FOR EDEXCEL

GCE Examinations
Advanced Subsidiary

Core Mathematics C3

Paper D

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks could be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

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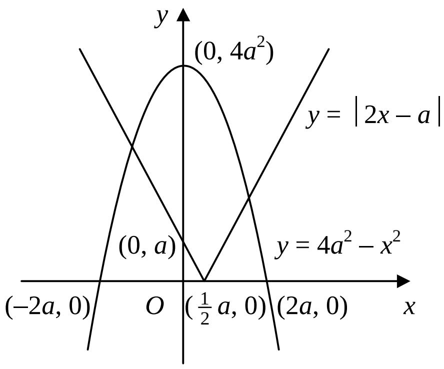
C3 Paper D – Marking Guide

1.	*(a)* $= \mathrm{f}(2) = 2 + \ln 4$	M1 A1	
	(b) $y = 2 + \ln(3x - 2)$, $\quad 3x - 2 = \mathrm{e}^{y-2}$	M1	
	$x = \frac{1}{3}(2 + \mathrm{e}^{y-2})$		
	$\mathrm{f}^{-1}(x) = \frac{1}{3}(2 + \mathrm{e}^{x-2})$	M1 A1	**(5)**
2.	$3(\mathrm{cosec}^2 x - 1) - 4\,\mathrm{cosec}\, x + \mathrm{cosec}^2 x = 0$	M1	
	$4\,\mathrm{cosec}^2 x - 4\,\mathrm{cosec}\, x - 3 = 0$		
	$(2\,\mathrm{cosec}\, x + 1)(2\,\mathrm{cosec}\, x - 3) = 0$	M1	
	$\mathrm{cosec}\, x = -\frac{1}{2}$ or $\frac{3}{2}$	A1	
	$\sin x = -2$ (no solutions) or $\frac{2}{3}$	M1	
	$x = 0.73, \pi - 0.7297$		
	$x = 0.73, 2.41$ (2dp)	A2	**(6)**
3.	*(a)* *(i)* $= \frac{\ln x}{\ln 2} = \frac{y}{\ln 2}$	M1 A1	
	(ii) $= \ln x^2 - \ln \mathrm{e} = 2\ln x - 1 = 2y - 1$	M1 A1	
	(b) $\frac{y}{\ln 2} = 4 - (2y - 1)$	M1	
	$y = (5 - 2y)\ln 2$		
	$y(2\ln 2 + 1) = 5\ln 2$	M1	
	$y = \frac{5\ln 2}{2\ln 2 + 1}$	A1	
	$x = \mathrm{e}^y = 4.27$ (2dp)	A1	**(8)**
4.	*(a)* LHS $\equiv \frac{2\sin(x+y)\cos(x-y)}{2\cos(x+y)\cos(x-y)}$	M1 A1	
	$\equiv \frac{\sin(x+y)}{\cos(x+y)} \equiv \tan(x+y) \equiv$ RHS	M1 A1	
	(b) let $x = 30°$, $y = 22.5°$ $\therefore \tan(30 + 22.5) = \frac{\sin 60 + \sin 45}{\cos 60 + \cos 45}$	M1	
	$\tan 52.5 = \frac{\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}}{\frac{1}{2} + \frac{1}{\sqrt{2}}} = \frac{\sqrt{3} + \sqrt{2}}{1 + \sqrt{2}}$	B1 A1	
	$= \frac{\sqrt{3} + \sqrt{2}}{1 + \sqrt{2}} \times \frac{1 - \sqrt{2}}{1 - \sqrt{2}}$	M1	
	$= \frac{\sqrt{3} - \sqrt{6} + \sqrt{2} - 2}{1 - 2} = \sqrt{6} - \sqrt{3} - \sqrt{2} + 2$	A1	**(9)**
5.	*(a)* $\mathrm{f}(x) = 3 - \frac{x-1}{x-3} + \frac{x+11}{(2x+1)(x-3)}$	B1	
	$= \frac{3(2x^2 - 5x - 3) - (x-1)(2x+1) + (x+11)}{(2x+1)(x-3)}$	M1 A1	
	$= \frac{4x^2 - 13x + 3}{(2x+1)(x-3)} = \frac{(4x-1)(x-3)}{(2x+1)(x-3)} = \frac{4x-1}{2x+1}$	M1 A1	
	(b) $\mathrm{f}'(x) = \frac{4 \times (2x+1) - (4x-1) \times 2}{(2x+1)^2} = \frac{6}{(2x+1)^2}$	M1 A1	
	$x = -2 \Rightarrow y = 3$, grad $= \frac{2}{3}$	A1	
	$\therefore\ y - 3 = \frac{2}{3}(x + 2)$	M1	
	$3y - 9 = 2x + 4$		
	$2x - 3y + 13 = 0$	A1	**(10)**

6. (a)	$\frac{dy}{dx} = 3e^{3x} \times \cos 2x + e^{3x} \times (-2\sin 2x) = e^{3x}(3\cos 2x - 2\sin 2x)$	M1 A1	
(b)	$\frac{d^2y}{dx^2} = 3e^{3x} \times (3\cos 2x - 2\sin 2x) + e^{3x}(-6\sin 2x - 4\cos 2x)$	M1 A1	
	$= e^{3x}(5\cos 2x - 12\sin 2x)$	A1	
(c)	SP: $e^{3x}(3\cos 2x - 2\sin 2x) = 0$		
	$3\cos 2x = 2\sin 2x$	M1	
	$\tan 2x = \frac{3}{2}$	M1	
	$2x = 0.98279, \quad x = 0.491$ (3sf)	M1 A1	
(d)	when $x = 0.491$, $\frac{d^2y}{dx^2} = -31.5$, $\frac{d^2y}{dx^2} < 0 \therefore$ maximum	M1 A1	**(11)**
7. (a)	 	B3 B3	
(b)	$4 - x^2 = 2x - 1$	M1	
	$x^2 + 2x - 5 = 0, \quad x = \frac{-2 \pm \sqrt{4+20}}{2} = \frac{-2 \pm 2\sqrt{6}}{2}$	M1	
	$x > \frac{1}{2} \quad \therefore x = -1 + \sqrt{6}$	A1	
	$4 - x^2 = -(2x - 1)$	M1	
	$x^2 - 2x - 3 = 0$		
	$(x + 1)(x - 3) = 0$	M1	
	$x < \frac{1}{2} \quad \therefore x = -1, \qquad x = -1, -1 + \sqrt{6}$	A1	**(12)**
8. (a)	$\frac{dy}{dx} = 2 - \frac{3}{2x+5} \times 2 = 2 - \frac{6}{2x+5}$	M1	
	grad = –4, grad of normal = $\frac{1}{4}$	A1	
	$\therefore\ y + 4 = \frac{1}{4}(x + 2) \qquad [\, y = \frac{1}{4}x - \frac{7}{2}\,]$	M1 A1	
(b)	$\frac{1}{4}x - \frac{7}{2} = 2x - 3\ln(2x + 5)$	M1	
	$\frac{7}{4}x + \frac{7}{2} - 3\ln(2x + 5) = 0$, let $f(x) = \frac{7}{4}x + \frac{7}{2} - 3\ln(2x + 5)$		
	$f(1) = -0.59, \ f(2) = 0.41$	M1	
	sign change, $f(x)$ continuous $\therefore$ root	A1	
(c)	$\frac{7}{4}x + \frac{7}{2} - 3\ln(2x + 5) = 0$		
	$7x + 14 - 12\ln(2x + 5) = 0$		
	$7x = 12\ln(2x + 5) - 14$	M1	
	$x = \frac{12}{7}\ln(2x + 5) - 2$	A1	
(d)	$x_1 = 1.5648, x_2 = 1.5923, x_3 = 1.6039, x_4 = 1.6087, x_5 = 1.6107$	M1 A1	
	$q = 1.61$ (3sf)	A1	
	$f(1.605) = -0.0073, \ f(1.615) = 0.0029$	M1	
	sign change, $f(x)$ continuous $\therefore$ root $\therefore q = 1.61$ (3sf)	A1	**(14)**
		Total	**(75)**

Performance Record – C3 Paper D

Question no.	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	Total
Topic(s)	functions	trigonometry	exponentials and logarithms	trigonometry	rational expressions, differentiation	differentiation	functions	differentiation, numerical methods	
Marks	5	6	8	9	10	11	12	14	75
Student									