 1.25$  (3sf) M1  
 $\therefore a = 2$ ,  $b = \sqrt{10}$ ,  $c = 1.25$  A2
- (d) SP:  $2 + \sqrt{10} \cos(x - 1.249) = 0$   
 $\cos(x - 1.249) = -\frac{2}{\sqrt{10}}$  M1  
 $x - 1.249 = \pi - 0.8861$ ,  $\pi + 0.8861 = 2.256, 4.028$  M1  
 $x = 3.50, 5.28$  (2dp) A2 (14)

Total (75)

