

GCE Examinations
Advanced Subsidiary

Core Mathematics C4

Paper H

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks could be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.



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C4 Paper H – Marking Guide

1. (a) $= 1 + \left(\frac{3}{2}\right)(4x) + \frac{\left(\frac{3}{2}\right)\left(\frac{1}{2}\right)}{2}(4x)^2 + \frac{\left(\frac{3}{2}\right)\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{3 \times 2}(4x)^3 + \dots$ M1
 $= 1 + 6x + 6x^2 - 4x^3 + \dots$ A3
- (b) $|x| < \frac{1}{4}$ B1 (5)
-
2. $u = 1 + \sin x \Rightarrow \frac{du}{dx} = \cos x$ M1
 $x = 0 \Rightarrow u = 1, x = \frac{\pi}{2} \Rightarrow u = 2$ B1
- $I = \int_1^2 u^3 du$ A1
 $= \left[\frac{1}{4}u^4\right]_1^2$ M1
 $= 4 - \frac{1}{4} = \frac{15}{4}$ M1 A1 (6)
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3. (a) $\frac{x+11}{(x+4)(x-3)} \equiv \frac{A}{x+4} + \frac{B}{x-3}$ M1
 $x+11 \equiv A(x-3) + B(x+4)$ A1
 $x = -4 \Rightarrow 7 = -7A \Rightarrow A = -1$ A1
 $x = 3 \Rightarrow 14 = 7B \Rightarrow B = 2$ A1
- $\frac{x+11}{(x+4)(x-3)} \equiv \frac{2}{x-3} - \frac{1}{x+4}$
- (b) $= \int_0^2 \left(\frac{2}{x-3} - \frac{1}{x+4}\right) dx$ M1 A1
 $= [2 \ln|x-3| - \ln|x+4|]_0^2$ M1
 $= (0 - \ln 6) - (2 \ln 3 - \ln 4)$ M1 A1
 $= \ln \frac{2}{27}$ (8)
-
4. $= \pi \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (2 \sin x + \operatorname{cosec} x)^2 dx$ M1
 $= \pi \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (4 \sin^2 x + 4 + \operatorname{cosec}^2 x) dx$ A1
 $= \pi \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (2 - 2 \cos 2x + 4 + \operatorname{cosec}^2 x) dx$ M1
 $= \pi [6x - \sin 2x - \cot x]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$ M1 A2
 $= \pi \left\{ (3\pi + 0 + 0) - \left(\pi - \frac{\sqrt{3}}{2} - \sqrt{3} \right) \right\}$ M1
 $= \pi \left(2\pi + \frac{3}{2}\sqrt{3} \right) = \frac{1}{2} \pi (4\pi + 3\sqrt{3})$ A1 (8)
-
5. (a) $2x - 3y - 3x \frac{dy}{dx} - 2y \frac{dy}{dx} = 0$ M1 A2
 $\frac{dy}{dx} = \frac{2x-3y}{3x+2y}$ M1 A1
- (b) $\text{grad} = 5$ M1
 $\therefore y + 2 = 5(x - 2) \quad [y = 5x - 12]$ M1 A1 (8)
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6. (a) $= \frac{|1 \times 6 + 5 \times 3 + (-1) \times (-6)|}{\sqrt{1 + 25 + 1} \times \sqrt{36 + 9 + 36}}$ M1 A1
 $= \frac{27}{\sqrt{27} \times \sqrt{81}} = \frac{\sqrt{27}}{9} = \frac{3\sqrt{3}}{9} = \frac{1}{3}\sqrt{3}$ M1 A1
- (b) $\sin(\angle AOB) = \sqrt{1 - (\frac{1}{3}\sqrt{3})^2} = \sqrt{\frac{2}{3}}$ M1 A1
 $\text{area} = \frac{1}{2} \times 3\sqrt{3} \times 9 \times \sqrt{\frac{2}{3}} = \frac{27}{2}\sqrt{2}$ M1 A1
- (c) $= OA \times \sin(\angle AOB) = 3\sqrt{3} \times \sqrt{\frac{2}{3}} = 3\sqrt{2}$ M1 A1 (10)

7. (a) $\frac{dx}{dt} = 2t - 1, \quad \frac{dy}{dt} = \frac{4 \times (1-t) - 4t \times (-1)}{(1-t)^2} = \frac{4}{(1-t)^2}$ B1 M1
 $\frac{dy}{dx} = \frac{4}{(2t-1)(1-t)^2}$ M1 A1
- (b) $t = -1, x = 2, y = -2, \text{grad} = -\frac{1}{3}$ M1
 $\therefore y + 2 = -\frac{1}{3}(x - 2)$ M1
 $3y + 6 = -x + 2$
 $x + 3y + 4 = 0$ A1
- (c) $t(t-1) + 3 \times \frac{4t}{1-t} + 4 = 0$ M1
 $-t(t-1)^2 + 12t + 4(1-t) = 0$
 $t^3 - 2t^2 - 7t - 4 = 0$ A1
 $t = -1$ is a solution $\therefore (t+1)$ is a factor M1
 $(t+1)(t^2 - 3t - 4) = 0$ M1
 $(t+1)(t+1)(t-4) = 0$ A1
 $t = -1$ (at P) or $t = 4 \therefore Q(12, -\frac{16}{3})$ M1 A1 (14)

8. (a) $\int \frac{1}{P} dP = \int k dt$ M1
 $\ln|P| = kt + c$ A1
 $t = 0, P = 300 \Rightarrow \ln 300 = c$ M1
 $\ln|P| = kt + \ln 300$
 $\ln \left| \frac{P}{300} \right| = kt, \quad \frac{P}{300} = e^{kt}, \quad P = 300e^{kt}$ M1 A1
- (b) $t = 1, P = 360 \Rightarrow 360 = 300e^k$ M1
 $k = \ln \frac{6}{5} = 0.182$ (3sf) A1
- (c) $P = 300e^{0.1823t}$
 when $t = 2, P = 432$; when $t = 3, P = 518$ B1
 model does not seem suitable as data diverges from predictions B1
- (d) $\int \frac{1}{P} dP = \int (0.4 - 0.25 \cos 0.5t) dt$ M1
 $\ln|P| = 0.4t - 0.5 \sin 0.5t + c$ A1
 $t = 0, P = 300 \Rightarrow \ln 300 = c$
 $\ln \left| \frac{P}{300} \right| = 0.4t - 0.5 \sin 0.5t \quad [P = 300e^{0.4t - 0.5 \sin 0.5t}]$ M1 A1
- (e) second model: $t = 1, 2, 3 \Rightarrow P = 352, 438, 605$ M1 A1
 the second model seems more suitable as it fits the data better B1 (16)

Total (75)

