FOR EDEXCEL

GCE Examinations
Advanced Subsidiary

Core Mathematics C3

Paper H

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks could be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

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C3 Paper H – Marking Guide

1. *(a)* $= \mathrm{f}(2) = -2$ — M1 A1

(b) $\mathrm{gf}(x) = \mathrm{g}(2 - x^2) = \frac{3(2-x^2)}{2(2-x^2)-1} = \frac{6-3x^2}{3-2x^2}$ — M1 A1

$\therefore\ \frac{6-3x^2}{3-2x^2} = \frac{1}{2},\quad 2(6 - 3x^2) = 3 - 2x^2$

$x^2 = \frac{9}{4},\quad x = \pm\frac{3}{2}$ — M1 A1 **(6)**

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2. $5(\sec^2 2\theta - 1) - 13\sec 2\theta = 1$ — M1

$5\sec^2 2\theta - 13\sec 2\theta - 6 = 0$

$(5\sec 2\theta + 2)(\sec 2\theta - 3) = 0$ — M1

$\sec 2\theta = -\frac{2}{5}$ or 3 — A1

$\cos 2\theta = -\frac{5}{2}$ (no solutions) or $\frac{1}{3}$

$2\theta = 70.529,\ 360 - 70.529,\ 360 + 70.529,\ 720 - 70.529$ — B1 M1

$= 70.529,\ 289.471,\ 430.529,\ 649.471$

$\theta = 35.3°,\ 144.7°,\ 215.3°,\ 324.7°$ (1dp) — A2 **(7)**

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3. *(a)* $= \frac{(2x-3)(x+3)}{(2x-3)(x-2)} = \frac{x+3}{x-2}$ — M1 A2

(b) $\ln(2x^2 + 3x - 9) - \ln(2x^2 - 7x + 6) = 2,\quad \ln\frac{2x^2+3x-9}{2x^2-7x+6} = 2$ — M1

$\ln\frac{x+3}{x-2} = 2,\quad \frac{x+3}{x-2} = \mathrm{e}^2$ — A1

$x + 3 = \mathrm{e}^2(x - 2)$

$3 + 2\mathrm{e}^2 = x(\mathrm{e}^2 - 1)$ — M1

$x = \frac{2\mathrm{e}^2+3}{\mathrm{e}^2-1}$ — A1 **(7)**

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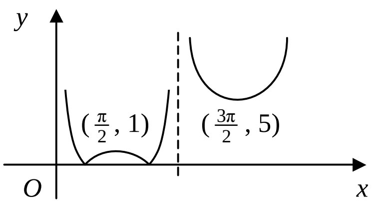
4. *(a)*



— B3

(b) $(\frac{\pi}{2}, -1) \Rightarrow -1 = a + b$

$(\frac{3\pi}{2}, -5) \Rightarrow -5 = a - b$ — B1

adding, $-6 = 2a\ \therefore\ a = -3,\ b = 2$ — M1 A1

(c) $-3 + 2\operatorname{cosec} x = 0,\quad \operatorname{cosec} x = \frac{3}{2},\quad \sin x = \frac{2}{3}$ — M1

$x = 0.73,\ \pi - 0.7297,\quad x = 0.73,\ 2.41$ (2dp) — A2 **(9)**

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5. *(a)* $t = 3,\ N = 18\,000 \Rightarrow 18\,000 = 2000\mathrm{e}^{3k},\quad \mathrm{e}^{3k} = 9$ — M1

$k = \frac{1}{3}\ln 9 = 0.732$ (3sf) — M1 A1

(b) $4000 = 2000\mathrm{e}^{0.7324t}$ — M1

$t = \frac{1}{0.7324}\ln 2 = 0.9464$ hours — M1 A1

$\therefore$ doubles in 57 minutes (nearest minute) — A1

(c) $N = 2000\mathrm{e}^{0.7324t},\quad \frac{\mathrm{d}N}{\mathrm{d}t} = 0.7324 \times 2000\mathrm{e}^{0.7324t} = 1465\mathrm{e}^{0.7324t}$ — M1 A1

when $t = 3$, $\frac{\mathrm{d}N}{\mathrm{d}t} = 13\,200$ $\therefore$ increasing at rate of 13 200 per hour (3sf) — A1 **(10)**

6.	*(a)*	$\frac{\text{d}}{\text{d}x}(\sec x) = \frac{\text{d}}{\text{d}x}[(\cos x)^{-1}]$		
		$= -(\cos x)^{-2} \times (-\sin x)$	M1 A1	
		$= \frac{\sin x}{\cos^2 x} = \frac{1}{\cos x} \times \frac{\sin x}{\cos x}$	M1	
		$= \sec x \tan x$	A1	
	(b)	$\frac{\text{d}y}{\text{d}x} = 2\text{e}^{2x} \times \sec x + \text{e}^{2x} \times \sec x \tan x = \text{e}^{2x} \sec x\,(2 + \tan x)$	M1 A1	
		$x = 0,\ y = 1,\ \text{grad} = 2$	M1	
		$\therefore\ y = 2x + 1$	A1	
	(c)	SP: $\text{e}^{2x} \sec x\,(2 + \tan x) = 0$	M1	
		$\tan x = -2$	M1	
		$x = -1.11$ (2dp)	A1	**(11)**
7.	*(a)*	$\text{f}(x) = (x-1)^2 - 1 + 5 = (x-1)^2 + 4$	M1 A1	
	(b)	$\text{f}(x) \geq 4$	B1	
	(c)	$y = (x-1)^2 + 4$		
		$(x-1)^2 = y - 4$		
		$x - 1 = \pm\sqrt{y-4}$	M1	
		$x = 1 \pm \sqrt{y-4}$		
		$\text{f}^{-1}(x) = 1 + \sqrt{x-4}$	M1 A1	
	(d)	translation by 4 units in negative x direction translation by 1 unit in negative y direction (either first)	B2	
	(e)	$\frac{\text{d}y}{\text{d}x} = \frac{1}{2}(x-4)^{-\frac{1}{2}}$	M1	
		$x = 8,\ y = 3,\ \text{grad} = \frac{1}{4}$	A1	
		$\therefore$ grad of normal $= -4$		
		$\therefore\ y - 3 = -4(x - 8)$ $[\,y = 35 - 4x\,]$	M1 A1	**(12)**
8.	*(a)*	$\frac{\text{d}y}{\text{d}x} = -\text{e}^2 x^{-2} + \text{e}^x$	M1 A1	
	(b)	SP: $-\text{e}^2 x^{-2} + \text{e}^x = 0$	M1	
		let $\text{f}(x) = -\text{e}^2 x^{-2} + \text{e}^x$		
		$\text{f}(1.3) = -0.70,\ \text{f}(1.4) = 0.29$	M1	
		sign change, f(x) continuous $\therefore$ root	A1	
	(c)	$x = 2,\ y = \frac{3}{2}\text{e}^2,\ \text{grad} = \frac{3}{4}\text{e}^2$	M1	
		$\therefore\ y - \frac{3}{2}\text{e}^2 = \frac{3}{4}\text{e}^2(x - 2)$	M1 A1	
		$y = \frac{3}{4}\text{e}^2 x$		
		$\therefore\ x = 0 \Rightarrow y = 0$ so passes through origin	A1	
	(d)	$x_1 = -1.125589,\ x_2 = -1.125803,\ x_3 = -1.125804$ (7sf)	M1 A2	
		$\therefore$ x-coordinate of $B = -1.1258$ (5sf)	A1	**(13)**
			Total	**(75)**

Performance Record – C3 Paper H

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	Total
Topic(s)	functions	trigonometry	rational expressions, exponentials and logarithms	functions, trigonometry	exponentials and logarithms, differentiation	differentiation	functions, differentiation	differentiation, numerical methods	
Marks	6	7	7	9	10	11	12	13	75
Student									