



Cambridge International AS & A Level

MATHEMATICS

9709/43

Paper 4 Mechanics

October/November 2023

MARK SCHEME

Maximum Mark: 50

<p>Published</p>

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Due to a series-specific issue during the live exam series, all candidates were awarded full marks for questions 1 and 2a. This published mark scheme for these questions was created alongside the question paper, but has not been used by examiners.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **20** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

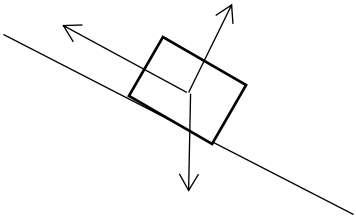
Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Use of suvat to find expressions for s	M1	
	$s = 3u - 5 \times 9$ $s = 4u - 5 \times 16$	A1	
	$u = 35, s = 60$	A1	
	Alternative method for Question 1:		
	Use of suvat to find expressions for u at max height	M1	
	$0 = u - 10 \times 3.5$	A1	
	$u = 35, s = 60$	A1	
		3	

Question	Answer	Marks	Guidance
2(a)	Attempt at conservation of momentum $[1.2v = (1.2 + 0.004) \times 40]$	M1	
	$v = \frac{602}{15}$	A1	oe
		2	
2(b)	$0^2 = (40)^2 + 2 \times 0.04 \times a$ [$a = -20000$] or $0.04 = \frac{0 + 40}{2}t$ gets $t = 0.002$, so $0 = 40 + 0.002a$ [$a = -20000$]	M1	Use of a 'suvat' method to get an equation in a . Allow sign errors. Allow ± 20000 . Do not allow 4 in place of 0.04. Allow use of 40.1 or $\frac{602}{15}$ for velocity in place of 40.
	Attempt to use Newton's Second Law vertically. $[-R + (1.2 + 0.004)g = (1.2 + 0.004) \times a]$ $[-R + 12.04 = 1.204a]$	M1	Must have the correct number of relevant terms. Allow sign errors, but terms including masses must be effectively added. Do not allow any mass other than $(1.2 + 0.004)$.
	$R = 24\,100\text{ N}$ [$24\,092.04 = \frac{602301}{25}$]	A1	WWW. Note: use of wrong sign for g leads to answers 24 067.96 which gets max M1M1A0. Note: Missing weight term gets 24 080 which gets Max M1M0A0.
		3	

Question	Answer	Marks	Guidance
2(b)	Alternative method for Question 2(b) using energy		
	[Change in PE =] $1.204g \times 0.04 [= 0.4816]$ or [change in KE =] $\frac{1}{2} \times 1.204 \times (40)^2 [= 963.2]$	B1	Allow use of 40.1 or $\frac{602}{15}$ for velocity in place of 40. B0 for kinetic energy, if extra kinetic energy terms present.
	$1.204g \times 0.04 + \frac{1}{2} \times 1.204 \times (40)^2 = 0.04R$	M1	Attempt at work energy equation. Must have correct number of relevant terms. dimensionally correct; allow sign errors. Do not allow 4 in place of 0.04. Allow use of 40.1 or $\frac{602}{15}$ for velocity in place of 40.
	$R = 24\,100\text{ N } [24\,092.04 = \frac{602301}{25}]$	A1	WWW Note: use of wrong sign for g leads to answers 24 067.96 which gets max B1M1A0. Note: Missing potential energy term gets 24 080, which gets maximum of B1M0A0.

Question	Answer	Marks	Guidance
3(a)	Correct force diagram with 3 forces in the correct directions. 	B1	No labels required on the 3 forces and ignore wrong labels. Arrows needed. Allow either or both components of weight if fully labelled. Allow sin/cos mix. If forces are not connected to the block, then the line of action of each force must go through the block.
		1	
3(b)	$R = 8g \cos 30 \left[= 40\sqrt{3} = 69.282\dots \right]$	B1	Resolving perpendicular to the plane.
	Resolving parallel to the plane and attempt to apply Newton's second law. $[8g \sin 30 - F = 8 \times 2.4 \Rightarrow F = 20.8]$	M1*	3 terms. Allow sign errors, sin/cos mix. Allow g missing, otherwise dimensionally correct.
	Use of $F = \mu R$ to get an equation in μ only. $[8g \sin 30 - 8g\mu \cos 30 = 8 \times 2.4 \quad 40 - 40\sqrt{3}\mu = 19.2]$	DM1	Allow g missing in either or both of F and R . Allow sign errors, consistent sin/cos mix. R must be a single component of a force. Allow the 3 masses to be cancelled.
	$\mu = 0.3[0\dots] \quad \left[\text{May first see } \frac{20.8}{40\sqrt{3}} \text{ or } \frac{20.8}{69.282\dots} \right]$	A1	Allow exact value $\frac{13\sqrt{3}}{75}$ or $\frac{104\sqrt{3}}{600}$ oe.
		4	

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Question	Answer	Marks	Guidance
3(c)	$[v^2 = 2 \times 2.4 \times 3 \Rightarrow \text{greatest speed} =] 3.79 \text{ ms}^{-1} = \frac{6\sqrt{10}}{5}$	B1	3.79473... (3.8 without a more accurate value seen gets B0 and should be annotated SF).
		1	

Question	Answer	Marks	Guidance
4(a)	$P = 480 \times 24$ or, e.g. $\frac{P}{24} - 480 = 0$	M1	For $\frac{P}{v} - F = 0$ or $P = Fv$ oe.
	$P = 11.52 \text{ [kW]}$	A1	Allow 11.5 M1A0 for 11 520 or 11 500.
		2	

Question	Answer	Marks	Guidance
4(b)	$[KE_{before}] = \frac{1}{2} \times 1600 \times 24^2 [= 460800]$ $[KE_{after}] = \frac{1}{2} \times 1600 \times 32^2 [= 819200]$	B1	For either correct. Do not allow $\frac{1}{2} \times 1600 \times (32 - 24)^2$.
	$[PE_{loss}] = 1600g \times 280 \times 0.09 [= 1600g \times 25.2 = 403200]$	B1	Allow $1600g \times 280 \times \sin 5.16^\circ$ or $1600g \times 280 \times \sin 5.2^\circ$ but not simply $1600g \times 280 \times \sin \theta$ (unless implied by correct final answer).
	Total WD = $12000 \times 10 [= 120000]$	B1	oe, e.g. $12000 = \frac{WD}{10}$.
4(b)	Work done against resistance = or $280F$ = or WD = or W = oe $12000 \times 10 + 1600g \times 280 \times 0.09 - \frac{1}{2} \times 1600 \times 32^2 + \frac{1}{2} \times 1200 \times 24^2$ $[= 120000 + 403200 - 819200 + 460800]$	M1	Attempt at work energy equation with 5 relevant terms (4 relevant terms plus work done against resistance); dimensionally correct. Allow sign errors. M0 for use of constant acceleration. Do not allow $\frac{1}{2} \times 1600 \times (32 - 24)^2$.
	WD = 164 800 [J]	A1	Or 164.8 kJ CAO but condone 165 kJ or 165 000 [J] Not from use of constant acceleration or Newton's second law. ISW attempt to find force after correct WD found.
		5	

Question	Answer	Marks	Guidance
5(a)	Attempt to resolve in one direction and form equation.	M1	Must have correct number of relevant terms (forces must have components as required). Allow sin/cos mix. Allow sign errors. If only one equation shown and it involves 32, it must be 32, not P .
	$T \sin \theta = 32$ and $T \cos \theta = 80$ or $0 = 80 \sin \theta - 32 \cos \theta$ and $T = 80 \cos \theta + 32 \sin \theta$	A1	For both horizontal and vertical, or both parallel and perpendicular.
	Attempt to solve for T or θ	M1	Must get to T or θ ; e.g. $T = \sqrt{32^2 + 80^2}$ or $\theta = \tan^{-1} \left(\frac{32}{80} \right)$. Condone, e.g. $\theta = \tan^{-1} \left(\frac{80}{32} \right)$. Must come from equations with correct number of relevant terms.
	$T = 86.2$ [N 86.1626....] or $16\sqrt{29}$ or $\sqrt{7424}$ and $\theta = 21.8$ [21.801...]	A1	For both.
		4	

Question	Answer	Marks	Guidance
5(a)	Alternative method using triangle of forces		
	$T^2 = 80^2 + 32^2 [-2 \times 80 \times 32 \cos 90]$ or $T \sin \theta = 32$ or $T \cos \theta = 80$	M1	For any of the five; allow sign errors.
	$80 \tan \theta = 32$ or $T = 80 \cos \theta + 32 \sin \theta$ oe	A1	For any two equations.
	Attempt to solve for T or θ	M1	Must get to T or θ ; e.g. $T = \sqrt{32^2 + 80^2}$ or $\theta = \tan^{-1}\left(\frac{32}{80}\right)$.
	$T = 86.2$ [N 86.1626....] or $16\sqrt{29}$ or $\sqrt{7424}$ and $\theta = 21.8$ [21.801...]	A1	
	Alternative Triangle of forces method using sine rule		
	$\frac{T}{\sin 90} = \frac{32}{\sin(\theta)} = \frac{80}{\sin(90 - \theta)}$	M1	For any two.
		A1	For all three.
	Attempt to solve for T or θ	M1	e.g. $\theta = \tan^{-1}\left(\frac{32}{80}\right)$.
	$T = 86.2$ [N 86.1626....] or $16\sqrt{29}$ or $\sqrt{7424}$ and $\theta = 21.8$ [21.801...]	A1	For both.

Question	Answer	Marks	Guidance
5(b)	Attempt to resolve in one direction and form equation	M1	Must have correct number of relevant terms (forces must have components as required). Allow sin/cos mix. Allow sign errors. Must use 120, not T .
	$120\sin\theta = P$ and $120\cos\theta = 80$ or $0 = 80\sin\theta - P\cos\theta$ and $120 = 80\cos\theta + P\sin\theta$	A1	For both horizontal and vertical, or both parallel and perpendicular.
	Attempt to solve for P or θ	M1	Must get to P or θ ; e.g. $P = \sqrt{120^2 - 80^2}$ or $\theta = \cos^{-1}\left(\frac{80}{120}\right)$. Must come from equations with correct number of relevant terms.
	$P = 89.4$ [89.4427] or $40\sqrt{5}$ or $\sqrt{8000}$ $\theta = 48.2$ [48.1896...]	A1	For both; allow $P = 89.5$ (from $120\sin 48.2$).
		4	
	Alternative method using triangle of forces		
	$120^2 = P^2 + 80^2 [-2 \times 80 \times P \cos 90]$ or $120\sin\theta = P$ or $120\cos\theta = 80$ or $80 \tan\theta = P$ or $120 = 80\cos\theta + P\sin\theta$ oe	M1	For any of the five; allow sign errors.
		A1	For any two equations.
	Attempt to solve for P or θ	M1	Must get to P or θ ; e.g. $P = \sqrt{120^2 - 80^2}$ or $\theta = \cos^{-1}\left(\frac{80}{120}\right)$, oe.
	$P = 89.4$ [89.4427] or $40\sqrt{5}$ or $\sqrt{8000}$ $\theta = 48.2$ [48.1896...]	A1	For both; allow $P = 89.5$ (from $120\sin 48.2$).

Question	Answer	Marks	Guidance
5(b)	Alternative Triangle of forces method using sine rule		
	$\frac{120}{\sin 90} = \frac{P}{\sin(\theta)} = \frac{80}{\sin(90-\theta)}$	M1	For any two.
		A1	For all three.
	Attempt to solve for P or θ	M1	Must get to P or θ ; e.g. $\theta = 90 - \sin^{-1}\left(\frac{80}{120}\right)$ or $\theta = \cos^{-1}\left(\frac{80}{120}\right)$.
	$P = 89.4$ [89.4427] or $40\sqrt{5}$ or $\sqrt{8000}$ $\theta = 48.2$ [48.1896...]	A1	For both; allow $P = 89.5$ (from $120\sin 48.2$).
6(a)	Attempt to integrate a	M1*	The power of t must increase by 1 with a change of coefficient in the t^2 term. Do not penalise missing c . Use of $v = at$ scores M0.
	$[v =] 36t - 3t^2 [+c]$ or $[v =] 36t - \frac{6t^2}{2} [+c]$	A1	Condone an integral sign in front of correct answer.
	$0 = 36t - 3t^2 - 33$ $[27 = 36 \times 2 - 3 \times 2^2 + c \Rightarrow c = -33]$	DM1	Use $t = 2$ and $v = 27$ to find c . Must get to $c =$ and set 3 term quadratic equal to zero.
	Solve $0 = 36t - 3t^2 - 33$ to get $t = 1$ and $t = 11$	A1	Allow $t = 1$ or $t = 11$; $t = 1, t = 11$ oe.
		4	

Question	Answer	Marks	Guidance
6(b)	Attempt to integrate an expression of the form $at + bt^2 [+c]$ with non-zero a and b . If correct $[s =] \frac{36t^2}{2} - \frac{3t^3}{3} - 33t [+c']$ or $[s =] 18t^2 - t^3 - 33t [+c']$	M1*	The power of t must increase by 1 with a change of coefficient in the same term. Use of $s = vt$ scores M0.
	Attempt to evaluate <i>their</i> $[18t^2 - t^3 - 33t]$ for $t = 0$ to $t = 1$ or $t = 1$ to $t = 11$ or $t = 11$ to $t = 12$ 0 to 1: $-16 - 0 = -16$ 1 to 11: $484 - (-16) = 500$ 11 to 12: $468 - 484 = -16$	DM1	Attempt using their limits (at least one strictly between 0 and 12) correctly.
	For all three	DM1	Allow 11 to 12 implied by symmetry instead of found separately.
	Distance = $[16 + 500 + 16] = 532\text{m}$	A1	
		4	

Question	Answer	Marks	Guidance
7(a)	Resolving for both particles or for the system to form equation(s)	M1*	Must have correct number of terms. Allow sign errors. Allow sin/cos mix. Allow g missing. M0 if acceleration included unless subsequently equated to zero. Masses must be appropriate for their equation(s). Forces must have components (or not) as required.
	Either $T - F - 2.4g \sin 30 = 0$ AND $3.3g - T = 0$ Or $3.3g - F - 2.4g \sin 30 = 0$	A1	Both correct or system correct. May get $F = 21$. Can be with a wrong non-zero F .
	$R = 2.4g \cos 30 \left[= 12\sqrt{3} = 20.7846... \right]$	B1	
	Use of $F = \mu R$ to get an equation in μ only $[3.3g - 2.4g\mu \cos 30 - 2.4g \sin 30 = 0]$	DM1	Must be from F dimensionally correct and single term R which is equal to a component the 2.4 kg weight. Allow consistent sin/cos mix but must be different components of weight. F and R must be numerical expressions.
	$\mu = 1.01$ [sight of 1.01036... or 1.0104]	A1	AG perhaps from one of $\mu = \frac{3.3g - 2.4g \sin 30}{2.4g \cos 30} = \frac{33 - 12}{12\sqrt{3}} = \frac{21}{12\sqrt{3}} = \frac{7\sqrt{3}}{12} = \frac{21}{20.7846...}$ $= \frac{21}{20.8}$ Do not allow unless evidence of 30 substituted for θ . E.g.: sight of 1.01036... or 1.0104.
		5	

Question	Answer	Marks	Guidance
7(b)	Using Newton's second law for both particles or the system	M1*	Must have correct number of terms. Allow sign errors. Allow sin/cos mix. Allow g missing. Masses must be appropriate for their equation(s). Forces must have components (or not) as required.
	Either $3.3g - T = 3.3a$ and $T - F - 2.4g \sin 20 = 2.4a$ $[T - 22.778 - 8.208 = 2.4a]$ or $[T - 30.986 = 2.4a]$ or $3.3g - F - 2.4g \sin 20 = (2.4 + 3.3)a$ $[2.013367... = 5.7a]$	A1	Both correct or system equation correct. Can be with a wrong non-zero F .
	$F = 1.01 \times 2.4g \cos 20$ $[= 22.778]$	B1	For correct expression for F .
	Attempt to solve for a $a = 0.353$ $[0.353222...]$	DM1	Using their F Must get to ' $a =$ '. If sin/cos mix must be consistent.
	$v^2 = 2 \times 0.353 \times 1$ $[= 0.706444...]$ or $[v = 0.841]$ Or $1 = 0 + \frac{1}{2} \times 0.353t^2 \Rightarrow t = 2.3795 \Rightarrow v = 0.353 \times 2.38$	A1FT	FT their value of $a \neq \pm g$ to get an expression for v^2 or v . Can be implied by awrt 0.84 for v or awrt 0.71 for v^2 . This mark does not depend on previous A or B mark, but both Ms must have been awarded.
	Using Newton's second law on A after B reaches the ground $-F - 2.4g \sin 20 = 2.4a$ $[-1.01 \times 2.4g \cos 20 - 2.4g \sin 20 = 2.4a]$ $[-22.78814... - 8.20848... = 2.4a]$	M1*	Must have correct number of terms. Allow sign errors. Allow sin/cos mix. Allow g missing. $[\Rightarrow a = -12.911...]$
	Use of suvat to find s $[0 = their 0.841^2 + 2 \times their -12.911... \times s \Rightarrow s = 0.027358...]$	DM1	Using their $a \neq \pm g$. Must get to ' $s =$ '. May find and use $t = 0.0651$.

Question	Answer	Marks	Guidance
7(b)	Total distance = 1.03 m	A1	
		8	
	Alternative method using energy for first 5 marks		
	$\left[\text{KE gained} = \right] \frac{1}{2} \times (2.4 + 3.3) v^2 \left[= 2.85 v^2 \right]$	B1	
	$\left[\text{PE lost} = \right] 3.3g \times 1 - 2.4g \times 1 \sin 20 \left[= 24.791 \dots \right] =$	B1	Allow omission of 1 in either or both terms.
	$\left[\text{Friction} = \right] 1.01 \times 2.4g \cos 20 \left[= 22.778 \dots \right]$	B1	For correct expression for F .
	$\frac{1}{2} \times (2.4 + 3.3) v^2 = 3.3g \times 1 - 2.4g \times 1 \sin 20 - 1.01 \times 2.4g \cos 20 \times 1$ Or $2.85 v^2 = 24.791 \dots - 22.778 \dots$	M1	For attempt at energy equation. Allow sign errors, allow sin/cos mix but must have sin/cos where needed. Correct number of terms, dimensionally correct. Allow omission of 1 in any or all the three relevant terms. Must have cos 20 and sin 20.
	To get a correct expression for v^2 $\left[v^2 = 0.706444 \dots \text{ or } v = 0.841 \dots \right]$	A1	Can be implied by awrt 0.84 for v or awrt 0.71 for v^2 if expression not seen.

Question	Answer	Marks	Guidance
7(b)	Alternative method using energy for final 3 marks		
	$KE = \frac{1}{2} \times 2.4 \times 0.841^2$	M1	Using their v^2 .
	$1.01 \times 2.4g \cos 20 \times s + 2.4g \sin 20 \times s = \frac{1}{2} \times 2.4 \times 0.841^2$ [$\Rightarrow s = 0.027358..$]	M1	For attempt at 3 term energy equation and solved to get to 's ='. Allow sign errors, allow consistent sin/cos mix but must have sin/cos where needed. Correct number of terms, dimensionally correct.
	Total distance = 1.03 m	A1	