FOR EDEXCEL

GCE Examinations

Advanced Subsidiary

Core Mathematics C4

Paper D

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and / or integration.

Full marks may be obtained for answers to ALL questions.

Mathematical formulae and statistical tables are available.

This paper has seven questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working may gain no credit.

Written by Shaun Armstrong

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1. *(a)* Find the binomial expansion of $(2 - 3x)^{-3}$ in ascending powers of x up to and including the term in x^3, simplifying each coefficient. **(5)**

(b) State the set of values of x for which your expansion is valid. **(1)**

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2. A curve has the equation

$$x^2 + 3xy - 2y^2 + 17 = 0.$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of x and y. **(5)**

(b) Find an equation for the normal to the curve at the point $(3, -2)$. **(3)**

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3. *(a)* Find the values of the constants A, B, C and D such that

$$\frac{2x^3 - 5x^2 + 6}{x^2 - 3x} \equiv Ax + B + \frac{C}{x} + \frac{D}{x-3}.$$ **(5)**

(b) Evaluate

$$\int_1^2 \frac{2x^3 - 5x^2 + 6}{x^2 - 3x}\, \mathrm{d}x,$$

giving your answer in the form $p + q \ln 2$, where p and q are integers. **(5)**

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4. A mathematician is selling goods at a car boot sale. She believes that the rate at which she makes sales depends on the length of time since the start of the sale, t hours, and the total value of sales she has made up to that time, £x.

She uses the model

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{k(5-t)}{x},$$

where k is a constant.

Given that after two hours she has made sales of £96 in total,

(a) solve the differential equation and show that she made £72 in the first hour of the sale. **(8)**

The mathematician believes that is it not worth staying at the sale once she is making sales at a rate of less than £10 per hour.

(b) Verify that at 3 hours and 5 minutes after the start of the sale, she should have already left. **(4)**

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5. Relative to a fixed origin, two lines have the equations

$$\mathbf{r} = \begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix} + s \begin{pmatrix} 1 \\ 4 \\ 5 \end{pmatrix}$$

and $$\mathbf{r} = \begin{pmatrix} -3 \\ 1 \\ -6 \end{pmatrix} + t \begin{pmatrix} 3 \\ a \\ b \end{pmatrix},$$

where a and b are constants and s and t are scalar parameters.

Given that the two lines are perpendicular,

(a) find a linear relationship between a and b. **(2)**

Given also that the two lines intersect,

(b) find the values of a and b, **(8)**

(c) find the coordinates of the point where they intersect. **(2)**

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6.

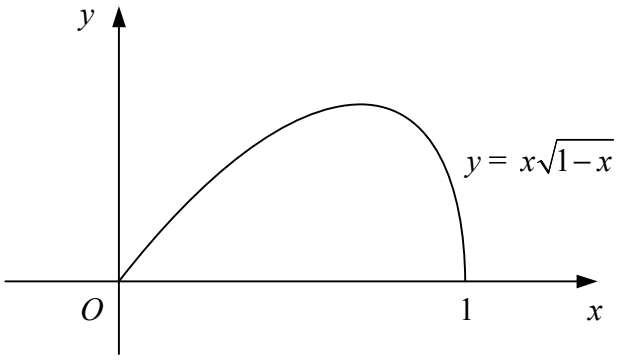

Figure 1

Figure 1 shows the curve with equation $y = x\sqrt{1-x}$, $0 \leq x \leq 1$.

(a) Use the substitution $u^2 = 1 - x$ to show that the area of the region bounded by the curve and the x-axis is $\frac{4}{15}$. **(8)**

(b) Find, in terms of π, the volume of the solid formed when the region bounded by the curve and the x-axis is rotated through 360° about the x-axis. **(5)**

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7. A curve has parametric equations

$$x = 3\cos^2 t, \quad y = \sin 2t, \quad 0 \leq t < \pi.$$

(a) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{2}{3}\cot 2t$. **(4)**

(b) Find the coordinates of the points where the tangent to the curve is parallel to the x-axis. **(3)**

(c) Show that the tangent to the curve at the point where $t = \frac{\pi}{6}$ has the equation

$$2x + 3\sqrt{3}\,y = 9.$$ **(3)**

(d) Find a cartesian equation for the curve in the form $y^2 = \mathrm{f}(x)$. **(4)**

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