FOR EDEXCEL

GCE Examinations

Advanced Subsidiary

# Core Mathematics C4

Paper G

Time: 1 hour 30 minutes

## Instructions and Information

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and / or integration.

Full marks may be obtained for answers to ALL questions.

Mathematical formulae and statistical tables are available.

This paper has eight questions.

## Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working may gain no credit.

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**1.** A curve has the equation

$$x^2 + 2xy^2 + y = 4.$$

Find an expression for $\dfrac{dy}{dx}$ in terms of $x$ and $y$. **(6)**

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**2.** Use integration by parts to find

$$\int x^2\, e^{-x}\ dx.$$

**(7)**

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3.	The first four terms in the series expansion of $(1 + ax)^n$ in ascending powers of $x$ are

$$1 - 4x + 24x^2 + kx^3,$$

where $a$, $n$ and $k$ are constants and $|ax| < 1$.

(a)	Find the values of $a$ and $n$.	**(6)**

(b)	Show that $k = -160$.	**(2)**

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**3.** continued

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4.  (a)  Use the trapezium rule with two intervals of equal width to find an estimate
        for the value of the integral

        $$\int_0^3 e^{\cos x} \, dx,$$

        giving your answer to 3 significant figures. **(5)**

    (b)  Use the trapezium rule with four intervals of equal width to find another
        estimate for the value of the integral to 3 significant figures. **(2)**

    (c)  Given that the true value of the integral lies between the estimates made in
        parts *(a)* and *(b)*, comment on the shape of the curve $y = e^{\cos x}$ in the interval
        $0 \le x \le 3$ and explain your answer. **(2)**

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**4.** continued

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5.  A straight road passes through villages at the points $A$ and $B$ with position vectors $(9\mathbf{i} - 8\mathbf{j} + 2\mathbf{k})$ and $(4\mathbf{j} + \mathbf{k})$ respectively, relative to a fixed origin.

The road ends at a junction at the point $C$ with another straight road which lies along the line with equation

$$\mathbf{r} = (2\mathbf{i} + 16\mathbf{j} - \mathbf{k}) + \mu(-5\mathbf{i} + 3\mathbf{j}),$$

where $\mu$ is a scalar parameter.

*(a)*   Find the position vector of $C$.   **(5)**

Given that 1 unit on each coordinate axis represents 200 metres,

*(b)*   find the distance, in kilometres, from the village at $A$ to the junction at $C$.   **(4)**

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**5.** continued

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**6.**   A small town had a population of 9000 in the year 2001.

In a model, it is assumed that the population of the town, $P$, at time $t$ years after 2001 satisfies the differential equation

$$\frac{\mathrm{d}P}{\mathrm{d}t} = 0.05 P \mathrm{e}^{-0.05t}.$$

(a)   Show that, according to the model, the population of the town in 2011 will be 13 300 to 3 significant figures.   **(7)**

(b)   Find the value which the population of the town will approach in the long term, according to the model.   **(3)**

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**6.** continued
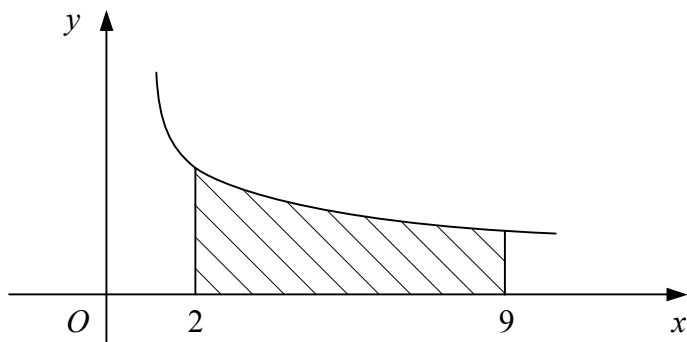
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**7.**



**Figure 1**

Figure 1 shows the curve with parametric equations

$$x = t^3 + 1, \quad y = \frac{2}{t}, \quad t > 0.$$

The shaded region is bounded by the curve, the $x$-axis and the lines $x = 2$ and $x = 9$.

*(a)* Find the area of the shaded region. **(5)**

*(b)* Show that the volume of the solid formed when the shaded region is rotated through $2\pi$ radians about the $x$-axis is $12\pi$. **(3)**

*(c)* Find a cartesian equation for the curve in the form $y = f(x)$. **(3)**

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**7.**    continued

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**Turn over**

**8.** *(a)* Show that the substitution $u = \sin x$ transforms the integral

$$\int \frac{6}{\cos x(2-\sin x)} \, dx$$

into the integral

$$\int \frac{6}{(1-u^2)(2-u)} \, du. \qquad \textbf{(4)}$$

*(b)* Express $\dfrac{6}{(1-u^2)(2-u)}$ in partial fractions. **(4)**

*(c)* Hence, evaluate

$$\int_0^{\frac{\pi}{6}} \frac{6}{\cos x(2-\sin x)} \, dx,$$

giving your answer in the form $a \ln 2 + b \ln 3$, where $a$ and $b$ are integers. **(7)**

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**8.** continued

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**8.** continued

**END**