

GCE Examinations  
Advanced Subsidiary

# Core Mathematics C4

Paper A

## MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks could be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.



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## C4 Paper A – Marking Guide

1.  $2x(2+y) + x^2 \frac{dy}{dx} - 2y \frac{dy}{dx} = 0$  M2 A2
- $$\frac{dy}{dx} = \frac{2x(2+y)}{2y-x^2}$$
- M1 A1 (6)
- 
2. (a)  $f\left(\frac{1}{10}\right) = \frac{3}{\sqrt{1-\frac{1}{10}}} = \frac{3}{\sqrt{\frac{9}{10}}} = \frac{3}{\left(\frac{3}{\sqrt{10}}\right)} = \sqrt{10}$  M1 A1
- (b)  $= 3(1-x)^{-\frac{1}{2}} = 3\left[1 + \left(-\frac{1}{2}\right)(-x) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}(-x)^2 + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{3 \times 2}(-x)^3 + \dots\right]$  M1
- $$= 3 + \frac{3}{2}x + \frac{9}{8}x^2 + \frac{15}{16}x^3 + \dots$$
- A2
- (c)  $\sqrt{10} = f\left(\frac{1}{10}\right) \approx 3 + \frac{3}{20} + \frac{9}{800} + \frac{15}{16000} = 3.1621875$  (8sf) B1
- (d)  $= \frac{\sqrt{10} - 3.1621875}{\sqrt{10}} \times 100\% = 0.003\%$  (1sf) M1 A1 (8)
- 
3. (a)  $1 + 3\lambda = -5 \quad \therefore \lambda = -2$  M1  
 $p - \lambda = 9 \quad \therefore p = 7$  A1  
 $-5 + 2\lambda = 11 \quad \therefore q = 2$  A1
- (b)  $1 + 3\lambda = 25 \quad \therefore \lambda = 8$  M1  
 when  $\lambda = 8$ ,  $\mathbf{r} = (\mathbf{i} + 7\mathbf{j} - 5\mathbf{k}) + 8(3\mathbf{i} - \mathbf{j} + 2\mathbf{k}) = (25\mathbf{i} - \mathbf{j} + 11\mathbf{k})$   
 $\therefore (25, -1, 11)$  lies on  $l$  A1
- (c)  $\overrightarrow{OC} = [(1 + 3\lambda)\mathbf{i} + (7 - \lambda)\mathbf{j} + (-5 + 2\lambda)\mathbf{k}]$   
 $\therefore [(1 + 3\lambda)\mathbf{i} + (7 - \lambda)\mathbf{j} + (-5 + 2\lambda)\mathbf{k}] \cdot (3\mathbf{i} - \mathbf{j} + 2\mathbf{k}) = 0$  M1  
 $3 + 9\lambda - 7 + \lambda - 10 + 4\lambda = 0$  A1  
 $\lambda = 1 \quad \therefore \overrightarrow{OC} = (4\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}), C(4, 6, -3)$  M1 A1
- (d)  $A: \lambda = -2, B: \lambda = 8, C: \lambda = 1 \quad \therefore AC:CB = 3:7$  M1 A1 (11)
- 
4. (a)  $\int \frac{1}{(x-6)(x-3)} dx = \int 2 dt$  M1
- $$\frac{1}{(x-6)(x-3)} \equiv \frac{A}{x-6} + \frac{B}{x-3}$$
- $$1 \equiv A(x-3) + B(x-6)$$
- M1
- 
- $x = 6 \Rightarrow A = \frac{1}{3}, x = 3 \Rightarrow B = -\frac{1}{3}$
- A2
- $$\frac{1}{3} \int \left( \frac{1}{x-6} - \frac{1}{x-3} \right) dx = \int 2 dt$$
- $$\ln|x-6| - \ln|x-3| = 6t + c$$
- M1 A1
- 
- $t = 0, x = 0 \quad \therefore \ln 6 - \ln 3 = c, \quad c = \ln 2$
- M1 A1
- 
- $x = 2 \Rightarrow \ln 4 - 0 = 6t + \ln 2$
- M1
- 
- $t = \frac{1}{6} \ln 2 = 0.1155 \text{ hrs} = 0.1155 \times 60 \text{ mins} = 6.93 \text{ mins} \approx 7 \text{ mins}$
- A1
- (b)  $\ln \left| \frac{x-6}{2(x-3)} \right| = 6t, \quad t = \frac{1}{6} \ln \left| \frac{x-6}{2(x-3)} \right|$   
 as  $x \rightarrow 3, t \rightarrow \infty \quad \therefore$  cannot make 3 g B2 (12)
-

5.	(a)	$x$ 0      0.5      1      1.5      2 $y$ 0      1.716      1.472      1.093      1.083 area $\approx \frac{1}{2} \times 0.5 \times [0 + 1.083 + 2(1.716 + 1.472 + 1.093)] = 2.41$ (3sf)	B2 B1 M1 A1
	(b)	volume $= \pi \int_0^2 16x e^{-2x} dx$ $u = 16x, u' = 16, v' = e^{-2x}, v = -\frac{1}{2} e^{-2x}$ $I = -8x e^{-2x} - \int -8e^{-2x} dx$ $= -8x e^{-2x} - 4e^{-2x} + c$ volume $= \pi[-8x e^{-2x} - 4e^{-2x}]_0^2$ $= \pi\{(-16e^{-4} - 4e^{-4}) - (0 - 4)\}$ $= 4\pi(1 - 5e^{-4})$	M1 M1 A2 A1 M1 A1

(12)

6.	(a)	$= \int (\cos x - \cos 5x) dx$ $= \sin x - \frac{1}{5} \sin 5x + c$	M1 A1 M1 A1
	(b)	$u^2 = x + 1 \Rightarrow x = u^2 - 1, \frac{dx}{du} = 2u$ $x = 0 \Rightarrow u = 1, x = 3 \Rightarrow u = 2$ $I = \int_1^2 \frac{(u^2 - 1)^2}{u} \times 2u du = \int_1^2 (2u^4 - 4u^2 + 2) du$ $= [\frac{2}{5} u^5 - \frac{4}{3} u^3 + 2u]_1^2$ $= (\frac{64}{5} - \frac{32}{3} + 4) - (\frac{2}{5} - \frac{4}{3} + 2) = 5\frac{1}{15}$	M1 B1 M1 A1 M1 A1 M1 A1

(12)

7.	(a)	$\cos 2t = \frac{1}{2}, 2t = \frac{\pi}{3}, t = \frac{\pi}{6}$	M1 A1
	(b)	$\frac{dx}{dt} = -2 \sin 2t, \frac{dy}{dt} = -\operatorname{cosec} t \cot t$ $\frac{dy}{dx} = \frac{-\operatorname{cosec} t \cot t}{-2 \sin 2t}$ $t = \frac{\pi}{6}, y = 2, \text{grad} = 2$ $\therefore y - 2 = 2(x - \frac{1}{2})$ $y = 2x + 1$	M1 M1 A1 M1 A1
	(c)	$x = 0 \Rightarrow t = \frac{\pi}{4}$ $\therefore \text{area} = \int_{\frac{\pi}{4}}^{\frac{\pi}{6}} \operatorname{cosec} t \times (-2 \sin 2t) dt$ $= - \int_{\frac{\pi}{4}}^{\frac{\pi}{6}} \operatorname{cosec} t \times 4 \sin t \cos t dt = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} 4 \cos t dt$	B1 M1 M1 A1
	(d)	$= [4 \sin t]_{\frac{\pi}{6}}^{\frac{\pi}{4}} = 2\sqrt{2} - 2 = 2(\sqrt{2} - 1)$	M2 A1

(14)

Total (75)

**Performance Record – C4 Paper A**

Question no.	1	2	3	4	5	6	7	Total
Topic(s)	differentiation	binomial series	vectors	differential equation, partial fractions	trapezium rule, integration	integration	parametric equations	
Marks	6	8	11	12	12	12	14	75
Student								