Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **October/November 2023**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **15** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M	Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
A	Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
B	Mark for a correct result or statement independent of method marks.
DM or **DB**	When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
FT	Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	$[y] = \left\{\frac{x^2}{2}\right\} \quad \left\{\frac{-3x^{\frac{1}{2}}}{\frac{1}{2}}\right\} [+c]$	**B1 B1**	Any unsimplified correct form, ISW for extra terms, allow lists.
	$1 = 8 - 12 + c$	**M1**	Substitute (into an integrated expression) $x = 4$, $y = 1$. c must be present. Expect $c = 5$.
	$y = \frac{1}{2}x^2 - 6x^{\frac{1}{2}} + 5$, allow $f(x) =$	**A1**	Condone $c = 5$ as the final line so long as '$y =$' present.
		4	

Question	Answer	Marks	Guidance
2(a)	$(0-3)^2 + (y-5)^2 = 40$	**M1**	OE. Substitute $x = 0$, may use $y^2 - 10y - 6 = 0$.
	$y = 5 \pm \sqrt{31}$	**A1**	OE. Must be surd form.
		2	
2(b)	$\left\{x^2 + (y-5)^2\right\} = \{31\}$ Allow $(x-0)^2$	**B1FT B1FT**	B1 FT for *their* 5 and B1 FT for *their* 31. Don't allow surd form.
		2	

Question	Answer	Marks	Guidance
3(a)	$5\cos^2\theta - \sin^2\theta + \cos\theta\ [=0]$	M1	Multiply by $\cos\theta$ and replace $\tan\theta$ by $\dfrac{\sin\theta}{\cos\theta}$.
	$5\cos^2\theta - \left(1-\cos^2\theta\right) + \cos\theta\ [=0]$	M1	
	$6\cos^2\theta + \cos\theta - 1 = 0$	A1	Missing '= 0' can be condoned if '= 0' appears earlier.
		3	
3(b)	$(3\cos\theta - 1)(2\cos\theta + 1) = 0$	M1	Must have 3 term quadratic, expect $\cos\theta = \dfrac{1}{3}, -\dfrac{1}{2}$. Factors (OE) must be shown.
	$\theta = \{1.23\}$; {2.09 or $\dfrac{2\pi}{3}$ } ; {5.05 and 4.19 (allow $\dfrac{4\pi}{3}$) }	A1 A1 A1 FT	For A1 FT is for <u>both</u> 2π – 1st solutions.
		4	

Question	Answer	Marks	Guidance
4(a)(i)	$1 + 10x + 40x^2$	B1	Ignore any additional terms (ISW). Allow x^0 or 1 x^0 for the first term, allow lists.
		1	
4(a)(ii)	$\{1\}\{-6ax\}\{+15a^2x^2\}$	B2, 1, 0	Ignore any additional terms (ISW). Allow x^0 or 1 x^0 for the first term, allow lists.
		2	

Question	Answer	Marks	Guidance
4(b)	$15a^2 - 60a + 40 = -5$	**M1 A1**	Correct 3 products from *their* expansions for M1. Condone inclusion of x^2 for M1.
	$[15](a-1)(a-3)[=0]$ OE	**DM1**	OE. Rearranging and solving a quadratic.
	$a = 1$ and 3	**A1**	**Special case:** If M1 A1 DM0 scored then **SC B1** can be awarded for correct answers.
		4	

Question	Answer	Marks	Guidance
5(a)	$\frac{5p}{2p+6} = \frac{8p+2}{5p}$	**M1**	OE. Setting up a valid relationship in terms of *p*.
	$9p^2 - 52p - 12$ [= 0]	**DM1**	OE. Simplifying to a 3 term quadratic equation, only condone sign errors.
	$[(9p+2)(p-6) = 0]$ leading to $p = \frac{-2}{9}$ and 6	**A1**	
		3	
5(b)	$a = 2\left(-\frac{2}{9}\right) + 6\left[= \frac{50}{9}\right]$	***M1**	FT *their* $-\frac{2}{9}$, allow any negative non-integer.
	$r = -\frac{10}{9} \div \frac{50}{9}\left[= -\frac{1}{5}\right]$	***M1**	Ft *their* $-\frac{2}{9}$, allow any negative non-integer.
	$S_\infty = \frac{50}{9} \div \left(1 - -\frac{1}{5}\right) = \frac{125}{27}$	**DM1 A1**	Can only get DM1 if |r| < 1. Accept AWRT 4.63 .
		4	

Question	Answer	Marks	Guidance
6	$cx^2+2x-3=6x-c$ leading to $cx^2-4x+(c-3)\ [=0]$	B1	3-term quadratic.
	$16-4c(c-3)=0$	*M1	Apply $b^2-4ac=0$ ('= 0' may be implied in subsequent work). *Their* coefficients must be substituted correctly
	$4c^2-12c-16[=0]$ leading to $[4](c-4)(c+1)\ [=0]$ leading to $c=4$ and -1	A1	Dependent on factorisation oe.
	When c = 4, $4x^2-4x+1[=0]$ $\left[(2x-1)^2\ =0\right]$	DM1	OE. Substituting *their* c = 4 into *their* quadratic equation.
	$x=\frac{1}{2}$, $y=-1$	A1	Both required.
	When $c=-1$, $x^2+4x+4[=0]$ $\left[(x+2)^2\ =0\right]$	DM1	OE. Substituting *their* c = -1 into *their* quadratic equation.
	$x=-2$, $y=-11$	A1	Both required.
	Alternative method for Question 6		
	$\frac{\mathrm{d}y}{\mathrm{d}x}=2cx+2$	B1	
	$2cx+2=6$	M1	Equating *the*ir curve gradient and 6.
	$c=\frac{2}{x}$	A1	SOI
	$2x^2+3x-2\ [=0]$	DM1	Substitute $c=\frac{2}{x}$ into $cx^2+2x-3=6x-c$. Simplify to 3-term quadratic.

Question	Answer	Marks	Guidance
6	$(2x-1)(x+2)[=0] \to x=\frac{1}{2}$ or -2	**A1**	Dependent on factorisation. Both required.
	$c=4$ and -1	**A1**	Both required, if DM0 given **SC B1** for both.
	$y=-1$ and -11	**A1**	Both required, if DM0 given **SC B1** for both. **SC** one correct (x, y). A1 only
		7	

Question	Answer	Marks	Guidance
7(a)	Range is $[y] > 1$	**B1**	Allow f, f(x), $(1,\infty)$, etc.
		1	
7(b)	$y=\frac{3}{x-2}+1$ leading to $y-1=\frac{3}{x-2}$ leading to $(x-2)(y-1)=3$	**M1**	Clearing the fraction.
	$x-2=\frac{3}{y-1}$	**DM1**	Reaching a stage which only requires one further operation.
	$x=\frac{3}{y-1}+2$ leading to $f^{-1}(x)=\frac{3}{x-1}+2$	**A1**	OE. Slightly longer routes lead to $f^{-1}(x)=\frac{2x+1}{x-1}$.
	[Domain is] $x>1$	**B1FT**	Must use x FT *their* **(a)**, must be a range.
		4	

Question	Answer	Marks	Guidance
7(c)	$\text{gf}(x) = 2\left(\frac{3}{x-2}+1\right)-2$ or $2\left(\frac{x+1}{x-2}\right)-2$	**M1**	Substitute f(*x*) into g(*x*).
	$\frac{6}{x-2}$	**A1**	
		2	

Question	Answer	Marks	Guidance
8(a)	$a = \frac{1}{2}$	**B1**	
	$b = \frac{\pi}{3}$	**B1**	$b = \frac{\pi}{3} + 4n\pi, n \geqslant 0$.
		2	
8(b)	*x*-coordinate = {$4p$} {-8}	**B1 B1**	OE, e.g. $4(p-2)$.
	y-coordinate = $-3q$	**B1**	
		3	

Question	Answer	Marks	Guidance
9(a)	$\frac{dy}{dx} = x^{-\frac{1}{2}} \to m = \frac{1}{2}$ at $x = 4$	B1	
	Equation of normal is $y - 3 = -2(x - 4)$	M1	Through (4, 3) with gradient $-\frac{1}{their\ m}$. (Dependent on differentiation used).
	$y = -2x + 11$	A1	Only acceptable answer.
		3	
9(b)	$\frac{dy}{dt} = their\frac{1}{2} \times 3$	M1	Correct use of the chain rule with a numerical gradient.
	$\frac{3}{2}$	A1	
		2	
9(c)	Required gradient $\left[= \frac{dy}{dx}\right] = -2$	B1FT	SOI. FT from *their* part **(a)** if a normal gradient has been found from $m_1m_2 = -1$ and differentiation.
	$\frac{dx}{dt} = \frac{1}{their\ normal\ gradient} \times -5$	M1	Correct use of the chain rule . Allow method mark also for +5, must be numerical. *Their* normal gradient must come from $m_1m_2 = -1$ and differentiation in part(a) unless 'restarted' here.
	$\frac{5}{2}$	A1	
		3	

Question	Answer	Marks	Guidance
10(a)	Angle $ACO = 0.7$	B1	Don't allow AWRT 0.7 .
		1	
10(b)	$[R =]\ 1.53\,r$	B1	Allow AWRT 1.53*r*.
		1	
10(c)	Sector $OAB = \frac{1}{2}r^2 \times 2.8 \quad [=1.4r^2]$	B1	
	Sector $CAB = \frac{1}{2}(their\ R)^2 \times 2 \times their\ 0.7$	*M1	
	$1.638\,r^2$	A1	Allow AWRT $1.64\,r^2$.
	$[2]\times\frac{1}{2}r^2\sin(\pi - 1.4)$ OR $[2]\times\frac{1}{2}r\times their R\sin 0.7$	*M1	
	$2\times 0.4927r^2$	A1	Allow AWRT $0.98\,r^2$ to $0.99\,r^2$.
	$1.4r^2 - \left(their\,1.638r^2 - their\,0.985r^2\right)$	DM1	
	$0.747r^2$ to $0.748r^2$	A1	
		7	

Question	Answer	Marks	Guidance
10(c)	**General guidance for alternative methods**		
	Finding any useful sector area of the circle radius, r	**B1**	May be 'nested' in a segment.
	Finding the area of sector CAB	***M1A1**	May be 'nested' in a segment.
	Finding the area of one useful triangle	***M1**	May be 'nested' in a segment.
	Finding the total area of useful triangles	**A1**	May be 'nested' in a segment.
	A correct plan for the shaded area	**DM1**	
	$0.747r^2$ to $0.748r^2$	**A1**	
		7	

Question	Answer	Marks	Guidance
11(a)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\left\{-2\times 2\times (2x-1)^{-3}\times 2\right\}+\{1\}$	**B1B1**	Expect $\frac{-8}{(2x-1)^3}+1$.
	Substitute $x=\frac{3}{2}$ leading to $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-8}{8}+1=0$. Hence x-coordinate of R is $\frac{3}{2}$	**DB1**	AG. Or correct solution of $\frac{\mathrm{d}y}{\mathrm{d}x}=0$.
	When $x=\frac{3}{2}, y=\frac{2}{4}+\frac{3}{2}=2$	**B1**	Answer only is acceptable.
		4	

Question	Answer	Marks	Guidance
11(b)	y-coordinate of $P = 3$, y-coordinate of $Q = \frac{20}{9}$	B1	Both required.
	$\left\{\frac{2(2x-1)^{-1}}{-1\times 2}\right\}+\left\{\frac{1}{2}x^2\right\}$	B1 B1	Area below curve.
	$\left[\left(-\frac{1}{3}+2\right)-\left(-1+\frac{1}{2}\right)\right]=\frac{5}{3}-\left(-\frac{1}{2}\right)$	M1	Apply limits 1→2 to an integral. Expect $\frac{13}{6}$.
	$\frac{1}{2}\left(3+\frac{20}{9}\right)=\frac{47}{18}$	M1	Area of trapezium, only allow errors in y-coordinate of Q.
	$\frac{47}{18}-\frac{13}{6}=\frac{4}{9}$	A1	Shaded region.
		6	
	Alternative method 1: Changes the award of the first M1		
	Their equation of line PQ:[$y=\frac{-7}{9}x+\frac{34}{9}$]. Integrating between 1 and 2.	M1	Must be some evidence of use of limits.
	Alternative method 2: Changes the award of the first M1, a B1 and the second M1		
	Combining line and curve: $\int\left(\frac{-16}{9}x+\frac{34}{9}-\frac{2}{(2x-1)^2}\right)dx$	M1	For area under the line if *their* $\frac{34}{9}$ is seen integrated correctly and limits used. Correct first and 3rd terms.
	$=\frac{-8}{9}x^2+\frac{34}{9}x+\frac{1}{(2x-1)}$	B1 B1	
	Use of limits on the whole integral	M1	