FOR EDEXCEL

GCE Examinations

Advanced Subsidiary

Core Mathematics C3

Paper H

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and / or integration.

Full marks may be obtained for answers to ALL questions.

Mathematical formulae and statistical tables are available.

This paper has eight questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working may gain no credit.

Written by Shaun Armstrong

1. The functions f and g are defined by

$$f : x \to 2 - x^2, \quad x \in \mathbb{R},$$

$$g : x \to \frac{3x}{2x-1}, \quad x \in \mathbb{R}, \quad x \neq \tfrac{1}{2}.$$

(a) Evaluate fg(2). **(2)**

(b) Solve the equation gf(x) = $\tfrac{1}{2}$. **(4)**

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2. Giving your answers to 1 decimal place, solve the equation

$$5\tan^2 2\theta - 13\sec 2\theta = 1,$$

for θ in the interval $0 \leq \theta \leq 360°$. **(7)**

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3. *(a)* Simplify

$$\frac{2x^2+3x-9}{2x^2-7x+6}.$$ **(3)**

(b) Solve the equation

$$\ln(2x^2 + 3x - 9) = 2 + \ln(2x^2 - 7x + 6),$$

giving your answer in terms of e. **(4)**

4.

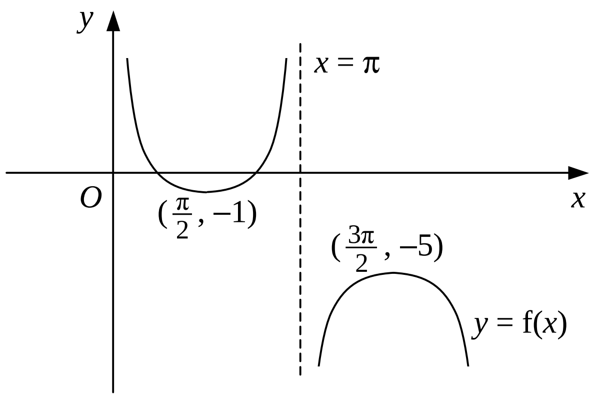

Figure 1

Figure 1 shows the graph of $y = \text{f}(x)$. The graph has a minimum at $(\frac{\pi}{2}, -1)$, a maximum at $(\frac{3\pi}{2}, -5)$ and an asymptote with equation $x = \pi$.

(a) Showing the coordinates of any stationary points, sketch the graph of $y = |\text{f}(x)|$. **(3)**

Given that

$$\text{f} : x \rightarrow a + b\,\text{cosec}\,x, \quad x \in \mathbb{R}, \quad 0 < x < 2\pi, \quad x \neq \pi,$$

(b) find the values of the constants a and b, **(3)**

(c) find, to 2 decimal places, the x-coordinates of the points where the graph of $y = \text{f}(x)$ crosses the x-axis. **(3)**

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5. The number of bacteria present in a culture at time t hours is modelled by the continuous variable N and the relationship

$$N = 2000\text{e}^{kt},$$

where k is a constant.

Given that when $t = 3$, $N = 18\,000$, find

(a) the value of k to 3 significant figures, **(3)**

(b) how long it takes for the number of bacteria present to double, giving your answer to the nearest minute, **(4)**

(c) the rate at which the number of bacteria is increasing when $t = 3$. **(3)**

Turn over

6. *(a)* Use the derivative of $\cos x$ to prove that

$$\frac{\mathrm{d}}{\mathrm{d}x}(\sec x) = \sec x \tan x.$$ **(4)**

The curve C has the equation $y = \mathrm{e}^{2x} \sec x$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

(b) Find an equation for the tangent to C at the point where it crosses the y-axis. **(4)**

(c) Find, to 2 decimal places, the x-coordinate of the stationary point of C. **(3)**

7.

$$\mathrm{f}(x) = x^2 - 2x + 5, \quad x \in \mathbb{R}, \quad x \geq 1.$$

(a) Express $\mathrm{f}(x)$ in the form $(x + a)^2 + b$, where a and b are constants. **(2)**

(b) State the range of f. **(1)**

(c) Find an expression for $\mathrm{f}^{-1}(x)$. **(3)**

(d) Describe fully two transformations that would map the graph of $y = \mathrm{f}^{-1}(x)$ onto the graph of $y = \sqrt{x}$, $x \geq 0$. **(2)**

(e) Find an equation for the normal to the curve $y = \mathrm{f}^{-1}(x)$ at the point where $x = 8$. **(4)**

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8. A curve has the equation $y = \frac{\mathrm{e}^2}{x} + \mathrm{e}^x$, $x \neq 0$.

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. **(2)**

(b) Show that the curve has a stationary point in the interval $[1.3, 1.4]$. **(3)**

The point A on the curve has x-coordinate 2.

(c) Show that the tangent to the curve at A passes through the origin. **(4)**

The tangent to the curve at A intersects the curve again at the point B.

The x-coordinate of B is to be estimated using the iterative formula

$$x_{n+1} = -\tfrac{2}{3}\sqrt{3 + 3x_n\mathrm{e}^{x_n - 2}},$$

with $x_0 = -1$.

(d) Find x_1, x_2 and x_3 to 7 significant figures and hence state the x-coordinate of B to 5 significant figures. **(4)**

END