FOR EDEXCEL

GCE Examinations
Advanced Subsidiary

Core Mathematics C4

Paper C

MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks could be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.

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C4 Paper C – Marking Guide

1. $u = \ln x,\ u' = \frac{1}{x},\ v' = x,\ v = \frac{1}{2}x^2$ — M1

$I = [\frac{1}{2}x^2 \ln x]_1^2 - \int_1^2 \frac{1}{2}x \ dx$ — A1

$= [\frac{1}{2}x^2 \ln x - \frac{1}{4}x^2]_1^2$ — M1 A1

$= (2 \ln 2 - 1) - (0 - \frac{1}{4}) = 2 \ln 2 - \frac{3}{4}$ — M1 A1 **(6)**

2.

x	0	0.5	1	1.5	2
arctan x	0	0.4636	0.7854	0.9828	1.1071

B2

(a) $\approx \frac{1}{2} \times 1 \times [0 + 1.1071 + 2(0.7854)] = 1.34$ (3sf) — B1 M1 A1

(b) $\approx \frac{1}{2} \times 0.5 \times [0 + 1.1071 + 2(0.4636 + 0.7854 + 0.9828)] = 1.39$ (3sf) — M1 A1 **(7)**

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3. *(a)* $6x - 2 + y + x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$ — M1 A2

$(-1, 3) \Rightarrow -6 - 2 + 3 - \frac{dy}{dx} + 6\frac{dy}{dx} = 0, \quad \frac{dy}{dx} = 1$ — M1 A1

grad of normal = -1

$\therefore\ y - 3 = -(x + 1)$ — M1

$y = 2 - x$ — A1

(b) sub. $\Rightarrow\ 3x^2 - 2x + x(2 - x) + (2 - x)^2 - 11 = 0$ — M1

$3x^2 - 4x - 7 = 0$ — A1

$(3x - 7)(x + 1) = 0$ — M1

$x = -1$ (at P) or $\frac{7}{3}$ $\therefore (\frac{7}{3}, -\frac{1}{3})$ — A1 **(11)**

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4. *(a)* $\overrightarrow{AB} = \begin{pmatrix} 10 \\ -15 \\ 5 \end{pmatrix}, \quad \therefore\ \mathbf{r} = \begin{pmatrix} 3 \\ 9 \\ -7 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$ — M1 A1

(b) $3 + 2\lambda = 9 \ \therefore\ \lambda = 3$ — M1

when $\lambda = 3$, $\mathbf{r} = \begin{pmatrix} 3 \\ 9 \\ -7 \end{pmatrix} + 3\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} = \begin{pmatrix} 9 \\ 0 \\ -4 \end{pmatrix}$ $\therefore$ (9, 0, –4) lies on l — A1

(c) $\overrightarrow{OD} = \begin{pmatrix} 3+2\lambda \\ 9-3\lambda \\ -7+\lambda \end{pmatrix} \quad \therefore \begin{pmatrix} 3+2\lambda \\ 9-3\lambda \\ -7+\lambda \end{pmatrix} \cdot \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} = 0$ — M1

$6 + 4\lambda - 27 + 9\lambda - 7 + \lambda = 0$ — A1

$\lambda = 2 \ \therefore\ \overrightarrow{OD} = \begin{pmatrix} 7 \\ 3 \\ -5 \end{pmatrix}$, D (7, 3, –5) — M1 A1

(d) $AB = \sqrt{100+225+25} = \sqrt{350}$, $OD = \sqrt{49+9+25} + \sqrt{83}$ — M1

area = $\frac{1}{2} \times \sqrt{350} \times \sqrt{83} = 85.2$ (3sf) — M1 A1 **(11)**

5.	*(a)*	$\frac{d\theta}{dt} = -k(\theta - 20)$	B2	
	(b)	$\int \frac{1}{\theta - 20}\ d\theta = \int -k\ dt$	M1	
		$\ln\lvert \theta - 20 \rvert = -kt + c$	M1 A1	
		$t = 0,\ \theta = 37 \Rightarrow c = \ln 17$	M1	
		$\ln\left\lvert \frac{\theta - 20}{17} \right\rvert = -kt, \quad \theta = 20 + 17e^{-kt}$	A1	
		$t = 4,\ \theta = 36 \Rightarrow \quad 36 = 20 + 17e^{-4k}$	M1	
		$k = -\frac{1}{4}\ln\frac{16}{17} = 0.01516$	A1	
		$t = 10,\ \theta = 20 + 17e^{-0.01516 \times 10} = 34.6$°C (3sf)	A1	
	(c)	$33 = 20 + 17e^{-0.01516t}$	M1	
		$t = -\frac{1}{0.01516}\ln\frac{13}{17} = 17.70$ minutes = 17 mins 42 secs	M1 A1	**(13)**
6.	*(a)*	$x = 0 \Rightarrow t = 0$ at O	B1	
		$y = 0 \Rightarrow t = 0$ (at O) or $\frac{\pi}{2}$ $\therefore t = \frac{\pi}{2}$ at A	B1	
	(b)	= volume when region above x-axis is rotated through 2π		
		$\frac{dx}{dt} = 3\cos t$	M1	
		$\therefore$ volume $= \pi\int_0^{\frac{\pi}{2}} (2\sin 2t)^2 \times 3\cos t\ dt = \int_0^{\frac{\pi}{2}} 12\pi \sin^2 2t \cos t\ dt$	M1 A1	
	(c)	$t = 0 \Rightarrow u = 0,\ t = \frac{\pi}{2} \Rightarrow u = 1,\ \frac{du}{dt} = \cos t$	B1	
		$\sin^2 2t = 4\sin^2 t\cos^2 t = 4\sin^2 t\,(1 - \sin^2 t)$	M1	
		$\therefore = \int_0^1 12\pi \times 4u^2(1 - u^2)\ du$	M1	
		$= 48\pi\int_0^1 (u^2 - u^4)\ du$	A1	
		$= 48\pi[\frac{1}{3}u^3 - \frac{1}{5}u^5]_0^1$	M1 A1	
		$= 48\pi[(\frac{1}{3} - \frac{1}{5}) - (0)] = \frac{32}{5}\pi$	M1 A1	**(13)**
7.	*(a)*	$\frac{8 - x}{(1 + x)(2 - x)} \equiv \frac{A}{1 + x} + \frac{B}{2 - x}$		
		$8 - x \equiv A(2 - x) + B(1 + x)$	M1	
		$x = -1 \Rightarrow 9 = 3A \Rightarrow A = 3$	A1	
		$x = 2 \Rightarrow 6 = 3B \Rightarrow B = 2 \ \therefore f(x) = \frac{3}{1 + x} + \frac{2}{2 - x}$	A1	
	(b)	$= \int_0^{\frac{1}{2}} (\frac{3}{1 + x} + \frac{2}{2 - x})\ dx = [3\ln\lvert 1 + x\rvert - 2\ln\lvert 2 - x\rvert]_0^{\frac{1}{2}}$	M1 A1	
		$= (3\ln\frac{3}{2} - 2\ln\frac{3}{2}) - (0 - 2\ln 2)$	M1	
		$= \ln\frac{3}{2} + \ln 4 = \ln 6$	M1 A1	
	(c)	$f(x) = 3(1 + x)^{-1} + 2(2 - x)^{-1}$		
		$(1 + x)^{-1} = 1 - x + x^2 - x^3 + \ldots$	B1	
		$(2 - x)^{-1} = 2^{-1}(1 - \frac{1}{2}x)^{-1}$	M1	
		$= \frac{1}{2}[1 + (-1)(-\frac{1}{2}x) + \frac{(-1)(-2)}{2}(-\frac{1}{2}x)^2 + \frac{(-1)(-2)(-3)}{3\times 2}(-\frac{1}{2}x)^3 + \ldots]$	M1	
		$= \frac{1}{2}(1 + \frac{1}{2}x + \frac{1}{4}x^2 + \frac{1}{8}x^3 + \ldots)$	A1	
		$\therefore f(x) = 3(1 - x + x^2 - x^3 + \ldots) + (1 + \frac{1}{2}x + \frac{1}{4}x^2 + \frac{1}{8}x^3 + \ldots)$	M1	
		$= 4 - \frac{5}{2}x + \frac{13}{4}x^2 - \frac{23}{8}x^3 + \ldots$	A1	**(14)**

Total	**(75)**

Performance Record – C4 Paper C

Question no.	1	2	3	4	5	6	7	Total
Topic(s)	integration	trapezium rule	differentiation	vectors	differential equation	parametric equations	partial fractions, binomial series	
Marks	6	7	11	11	13	13	14	75
Student								