

GCE Examinations  
Advanced Subsidiary

# Core Mathematics C3

Paper A

## MARKING GUIDE

This guide is intended to be as helpful as possible to teachers by providing concise solutions and indicating how marks could be awarded. There are obviously alternative methods that would also gain full marks.

Method marks (M) are awarded for knowing and using a method.

Accuracy marks (A) can only be awarded when a correct method has been used.

(B) marks are independent of method marks.



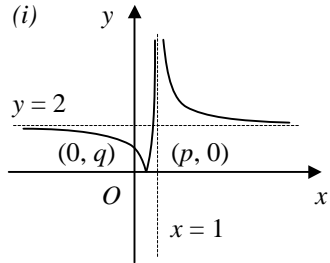
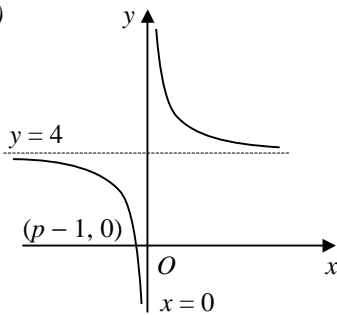
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## C3 Paper A – Marking Guide

1.  $\frac{dx}{dy} = 2 \sec y \times \sec y \tan y + \sec^2 y = \sec^2 y (2 \tan y + 1) = \frac{2 \tan y + 1}{\cos^2 y}$  M1 A1  
 $\frac{dy}{dx} = 1 \div \frac{dx}{dy} = \frac{\cos^2 y}{2 \tan y + 1}$  M1 A1 (4)
- 
2. (a)  $= f\left(\frac{1}{2}\right) = -\frac{5}{2}$  M1 A1  
 (b)  $gf(x) = \frac{2}{(3x-4)+3} = \frac{2}{3x-1}$  M1 A1  
 $\therefore \frac{2}{3x-1} = 6$   
 $2 = 6(3x-1)$  M1  
 $x = \frac{4}{9}$  A1 (6)
- 
3.  $e^{2y} - x + 2 = 0 \Rightarrow e^{2y} = x - 2$   
 $2y = \ln(x-2)$  M1  
 sub.  $\Rightarrow \ln(x+3) - \ln(x-2) - 1 = 0$  A1  
 $\ln \frac{x+3}{x-2} = 1$  M1  
 $\frac{x+3}{x-2} = e$  A1  
 $x+3 = e(x-2)$  M1  
 $3+2e = x(e-1)$  M1  
 $x = \frac{2e+3}{e-1} = 4.91$  (2dp),  $y = \frac{1}{2} \ln\left(\frac{2e+3}{e-1} - 2\right) = 0.53$  (2dp) A2 (8)
- 
4. (a)  $\frac{d}{dx}(\tan x) = \frac{d}{dx}\left(\frac{\sin x}{\cos x}\right)$   
 $= \frac{\cos x \times \cos x - \sin x \times (-\sin x)}{\cos^2 x}$  M1 A1  
 $= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$  M1 A1  
 (b)  $\frac{dy}{dx} = 2 \times \tan x + 2x \times \sec^2 x = 2 \tan x + 2x \sec^2 x$  M1 A1  
 $x = \frac{\pi}{4}, y = \frac{\pi}{2}, \text{grad} = 2 + \pi$  B1  
 $\therefore y - \frac{\pi}{2} = (2 + \pi)(x - \frac{\pi}{4})$  M1  
 at P,  $x = 0$   
 $\therefore y = \frac{\pi}{2} - \frac{\pi}{4}(2 + \pi) = -\frac{1}{4}\pi^2$  M1 A1 (10)
- 
5. (a)  $3 \cos x + \sin x = R \cos x \cos \alpha + R \sin x \sin \alpha$   
 $R \cos \alpha = 3, R \sin \alpha = 1$   
 $\therefore R = \sqrt{3^2 + 1^2} = \sqrt{10}$  M1 A1  
 $\tan \alpha = \frac{1}{3}, \alpha = 18.4$  (3sf) M1 A1  
 $\therefore 3 \cos x + \sin x = \sqrt{10} \cos(x - 18.4)^\circ$   
 (b)  $6 \cos^2 x + 2 \sin x \cos x = 0$  M1  
 $2 \cos x(3 \cos x + \sin x) = 0$  M1  
 $\cos x = 0$  or  $3 \cos x + \sin x = \sqrt{10} \cos(x - 18.4) = 0$  A1  
 $x = 90, 270$  or  $x - 18.4 = 90, 270$  B1 M1  
 $x = 90, 108.4$  (1dp),  $270, 288.4$  (1dp) A1 (10)
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6. (a) (i)  (ii) 

M1 A1  
M2 A2

(b)  $y = 0 \Rightarrow 2x - 1 = 0 \Rightarrow x = \frac{1}{2} \therefore p = \frac{1}{2}$  M1 A1  
 $x = 0 \Rightarrow y = 1 \therefore q = 1$  B1

(c)  $y = \frac{2x-1}{x-1}, \quad y(x-1) = 2x-1$  M1  
 $x(y-2) = y-1, \quad x = \frac{y-1}{y-2}$  M1  
 $\therefore f^{-1}(x) = \frac{x-1}{x-2}$  A1 (12)

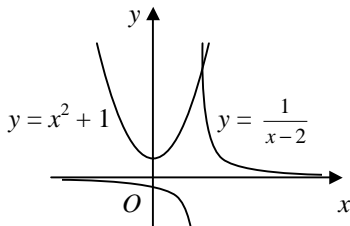
7. (a) (i) LHS =  $\sin x \cos 30 + \cos x \sin 30 + \sin x \cos 30 - \cos x \sin 30$  M1 A1  
 $= 2 \sin x \cos 30 = \sqrt{3} \sin x \quad [a = \sqrt{3}]$  A1

(ii) let  $x = 45, \sin 75 + \sin 15 = \sqrt{3} \sin 45 = \sqrt{3} \times \frac{1}{\sqrt{2}} = \frac{1}{2}\sqrt{6}$  M2 A1

(b)  $2(\operatorname{cosec}^2 y - 1) + 5 \operatorname{cosec} y + \operatorname{cosec}^2 y = 0$  M1  
 $3 \operatorname{cosec}^2 y + 5 \operatorname{cosec} y - 2 = 0, \quad (3 \operatorname{cosec} y - 1)(\operatorname{cosec} y + 2) = 0$  M1  
 $\operatorname{cosec} y = -2$  or  $\frac{1}{3}$  (no solutions) A1  
 $\sin y = -\frac{1}{2}$   
 $y = 180 + 30, 360 - 30$  B1 M1  
 $y = 210, 330$  A1 (12)

8. (a) 
$$\begin{array}{r} x^2 + 0x + 1 \\ x^2 + x - 6 \overline{) x^4 + x^3 - 5x^2 + 0x - 9} \\ \underline{x^4 + x^3 - 6x^2} \phantom{+ 0x - 9} \\ x^2 + 0x - 9 \\ \underline{x^2 + x - 6} \\ -x - 3 \end{array}$$
 M1 A1

$\therefore f(x) = x^2 + 1 + \frac{-x-3}{x^2+x-6}$  A1  
 $= x^2 + 1 - \frac{x+3}{(x-2)(x+3)} = x^2 + 1 - \frac{1}{x-2} \quad [A = 1, B = -1, C = -2]$  M1 A1

(b)  B2

$f(x) = 0 \Rightarrow x^2 + 1 = \frac{1}{x-2},$  graphs intersect once  $\therefore$  exactly one real root B1

(c) e.g.  $x_0 = 3, \quad x_1 = 2.1, \quad x_2 = 2.1848, \quad x_3 = 2.1732, \quad x_4 = 2.1747$  M1 A1  
 $\therefore$  root = 2.17 (3sf) A1  
 $f(2.165) = -0.37, \quad f(2.175) = 0.016$  M1  
 sign change,  $f(x)$  continuous  $\therefore$  root A1 (13)

Total (75)

