MATHEMATICS

Paper 9709/11
Pure Mathematics 1 (11)

Key messages

Candidates generally seemed confident with topics such as integration and differentiation and could apply the appropriate techniques. Where many candidates unfortunately struggled is with some more basic algebraic techniques, including manipulation of trigonometric functions.

The previous three reports have each highlighted that the question paper contains a statement in the rubric on the front cover that 'no marks will be given for unsupported answers from a calculator.' Although this message has been taken on board by most candidates, there are still a significant minority for whom it hasn't. Clear working must always be shown to justify solutions. For quadratic equations, it would be necessary to show factorisation, use of the quadratic formula or completing the square as stated in the syllabus. Using calculators to solve equations and writing down only the solution is not sufficient. If using a factorisation method, the factors must always produce the coefficients of the quadratic when expanded. If using the quadratic formula, it is insufficient to quote only the formula: candidates need to show values substituted into it.

General comments

Some very good responses were seen but the paper proved challenging for many candidates. In AS and A Level Mathematics papers the knowledge and use of basic algebraic and trigonometric methods from IGCSE or O Level is expected, as stated in the syllabus.

Comments on specific questions

Question 1

(a) Most candidates used the binomial expansion formula and gave the correct answer. A common error was to find the 3rd term using $15\times3x^2$ rather than $15\times(3x)^2$.

(b) Stronger candidates found the sum of the three products, $(1 \times 1) + (-7 \times 18) + (1 \times 135) = 10$. Candidates should be aware that finding the full expansion with nine terms is unnecessary and time consuming.

Question 2

Most candidates equated the two equations and put all the terms to one side to give a 3-term quadratic, with occasional sign errors. Some candidates found the correct discriminant of the 3-term quadratic. Others made algebraic slips and sign errors and were unable to be awarded any further marks. Many candidates stated that *their* discriminant = 0, rather than *their* discriminant > 0. Candidates need to understand that if the discriminant is positive for all values of *c*, then the line and the curve intersect for all values of *c*.

Question 3

This question proved to be challenging for most candidates. Successful candidates found $\frac{\mathrm{d}V}{\mathrm{d}x} = 3x^2$ and then used the chain rule, $\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{dV}{dx} \times \frac{dx}{dt}$. However most candidates answered the question without using differentiation and were unable to gain any marks. Candidates should be aware that questions on rates of change will involve differentiation and use of the chain rule.

Question 4

Stronger candidates gained full marks on this question. Many candidates made sign errors.

(a) Candidates were more likely to be awarded the second mark rather than the first mark.

(b) Candidates were more likely to be awarded the second mark rather than the first mark.

(c) Candidates who gave the correct answers to **(a)** and **(b)** usually gave the correct answer for **(c)**. Some answers were awarded the follow through marks that were available for a translation along the *y*-axis.

Question 5

(a) Successful candidates wrote $\frac{5}{\tan x}$ as $\frac{5\cos x}{\sin x}$ and then multiplied by $\sin x$.

Most candidates who found that $4\sin^2 x + 5\cos x + 2 = 0$ then went on to use the substitution $\sin^2 x = 1 - \cos^2 x$ to find $4\cos^2 x - 5\cos x - 6 = 0$. Some candidates were unable to gain any marks due to errors in algebraic manipulation.

(b) Stronger candidates gained full marks on this question. Other candidates did not gain full marks due to incomplete methods. The first mark was gained for solving a quadratic in $\cos x$ using factorisation, the quadratic formula or completing the square. Many answers were not awarded this mark. The final two marks were gained for finding a correct angle and 360° – *their* angle. Candidates should be reminded to check their solutions by substituting into the original equation. Candidates should also be reminded to ensure they include all possible solutions within the range so that they are fully answering the question.

Question 6

(a) Successful candidates understood that *AOC* and *BOC* are equilateral triangles, hence angle $ACB = \frac{2\pi}{3}$. Some candidates did not answer this question.

(b) Successful candidates understood that arcs *AOB* and *ACB* are equal in length, so that the perimeter of the shaded area is equal to the perimeter of the circle. Candidates should understand that if a question is only worth one mark it should not need a lot of work to solve it. Many candidates did not answer this question.

(c) This question proved to be challenging for most candidates. It required candidates to devise a strategy and apply their knowledge of sectors and circles. In a question of this type, candidates are advised to first write down an overall strategy to help structure their answer. There were several ways to solve this question, all of which involved finding the area of a segment. Candidates should understand the need to know the formula for the area of a segment, as it is often needed in questions on this topic. A fully correct answer was rare, but many attempts at the area of a sector, segment or triangle were seen.

Question 7

Stronger candidates gained full marks on this question. Many answers were awarded the first two marks for the sum of the first two terms and the sum to infinity. Candidates who used the sum of the first two terms as $a + ar$ could eliminate a and find a quadratic in a few steps. Candidates who used the sum of the first two terms as $\frac{a(1-r^2)}{1-r}$, and did not simplify this, needed several steps to find a cubic, which often contained errors. Candidates who solved a quadratic or cubic usually took note of the question and used the negative value.

Question 8

(a) Successful candidates substituted $u = 2x - 3$, rearranged to get $2u^4 - u^2 - 1 = 0$, then solved using $(2u^2 + 1)(u^2 - 1) = 0$ or another method. Candidates gained a mark for solving a quadratic in u^2 using factorisation, the quadratic formula or completing the square, and many candidates were not awarded this mark. Some candidates used the substitution $u = u^2$, but often then forgot to square root. Those who used a letter other than u were usually more successful. Some candidates used their solution to u as x and were unable to gain the final two marks.

(b) Candidates who attempted this part of the question realised the need to integrate the functions and substitute in both limits. Subtraction was done either before or after the integration and substitution. Most candidates showed their substitution of the limits clearly, though candidates who used a calculator to obtain a numerical answer without showing the substitution could not be awarded a mark for the substitution.

Question 9

(a) Many candidates gained full marks on this question by giving the expression in the specified form. Other candidates rearranged to get $4\left(x - \frac{3}{2}\right)^2 + 4$ and did not gain full marks.

(b) This question proved to be challenging for most candidates. Successful candidates wrote $(2x-3)^2 + 4 < 8$, or an equivalent, and solved to find the two critical values. Many others used the composite function gf and were unable to gain any marks.

(c) Most candidates gained full marks on this question, giving the answer in a variety of forms.

(d) Candidates who used the form specified in part **(a)** were able to gain two marks for finding $\mathrm{h}^{-1}(x) = \frac{3}{2} + \frac{\sqrt{x-4}}{2}$. Few candidates gained full marks by considering the domain of h(x) to find the correct answer of $\mathrm{h}^{-1}(x) = \frac{3}{2} - \frac{\sqrt{x-4}}{2}$.

Candidates who attempted to find $\mathrm{h}^{-1}(x)$ from h(x) = $4x^2 - 12x + 13$ gained no marks, unless they were able to first complete the square to reduce to x appearing only once.

Question 10

(a) Successful candidates used integration to find $\frac{dy}{dx} = 3x^2 + c$ and then substituted $x = 2$, $\frac{dy}{dx} = 0$ to get $\frac{dy}{dx} = 3x^2 - 12$. A common error was to substitute $x = 2$, $y = -10$.

(b) Successful candidates integrated their answer to part **(a)**, substituted $x = 2$, $y = -10$ to find the constant, and gave the correct answer. Most candidates who answered this question integrated their answer to part **(a)** and substituted $x = 2$, $y = -10$ to find the constant.

(c) Most candidates who answered this question set their $\frac{dy}{dx} = 0$ and solved to find x. Candidates who attempted to find the nature of the stationary point usually used $\frac{d^2y}{dx^2}$.

(d) Successful candidates found the gradient from their answer to part **(a)** and the y-intercept from their answer to part **(b)**

Question 11

(a) Stronger candidates identified the centre of the circle, found the gradient of AB, and substituted into $y = mx + c$ or $(y - y_1) = m(x - x_1)$ to find the equation of the line AB. Some candidates identified the centre of the circle only.

(b) Candidates who found an equation for the line AB in part **(a)** were generally able to substitute their y value into the equation of the circle. Candidates who formed a 3-term quadratic in x generally solved the equation and rejected one solution.

(c) Candidates who had a solution to part **(b)** were often able to gain at least one mark by using the equation y – *their* y-coordinate of A = 1(x – *their* x-coordinate of A), or equivalent.

MATHEMATICS

Paper 9709/12
Pure Mathematics 1 (12)

Key messages

Candidates generally seem very confident with topics such as Integration and Differentiation and could apply the appropriate techniques to quite complicated functions. Many candidates struggled with more basic algebraic techniques; common errors included:

- Squaring each term of a 3 or 4 term expression.
- When two expressions multiplied together equalled 1, each expression was then set to equal to 1.
- Mixing up the techniques for adding and multiplying algebraic fractions.
- Working with fractional powers – particularly $x^{-\frac{1}{2}}$.

Candidates would benefit from consolidating the algebraic techniques required at this level.

The previous three reports have each highlighted that on the front of the question paper, in the list of instructions, there is a statement 'You must show all necessary working clearly, no marks will be given for unsupported answers from a calculator.' Although this message has been taken on board by most candidates, there is still a minority for whom it has not. For the solution of quadratic equations, it would be necessary to show factorisation, use of the quadratic formula or completing the square – as stated in the syllabus. Using calculators to solve equations and writing down the solutions is not sufficient. Neither is it sufficient only to quote the formula: candidates need to show values substituted into it. Factors must also always produce the coefficients of the quadratic when expanded.

General comments

The paper was found to be accessible for candidates and many very good scripts were seen. Candidates generally seemed to have sufficient time to finish the paper. Presentation of work was mostly good, although some answers still seem to be written in pencil and then overwritten with ink. This practice produces a very unclear image when the script is scanned and makes it difficult to mark. Centres should strongly advise candidates not to do this.

Comments on specific questions

Question 1

This question was a good start to the paper for most candidates. Many were able to obtain the required coefficients, form a correct equation and solve it. Errors included retaining x^2 or x^3 in the equation and multiplying the wrong coefficient by 6.

Question 2

Some candidates found this question challenging. The main point of confusion seemed to be the meaning of $\cos^{-1}$. Weaker candidates thought that it meant the reciprocal of cos. A good number of others understood it's meaning but used degree mode in their calculators and then combined 30 with $\frac{\pi}{6}$. Good candidates were able to use their calculators in radian mode to obtain the inverse cosine, combine it correctly with $\frac{\pi}{6}$, take the tangent of this new angle and divide by 4. Other common errors were combining the angles incorrectly, losing the minus sign and working out a non-exact answer.

Question 3

Many fully complete answers were seen for this question. However, weaker candidates sometimes put the working for **part (b)** as their answer for **part (a)**. Others were confused as to how to find the gradient of the normal with some using the negative reciprocal of $\frac{1}{2}$ from $\frac{1}{2}x$. In **part (b)** the vast majority realised that integration was required and were able to complete this successfully although the minus sign was sometimes lost and $-\frac{72}{3}$ was not always cancelled down to give the coefficient as a whole number.

Question 4

Some weaker candidates were unsure as to how to start this question and it was omitted by 6%. Most candidates realised that the angles needed were all $\frac{\pi}{3}$, and used the required formulae for the arc length, and the area of a sector and a triangle correctly. A significant number used the circle formulae with 60° rather than $\frac{\pi}{3}$. It is important that candidates understand that the formulae for arc length and sector area are only valid if radians are used. In **part (b)** most candidates used one of the correct combinations of sector, segment, and triangle areas, usually a triangle added to 3 segments, but weaker candidates often used an incorrect one.

Question 5

Many fully correct answers were seen for this question. In **part (a)**, the most successful approach, was to set up the equation $\frac{\cos\theta}{\sin\theta} = \frac{2-\sin\theta}{\cos\theta}$. This approach should be encouraged for this type of question, rather than forming equations in *a* and *r* and attempting to eliminate these. A good number of candidates gave the answer as 30° and therefore were not awarded the final mark. Weaker candidates sometimes used the formulae for an arithmetic progression although this was less common than in previous series. In **part (b)** most were able to use the sum formulae correctly, although the formula for the tenth term was sometimes used or the value of *n* taken as 9 and some struggled to give the answer in the requested form. About 20% omitted **part (b)**.

Question 6

Part (a) of this question was done very well by candidates with most either differentiating and setting to 0 or completing the square. Weaker candidates sometimes set the equation equal to 0 instead or only gave the *x*-coordinate. **Parts (b)** and **(c)** were more challenging with many attempting to find the equation of the transformed curve in **part (b)** rather than the new point. There was confusion over the effect of the translation and the required order. Those candidates who worked in 2 stages, stretching the point and the curve, writing down their answers and then applying the translation, were usually more successful and this approach should be encouraged rather than trying to do both together. Some candidates gave an expression rather than the equation requested in **part (c)**; candidates would benefit from a clear understanding of the difference between these. 24% omitted **part (c)**.

Question 7

Part (a) was generally completed successfully but some candidates did not manage to find the 6 required terms or simply checked the expressions for one or two values. In **part (b)** the vast majority started with the left-hand side of the identity and were able to replace $\tan^2\theta$ with $\frac{\sin^2\theta}{\cos^2\theta}$ but were often then unable to simplify the algebraic fraction successfully. It was common to see $\frac{1}{\tan^2\theta-1}$ being replaced by $\frac{\cos^2\theta}{\sin^2\theta}-1$. Many candidates would benefit from extra time being spent on these algebraic techniques. In **part (c)** most candidates used the result from **part (b)**, but the majority did not use the result from **part (a)** and therefore either struggled to solve the resultant cubic or used their calculators and received no credit for doing so.

Question 8

In **part (a)** most candidates kept the function in the given completed square form and were able to make progress towards the inverse function and obtain the first two marks, but many did not realise that it was the negative root only that was required. Weaker candidates expanded the bracket and therefore were unable to make any progress. **Part (b)** proved challenging for many. It is important for candidates to realise that domains should always be expressed in terms of *x* and ranges in terms of *y*, or in this case f^{-1}. In **part (c)** most candidates were confident using the techniques for composite functions, but many did not show the required working for solving the resultant quadratic, using their calculators instead. Only the most able candidates appreciated that only one of the resulting solutions satisfied the limitations placed on the domain.

Question 9

In **part (a)** most candidates realised that in order to find the points of intersection, the two curves needed to be solved simultaneously. Unfortunately, many were then unable to make any progress dealing with $10x^{-\frac{1}{2}}$. It was common for $x^{-\frac{1}{2}}$ to be set to *y*, for example, and then $x^{\frac{1}{2}}$ to be put as y^2 or vice versa. Candidates who wrote $x^{-\frac{1}{2}}$ as $\frac{1}{x^{\frac{1}{2}}}$ and then substituted were usually more successful. Good candidates were able to do this confidently but many more would benefit from extra time spent consolidating their understanding of algebraic techniques. 22% of candidates missed out **part (b)** having been unable to find the required coordinates in **part (a)**. Those who did integrate and subtract were able to obtain 3 of the available marks even if no limits were known. The integration was very often completed successfully although a few candidates divided by the coefficient of the *x* term.

Question 10

Nearly all candidates realised the need to differentiate and set to 0, in order to find the stationary points and this was nearly always done correctly, including correct use of the chain rule. Many, however, in spite of the plural in the question, only found one *x* value. The vast majority then attempted to find the second differential to determine the nature of the stationary points, this was often done less successfully. Many candidates failed to appreciate the link between the two parts of the question and either missed out **part (b)** altogether (36%) or set up an inequality to be solved. Candidates would benefit from considering the number of marks available and therefore the amount of work expected or understanding the meaning of 'state' more fully. Few correct answers were seen for this part.

Question 11

Candidates who drew a diagram for this question were invariably more successful and this approach should always be encouraged. There was a considerable amount of confusion, in **parts (b)** and **(c)** in particular, about which line was the diameter, and a diagram would no doubt have helped this. In **part (a)** good candidates often confidently multiplied the two gradients together and set this expression equal to -1. This approach was more successful than trying to work with the negative reciprocal. Weaker candidates were unable to solve the resulting equation, sometimes mixing up the techniques for multiplying algebraic fractions with those for adding. A number tried to work with the equations of the lines and put in a lot of work but made no meaningful progress. In **part (b)** most candidates knew the formula for the equation of a circle although some did use the equation of a straight line. Many seemed to not know, however, how to find the centre or the radius. It was common for one of the given points or one of the midpoints of the lines *AB* or *BC* to be used as the centre. The diameter was also sometimes found rather than the radius. It should be noted that $\left(\sqrt{65}\right)^2$ was not accepted for the radius squared in the final answer. A few candidates formed 3 simultaneous equations in 3 unknowns using the 3 given points. In **part (c)**, again, a diagram would have helped candidates and avoided errors such as using the gradient of *BC* rather than *AC*.

MATHEMATICS

Paper 9709/13
Pure Mathematics 1 (13)

Key messages

Candidates generally seemed very confident with topics involving calculus and could confidently apply the appropriate techniques, even to quite complicated functions. Much good algebraic manipulation was seen but this is an area of improvement for many candidates. Candidates should also be encouraged to check that their answers match the requirements of the question e.g. use of significant figures, using appropriate units for angles and supplying a range of or multiple answers rather than a single answer when the question asks for 'answers'.

The previous four reports have each highlighted that the question paper contains a statement in the rubric on the front cover that 'no marks will be given for unsupported answers from a calculator.' Although this message has been taken on board by many candidates, there was still a significant minority for whom it had not. Clear working must always be shown to justify solutions. For quadratic equations, it would be necessary to show factorisation, use of the quadratic formula or completing the square as stated in the syllabus. Using calculators to solve equations and writing down only the solution is not sufficient. It is also insufficient to quote only the formula: candidates need to show values substituted into it. Factors must also always produce the coefficients of the quadratic when expanded. The use of calculators to evaluate definite integrals is not permitted and even when the algebraic result of the integration is reached it is expected that candidates will show the substitution of limits or boundary conditions.

General comments

The paper was generally found to be accessible for nearly all candidates. Very many good scripts were seen and candidates generally seemed to have sufficient time to finish the paper. Presentation of work was mostly good although **Question 10(c)** was an exception to this. Candidates should be encouraged to make their methods clear in unstructured questions of this type.

Comments on specific questions

Question 1

The integration and subsequent substitution required to find the equation of the line was well understood and, for most candidates, this proved to be a very successful start to the examination.

Question 2

(a) Setting the value of *x* to zero and solving the resulting quadratic equation caused few problems for most candidates. Those who chose to solve $(y-5)^2 = 31$ rather than forming a 3-term quadratic equation quickly reached the required answers.

(b) Use of the general circle equation form given in the first line of the question was evident in most answers, as was the use of $(0,5)$ and $\sqrt{31}$ as the centre and radius of the circle. Occasionally seen errors were the use of the diameter rather than the radius and a centre *x*-coordinate other than zero.

Question 3

(a) The required trigonometric relationships and algebraic techniques were used in most answers. With very few errors using brackets and clearing fractions, the required result was reached in most answers.

(b) Although many correct answers were seen, the quadratic equation in $\cos\theta$, despite having straightforward factors, was often solved using calculator functions. In these cases, full marks cannot be awarded. Most answers were given in radians as required by the stated range of θ and most candidates were able to find the additional solutions in the third and fourth quadrants.

Question 4

(a) The correct use of the binomial expansion was seen in nearly all scripts for both of **parts (i)** and **(ii)**. A few errors in dealing with powers and the negative term were noted and there were still candidates who produced every term of both expansions or forgot that '1' is a term in x^0.

(b) With many correct expansions in **part (a)** the terms in x^2 were often found and presented correctly from the product of the two expansions. As in **3(b)** the resulting quadratic was often solved without any evidence of correct working thus depriving candidates of a mark which they could easily have gained.

Question 5

(a) The relationship between the successive terms of a geometric progression was generally used successfully to find two values of the common ratio, which were then equated to obtain an equation in *p*. When this led to a quadratic equation, the two possible values of *p* were usually found correctly. Those who formed a cubic equation often struggled to reach and then solve a correct equation.

(b) The correct formula for the sum to infinity was almost always used. The appropriate value of *p* was nearly always used correctly to find the first term of the series but less frequently to find the common ratio. Here some candidates chose to use their fractional value of *p* as *r*, rather than finding it from one of their expressions for *r* from **part (a)**.

Question 6

Nearly all candidates were able to attempt this unstructured question and many found the required two coordinates and associated values of *c*. The favoured approach was to equate the line and curve equations and solve for *c* in the case when the discriminant of the resulting quadratic equation was zero. Nearly as popular was equating the gradient of the curve and line to obtain a relationship between *c* and *x* and using this with the 2 equations to find *c* or *x*. Some candidates used a mixture of both methods. Using the properties of quadratic equations with equal roots enabled some candidates to produce answers with a minimum of working. With at least one quadratic equation requiring a solution, whichever method was chosen, it was expected that a genuine solution coming from factorisation, the quadratic formula or completion of the square, would be shown for this solution.

Question 7

(a) This part was generally well answered. Many candidates were able to deduce the range without any working and presented it in an acceptable form. A few were not awarded the available mark through stating the range in terms of *x*.

(b) The procedure for finding the inverse of this type of function appeared to be well understood. Some were able to quote it directly from observation of the function. Some marks were not awarded through sign errors but the algebraic processes used were carried out efficiently. The range found in **part (a)** was usually quoted as the domain. It was clear to most candidates that the domain of the inverse should be defined in terms of *x*.

(c) This was the most successfully answered question part on the paper. The order of application of the functions and the necessary algebra were nearly always in evidence.

Question 8

(a) In previous papers candidates had shown themselves able to state the transformations caused by addition of the constants *a* and *b* to $y = \sin x$ to give $y = \sin a(x + b)$, so it was expected that given the transformations, albeit in diagrammatic form, the values of *a* and *b* could be stated. Only a minority of candidates were able to give correct values of both *a* and *b*. The value of *a* was often given as 2 rather than $\frac{1}{2}$ and values of *b* frequently omitted π. Alternative acceptable values of *b* other than $\frac{\pi}{3}$ were never seen.

(b) The connection between the transformations and the maximum point was only seen by a minority. Of those who did appreciate this, the similarity between the function in **part (a)** and this one were often not noticed, with $4(p-8)$ presented as the new *x*-coordinate. More correct answers were seen for the *y*-coordinate than the *x*-coordinate.

Question 9

(a) The requirement for differentiation and substitution of *x* = 4 was well understood and carried out successfully by the majority. Most went on to find the normal gradient correctly and deduce it's equation in the required form.

(b) The chain rule was often used correctly to find the numerical value of the rate of change of the *y*-coordinate. Although the value of $\frac{dy}{dy}$ was available from their **part (a)** working, some candidates did not use a numerical value and left their answer in terms of *x*, which led to neither mark being awarded in the process.

(c) Here again candidates usually realised the chain rule was required but the use of their normal gradient from **part (a)** was not always seen. The use of a negative value for the rate of decrease of the *y*-coordinate was a feature of the better answers.

Question 10

(a) Those candidates who realised that 'state the value…' implies an exact answer from a brief deduction were usually able to find and state the value of $A\hat{C}O$ exactly.
Some were uncertain which angle they were finding.

(b) Those candidates who realised an answer of the form $R = kr$ required a numerical value of k usually produced the correct answer from simple trigonometry. Those who used more convoluted methods also gained the available mark from correct numerical answers.

(c) A variety of correct methods were seen with the required area defined in terms of the areas of sectors, segments and triangles and with relevant length calculations shown in the working. Examiners were left to try and interpret the working of a significant number of candidates who gave very little explanation of their methods. Nearly all attempts featured a correct value for the area of sector *AOB*. Marks were available for intermediate numerical answers. Those candidates who chose to define all areas in terms of sine and/or cosine and algebraic expressions required an accurate final answer to gain these marks. Many realised that to gain a final answer correct to 3 decimal places, a greater degree of accuracy may be needed in intermediate calculations.

Overall this question proved to be a good discriminator with a wide range of marks being awarded.

Question 11

(a) As in **Question 9(a)**, the requirement to differentiate was appreciated by most candidates. Many carried out the integration correctly and then made a convincing argument for why $x = \frac{3}{2}$ at $\frac{dy}{dx} = 0$. Since the value of *x* was given this was essential.

(b) The requirement to find the area under the line *PQ* and the arc *PRQ* was seen by most candidates and the necessary *y*-coordinates of *P* and *Q* were usually found correctly. Those who used the trapezium area formula (or a combination of rectangle and triangle area formulae) were more successful than those who found the equation of the line *PQ* and then used this to find the area under the line *PQ*. The integration process required to find the area under the arc *PRQ* was well understood and often carried out successfully. Those who combined two integrals were more likely to produce sign errors but generally a good standard of algebraic and arithmetic manipulation was seen. To gain full marks candidates had to show that they had used the limits correctly in their integrals. Some showed no working and were not awarded marks as a result.

MATHEMATICS

Paper 9709/21
Pure Mathematics 2 (21)

Key messages

It is essential that candidates read each question carefully and ensure that they give their answer in the required form. Some candidates seemed to not appreciate the meaning of 'exact form' and could be directed to past mark schemes for guidance.

Use of the word 'Hence' implies that work done in the previous part or parts of the question is needed.

General comments

Candidates who had prepared well for the examination were usually fairly successful. However, many candidates did not gain any meaningful marks. There appeared to be no timing issues and candidates had plenty of space in which to write their responses.

Comments on specific questions

Question 1

Very few correct solutions were seen for this question. Most candidates made use of their calculator to work out the value of θ and then substituted this value to evaluate $\sin(\theta + 60^\circ)$ again using their calculator. This highlights the need for candidates to read the question carefully and ensure that their answer is given in the required form. In this case, an exact answer was required with no calculator use intended or needed. Candidates were expected to deduce $\cos\theta = \frac{\sqrt{5}}{3}$, or an exact equivalent, and make use of the compound angle formula for $\sin(\theta + 60^\circ)$.

Question 2

Again, few completely correct solutions were seen. Provided candidates recognised that they needed to differentiate a product, most responses were awarded some marks. Most errors involved incorrect coefficients of differentiated terms, which then meant that a correct evaluation was not possible.

Question 3

(a) The form of the expected answer no doubt helped some candidates with the integration of $\frac{4}{2x-5}$. Most errors involved the coefficient of $\ln(2x-5)$, but candidates were still able to obtain a method mark for the correct use of the laws of logarithms. Many correct solutions were seen.

(b) Most candidates realised that the integral involved e^{2x-5}, with any errors usually involving the coefficient of this term. Some candidates chose to evaluate their integral using their calculator rather than giving their answer in the required exact form.

Question 4

(a) Although most candidates were able to produce a correct sketch of both $y = |3x - 5|$ and $y = 2x + 7$, it was surprising that some candidates were unable to position the straight line correctly and in some cases sketched $y = |2x + 7|$. There were some candidates who seemed to be unfamiliar with the graphs of modulus functions. It was essential that the sketch showed or implied that there were two points of intersection.

(b) Many correct solutions were seen with most candidates choosing to use the method of squaring both sides of the equation in order to obtain the two solutions. Some of the candidates considering two separate linear equations made sign errors when attempting to solve an equation where the signs of 2*x* and 3*x* are different.

(c) Candidates should be aware that the use of the word 'Hence' implies that work done in the previous part of the question is needed. Many candidates attempted this question part as a new separate question, unrelated to anything previously done. Provided the positive value obtained in **part (b)** was used, most candidates who appreciated the use of the word 'Hence', were able to obtain at least one mark.

Question 5

(a) Most candidates were able to apply both the factor theorem and remainder theorem correctly and obtain a fully correct solution. Any errors were usually sign errors in either the simplification of the equations or the solution of the simultaneous equations.

(b) Most candidates made a reasonable attempt to obtain a quadratic factor initially, by dividing their $\mathrm{p}(x)$ by $(x+2)$ either by algebraic long division, synthetic division or observation. Subsequent factorisation of this quadratic factor was straightforward for those candidates with a correct $\mathrm{p}(x)$. Candidates were more unsure of the next step to take when attempting to solve $\mathrm{p}(3x) = 0$, so fully correct solutions were less common.

Question 6

(a) Provided candidates knew that $\operatorname{cosec}\theta = \frac{1}{\sin\theta}$, most solutions seen were fully correct, with the correct use of the double angle formula for $\sin 2\theta$, correct use of $\sin^2\theta = 1 - \cos^2\theta$ and correct simplification usually seen.

(b) It was essential that candidates make the connection between **parts (a)** and **(b)** and attempt to solve the equation $4\cos^2\theta - 6\cos\theta - 7 = 0$. Few completely correct solutions were seen as candidates attempted to factorise the equation rather than use the quadratic formula which was essential in this case. The fact that an answer in radians such that $-\pi < \theta < 0$ also deterred some candidates, some of whom seemed to find it difficult to work in radians and also deal with negative angles.

(c) Again, it was essential that candidates make the connection between **parts (a)** and **(c)** and attempt to find $\int \left(4 + 6\cos\theta - 4\cos^2\theta\right) \mathrm{d}\theta$. Some candidates attempted to integrate the expression as it was written in the question, with no success. It was essential that the use of the double angle formula $\cos^2\theta = \frac{1}{2}(\cos 2\theta + 1)$ was made. However, few candidates were able to do this correctly. Many candidates do not recognise the need for the use of double angle formulae when attempting to integrate terms involving the square of sine and cosine.

Question 7

(a) It was essential that candidates recognised that implicit differentiation was needed, otherwise no marks could be obtained. Many made a reasonable attempt using implicit differentiation and then chose to rearrange their equation to obtain an expression for $\frac{dy}{dx}$. Sometimes there were errors in this simplification which was unfortunate as no simplification was needed. A substitution of $\frac{dy}{dx} = 0$ and $x = p$ into the original differentiated expression gave $2e^{2p} - 18 = 0$, which could then be solved to obtain $p = \ln 3$.

(b) Substitutions of $p = \ln 3$ and $y = q$ into the original equation were essential as a first step. Numerical simplification and isolating the term of q^3 were the next steps which would then lead to the given result. Unfortunately, few candidates were able to show the given result.

(c) Candidates who chose to consider either the values of $\sqrt[3]{2 + 18\ln 3 - q} - q$ or $q - \sqrt[3]{2 + 18\ln 3 - q}$ when q took the values of 2.5 and 3.0 usually had the most success at this question part as they were able to clearly show a change of sign. It was essential that a comment that the change of sign implied a value in the given interval was made. Other methods usually required more explanation which was seldom sufficient and hence less successful.

(d) It was evident that some candidates were not confident with the subject of iterative formulae and for those that were more confident, they were not able to make appropriate use of their calculator to aid the iterative process. For the candidates that were fully confident, most were able to choose a suitable starting point and perform sufficient iterations at the required level of accuracy to obtain the correct value of 2.673.

MATHEMATICS

Paper 9709/22
Pure Mathematics 2 (22)

Key messages

It is essential that candidates read each question carefully and ensure that they give their answer in the required form. Some candidates seemed to not appreciate the meaning of 'exact form' and could be directed to past mark schemes for guidance.

Use of the word 'Hence' implies that work done in the previous part or parts of the question is needed.

General comments

Candidates who had prepared well for the examination were usually fairly successful. However, many candidates did not gain any meaningful marks. There appeared to be no timing issues and candidates had plenty of space in which to write their responses.

Comments on specific questions

Question 1

Most candidates made use of the remainder theorem and were able to obtain the correct value for *a*. Some candidates substituted in $x = 2$, rather than $x = -2$. Other errors include equating to zero rather than 33 and arithmetic errors in evaluation. Candidates attempting algebraic long division rarely had a remainder of the correct form.

Question 2

To make any progress with this question, it was necessary to make use of $\sec\theta = \frac{1}{\cos\theta}$ and the correct compound angle formula for $\cos(\theta - 60^\circ)$. The usual method was to then obtain an equation in $\tan\theta$ only, although other methods were acceptable. Many candidates used this method and were successful in obtaining the angle 76.1°, with fewer candidates obtaining the angle -103.9°. Many candidates appeared to have difficulties when dealing with negative angles. Most errors occurred when candidates did not use the compound formula and the statement of $\cos(\theta - 60^\circ) = \cos\theta - \cos 60^\circ$ was unfortunately a very common error.

Question 3

(a) Most candidates were able to differentiate to obtain the form $k\mathrm{e}^{-\frac{1}{2}x}$, and many correct answers of $\frac{3}{\mathrm{e}}$ were seen. However, some candidates chose to make use of their calculator and evaluate their answer, giving it in decimal form rather than exact form as required. This highlights the need for candidates to read the question carefully.

(b) Most candidates were able to integrate and obtain the form $ke^{-\frac{1}{2}x}$, with many obtaining $12e^{-\frac{1}{2}x}$. Correct substitution of the limits to obtain the area under the curve lead to $12-\frac{12}{e}$, which some candidates took to be the final answer. Fewer completely correct responses were seen as many candidates did not take into account that they needed to subtract the area under the curve from the area of a rectangle as shown in the diagram. A few candidates attempted to use the complete method of $\int_0^2 \left(6-6e^{-\frac{1}{2}x}\right) dx$ and were, in general, successful in obtaining the correct answer. Again, some candidates chose to make use of their calculator and evaluate their answer, giving it in decimal form rather than exact form as required. This highlights the need for candidates to read the question carefully.

Question 4

(a) Although most candidates were able to produce a correct sketch of both $y=|3-x|$ and $y=9-2x$, it was surprising that some candidates were unable to position the straight line correctly and in some cases sketched $y=|9-2x|$. There were some candidates who seemed to be unfamiliar with the graphs of modulus functions. It was essential that the sketch showed that there was only one point of intersection.

(b) Most candidates used a correct method and were able to obtain to obtain critical values of 4 and 6. The sketching of the graphs had been intended to alert candidates to the fact that there was only one point of intersection, so only one critical value was valid. The sketch would also help with deciding which of the critical values was the valid one. Very few candidates gave the correct answer of $x > 4$, with most obtaining inequalities involving both 4 and 6.

(c) Many candidates were able to obtain the correct critical value of 6.32 by manipulating logarithms correctly, with most going on to obtain the correct final answer of $x < 6.32$.

(d) Very few correct solutions were seen with many candidates appearing not to understand the demand of the question and made no use of their answers to **parts (a)** and **(b)**. The fact that many candidates had the critical value of 6 included in their answer to **part (a)** meant that a correct answer of 5 and 6 was seldom seen.

Question 5

(a) Most candidates made use of algebraic long division and clearly showed the correct quotient and the correct remainder. Some candidates chose to form an identity and obtain the correct quotient and the correct remainder. This method was equally successful.

(b) Whilst most candidates obtained full marks in **part (a)**, very few managed to obtain any marks in **part (b)**. The word 'Hence' implies that the work done in **part (a)** is needed in the solution for **part (b)**. Many candidates attempted, without success to integrate the original function. Other candidates did attempt to make use of **part (a)** but were unable to write the integrand in the correct form of $3x^2-4x-10+\frac{6}{2x+1}$, choosing instead to multiply by $\frac{6}{2x+1}$. Of the candidates that did obtain the correct integrand, most were able to obtain a correct answer in the required form.

Question 6

(a) Most candidates realised that they needed to find $\frac{dx}{dt}$ and $\frac{dy}{dt}$. Errors occurred in the attempted differentiation of both *x* and *y* with respect to *t*. Too many candidates did not realise that the product rule was needed to find $\frac{dy}{dt}$, with many obtaining $\frac{dx}{dt}=\frac{3}{2t-3}$. Whilst many candidates did find an expression for $\frac{dy}{dx}$, most did not realise that they needed to find the value of t at the point A. Very few completely correct solutions were seen.

(b) It was essential that the gradient function obtained in **part (a)** be equated to 12 and rearranged to obtain the given result. Most candidates did not have a correct expression for $\frac{dy}{dx}$ and so were unable to obtain the given result. This should have alerted candidates that errors may have been made and to check their work in **part (a)**. Very few correct solutions were seen.

(c) It was evident that some candidates where not confident with the subject of iterative formulae and for some of those that were, could not make appropriate use of their calculator to aid the iterative process. For the candidates that were fully confident, most performed sufficient iterations at the required level of accuracy to obtain the correct value of 4.96.

Question 7

(a) Unfortunately, some candidates misread this question and attempted to use $2x(\cot x + 3\tan x)$ rather than the correct $\sin 2x(\cot x + 3\tan x)$. These candidates were given method marks if they dealt with $\cot x$ and $\tan x$ correctly and also if they attempted to use an appropriate double angle formula. This highlights the need to ensure that the question is read carefully and then checked if it is clear that a given result cannot be obtained from the working. For the candidates that did not misread the question, most were able to make use of the double angle formula for $\sin 2x$ and together with the correct use of $\cot x$ and $\tan x$ go on to obtain the intermediate expression of $2\cos^2 x + 6\sin^2 x$ or equivalent. Many candidates were unable to make use of the double angle formulae to re-write their correct intermediate expression in the required form.

(b) Again, the word 'Hence' implied that work done in **part (a)** was needed in the solution of **part (b)**. Too many candidates made use of their calculator to find the value of $\cot\frac{\pi}{12} + 3\tan\frac{\pi}{12}$. Calculators will give a correct exact answer. However, this type of solution was regarded as being an unsupported answer from a calculator as per the rubric on the front of the examination paper. The work from **part (a)** was not being used, so solutions of this type did not gain any marks.

It was intended that $\cot\frac{\pi}{12} + 3\tan\frac{\pi}{12}$ be written as $\frac{4 - 2\cos\frac{\pi}{6}}{\sin\frac{\pi}{6}}$. This can be evaluated without the use of a calculator to give the exact form required.

(c) Most candidates were able to find the x-coordinate of A using differentiation. Fewer candidates were able to obtain the correct x-coordinate of B. It should be noted that the x-coordinate of both points could be obtained by use of symmetry and considering transformations of trigonometric curves. Most were also able to integrate the given integrand correctly. Few were able to obtain the given result correctly due to an incorrect upper limit.

MATHEMATICS

Paper 9709/23
Pure Mathematics 2 (23)

Key messages

It is essential that candidates read each question carefully and ensure that they give their answer in the required form. Some candidates seemed to not appreciate the meaning of 'exact form' and could be directed to past mark schemes for guidance.

Use of the word 'Hence' implies that work done in the previous part or parts of the question is needed.

General comments

Candidates who had prepared well for the examination were usually fairly successful. However, many candidates did not gain any meaningful marks. There appeared to be no timing issues and candidates had plenty of space in which to write their responses.

Comments on specific questions

Question 1

Very few correct solutions were seen for this question. Most candidates made use of their calculator to work out the value of θ and then substituted this value to evaluate $\sin(\theta + 60°)$ again using their calculator. This highlights the need for candidates to read the question carefully and ensure that their answer is given in the required form. In this case, an exact answer was required with no calculator use intended or needed. Candidates were expected to deduce $\cos\theta = \frac{\sqrt{5}}{3}$, or an exact equivalent, and make use of the compound angle formula for $\sin(\theta + 60°)$.

Question 2

Again, few completely correct solutions were seen. Provided candidates recognised that they needed to differentiate a product, most responses were awarded some marks. Most errors involved incorrect coefficients of differentiated terms, which then meant that a correct evaluation was not possible.

Question 3

(a) The form of the expected answer no doubt helped some candidates with the integration of $\frac{4}{2x-5}$. Most errors involved the coefficient of $\ln(2x-5)$, but candidates were still able to obtain a method mark for the correct use of the laws of logarithms. Many correct solutions were seen.

(b) Most candidates realised that the integral involved e^{2x-5}, with any errors usually involving the coefficient of this term. Some candidates chose to evaluate their integral using their calculator rather than giving their answer in the required exact form.

Question 4

(a) Although most candidates were able to produce a correct sketch of both $y = |3x - 5|$ and $y = 2x + 7$, it was surprising that some candidates were unable to position the straight line correctly and in some cases sketched $y = |2x + 7|$. There were some candidates who seemed to be unfamiliar with the graphs of modulus functions. It was essential that the sketch showed or implied that there were two points of intersection.

(b) Many correct solutions were seen with most candidates choosing to use the method of squaring both sides of the equation in order to obtain the two solutions. Some of the candidates considering two separate linear equations made sign errors when attempting to solve an equation where the signs of $2x$ and $3x$ are different.

(c) Candidates should be aware that the use of the word 'Hence' implies that work done in the previous part of the question is needed. Many candidates attempted this question part as a new separate question, unrelated to anything previously done. Provided the positive value obtained in **part (b)** was used, most candidates who appreciated the use of the word 'Hence', were able to obtain at least one mark.

Question 5

(a) Most candidates were able to apply both the factor theorem and remainder theorem correctly and obtain a fully correct solution. Any errors were usually sign errors in either the simplification of the equations or the solution of the simultaneous equations.

(b) Most candidates made a reasonable attempt to obtain a quadratic factor initially, by dividing their $\mathrm{p}(x)$ by $(x + 2)$ either by algebraic long division, synthetic division or observation. Subsequent factorisation of this quadratic factor was straightforward for those candidates with a correct $\mathrm{p}(x)$. Candidates were more unsure of the next step to take when attempting to solve $\mathrm{p}(3x) = 0$, so fully correct solutions were less common.

Question 6

(a) Provided candidates knew that $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$, most solutions seen were fully correct, with the correct use of the double angle formula for $\sin 2\theta$, correct use of $\sin^2\theta = 1 - \cos^2\theta$ and correct simplification usually seen.

(b) It was essential that candidates make the connection between **parts (a)** and **(b)** and attempt to solve the equation $4\cos^2\theta - 6\cos\theta - 7 = 0$. Few completely correct solutions were seen as candidates attempted to factorise the equation rather than use the quadratic formula which was essential in this case. The fact that an answer in radians such that $-\pi < \theta < 0$ also deterred some candidates, some of whom seemed to find it difficult to work in radians and also deal with negative angles.

(c) Again, it was essential that candidates make the connection between **parts (a)** and **(c)** and attempt to find $\int \left(4 + 6\cos\theta - 4\cos^2\theta\right) \mathrm{d}\theta$. Some candidates attempted to integrate the expression as it was written in the question, with no success. It was essential that the use of the double angle formula $\cos^2\theta = \frac{1}{2}(\cos 2\theta + 1)$ was made. However, few candidates were able to do this correctly. Many candidates do not recognise the need for the use of double angle formulae when attempting to integrate terms involving the square of sine and cosine.

Question 7

(a) It was essential that candidates recognised that implicit differentiation was needed, otherwise no marks could be obtained. Many made a reasonable attempt using implicit differentiation and then chose to rearrange their equation to obtain an expression for $\frac{dy}{dx}$. Sometimes there were errors in this simplification which was unfortunate as no simplification was needed. A substitution of $\frac{dy}{dx} = 0$ and $x = p$ into the original differentiated expression gave $2e^{2p} - 18 = 0$, which could then be solved to obtain $p = \ln 3$.

(b) Substitutions of $p = \ln 3$ and $y = q$ into the original equation were essential as a first step. Numerical simplification and isolating the term of q^3 were the next steps which would then lead to the given result. Unfortunately, few candidates were able to show the given result.

(c) Candidates who chose to consider either the values of $\sqrt[3]{2 + 18\ln 3 - q} - q$ or $q - \sqrt[3]{2 + 18\ln 3 - q}$ when q took the values of 2.5 and 3.0 usually had the most success at this question part as they were able to clearly show a change of sign. It was essential that a comment that the change of sign implied a value in the given interval was made. Other methods usually required more explanation which was seldom sufficient and hence less successful.

(d) It was evident that some candidates were not confident with the subject of iterative formulae and for those that were more confident, they were not able to make appropriate use of their calculator to aid the iterative process. For the candidates that were fully confident, most were able to choose a suitable starting point and perform sufficient iterations at the required level of accuracy to obtain the correct value of 2.673.

MATHEMATICS

Paper 9709/31
Pure Mathematics 3 (31)

Key messages

- It is important to prepare thoroughly for the whole specification.
- Take careful note of the rubric on the front of the paper and of any special instructions within each question.
- Set your working out clearly so that the examiners can follow your solution.
- Show all necessary working in full.

General comments

There was only a small number of candidates for this paper, several of whom demonstrated a good grounding in the key subject areas examined. Candidates were confident in using basic methods in calculus, algebra and trigonometry. The main issues in these questions were slips in processing the algebra and the arithmetic.

A high proportion of candidates did not attempt the questions on complex numbers (**Question 2** and **Question 4**). Almost 25 per cent of candidates did not offer a solution to all or part of the question on numerical methods (**Question 8**) and a similar number offered no response to the vector question (**Question 11**).

Comments on specific questions

Where numerical and other answers are given after the following comments on individual questions it should be understood that alternative forms are often acceptable and that the form given is not necessarily the only 'correct' answer.

Question 1

The candidates made a confident start, with the majority applying the quotient rule or the product rule correctly. Candidates who obtained the correct quadratic equation usually went on to obtain the correct answers. Most errors were due to slips in processing the algebra.

Question 2

Forty percent of the candidates offered no response to this question. The first two marks required candidates to identify the locus as the perpendicular bisector of the line joining the points representing 2i and –2 + i. The two points were often shown correctly, but a variety of lines were drawn. Those candidates who drew the correct lines usually identified the correct region.

Question 3

The question makes it clear that the coordinate pairs give values for x and $\ln y$, but many candidates treated them as values for x and y. Very few candidates followed the expected route of starting with the equation $\ln y = \ln a + x \ln b$. The majority tried to substitute values into $y = ab^x$, usually forgetting to start by finding the values of y.

Question 4

(a) Most of the candidates who attempted this question understood that they needed to multiply the numerator and denominator of u by $a+5\text{i}$. Several obtained the correct answer. Most errors were due to slips in the arithmetic. A few candidates did not use $\text{i}^2=-1$.

(b) Almost half of the candidates offered no solution. Of the remainder, several did not understand the implication of the value of arg u.

Question 5

(a) Most candidates started by expanding both sides of the given equation. There were several correct solutions. The majority of errors were due to using incorrect trigonometric formulae, or making slips in the algebra. The expansions of $\sin(x\pm\alpha)$ and $\cos(x\pm\alpha)$ are given in the formula booklet – candidates who are unsure should check the booklet before making up their own formulae.

(b) Candidates with a correct value for $\tan x$ usually obtained the correct answers. Candidates with an incorrect value for $\tan x$ were able to score one mark if they understood the periodicity of $\tan x$.

Question 6

(a) Many candidates were familiar with the method for differentiating a function expressed in parametric form and several fully correct solutions were seen.

(b) This part of the question was more challenging. Several candidates equated their gradient to -2, rather than to $\frac{1}{2}$.

Question 7

Most candidates recognised this as a question requiring separation of the variables. This process was often completed correctly, although the notation was not always fully correct, with missing integral signs and missing $\text{d}x$ or $\text{d}\theta$. The majority of candidates recognised at least one of the integrals. There were several slips in removing logarithms to obtain an expression for x^2 in terms of θ.

Question 8

(a) Several of the sketches drawn bore little resemblance to the expected functions. The sketch of $y=\sqrt{x}$ did not always pass through the origin, and the vertical intercept for $y=\text{e}^x-3$ was often incorrect. When candidates did have a pair of recognisable sketches, they did not always go on to mark the point of intersection or to comment on how their diagram demonstrated the required result.

(b) This is a familiar question, and candidates should be aware of the requirements. There were several possible approaches, leading a few candidates to make inappropriate comparisons, such as looking for a sign change when they should have been comparing their value with $\sqrt{x}$. Some candidates did not state which functions they were using, so it was often difficult to verify their results.

(c) A large proportion of candidates offered no response to this question. What was expected was to show that $x=\ln\left(3+\sqrt{x}\right)$ is a rearrangement of the original formula $\sqrt{x}=\text{e}^x-3$.

(d) Those candidates who attempted this question usually obtained the correct answer. The most common errors were due to not following the requested accuracy, or stopping the iterative process before they had sufficient values to confirm the root correct to 2 decimal places.

Question 9

(a) Most candidates attempted to use the product rule to differentiate the function. The most common error was to overlook the chain rule and to assume that $\frac{\mathrm{d}}{\mathrm{d}x}\mathrm{e}^{-\frac{1}{4}x^2} = \mathrm{e}^{-\frac{1}{4}x^2}$.

(b) The question offers one possible substitution. The alternatives are to substitute $u = -\frac{1}{4}x^2$, or to recognise the function as an exact derivative. Most candidates who offered a solution attempted to use the given substitution. The basic structure of the method was understood, but there were sign errors and factors of 2 did not always appear correctly.

Question 10

(a) Most candidates tackled the partial fractions with confidence and there were several fully correct solutions. A small minority made errors in dealing with the repeated factor, and there were some arithmetic errors.

(b) The expansion of f(x) proved to be more challenging. Attempts to use the binomial expansion were successful for a few candidates. The most common errors were in dealing with $(2+x)^{-1}$ and $(2+x)^{-2}$, especially when taking out the factor of 2. There were also several sign errors in the expansions.

(c) Several candidates offered no solution to this question. Some solutions did not reflect the factors in the denominator. Some solutions gave part, but not all of the solution, e.g. $0 < x < \frac{1}{2}$.

Question 11

(a) For those candidates who seemed to understand what they were doing, this proved to be very straight forward.

(b) Several candidates demonstrated a good understanding of how to use the scalar product. Some candidates used the vectors $\overrightarrow{PA}$ and $\overrightarrow{AM}$, rather than vectors $\overrightarrow{AP}$ and $\overrightarrow{AM}$, or vectors $\overrightarrow{PA}$ and $\overrightarrow{MA}$. A few obtained the correct value for the scalar product but then divided it by $\sqrt{5}+\sqrt{9}$.

(c) The small number of candidates who started by finding the vector from P to a general point on the line passing through O and M often went on to complete this task successfully. The majority of attempts did not demonstrate a correct strategy for finding the distance.

MATHEMATICS

Paper 9709/32
Pure Mathematics 3 (32)

Key messages

- Take careful note of the rubric on the front of the paper and of any special instructions within each question.
- Set working out clearly so that the examiners can follow solutions.
- Show all necessary working in full.
- Be aware of the contents of the formula booklet and always check a formula if uncertain.
- Prepare thoroughly so that you will know which methods you can apply in a particular situation.

General comments

There were some candidates who worked through the paper producing a succession of concise, elegant solutions. However this was not the case for the majority.

Formulae from the booklet supplied were often misquoted, such as the formula for the expansion of $\cos(A+B)$. Incorrect formulae such as $\cos x \cos 2x = \cos^2 x$ and $\int \cos^3 x \, dx = \frac{1}{4}\cos^4 x$ were common. When it came to the three questions on integration (**Question 5**, **Question 9(b)** and **Question 11**), many candidates did not seem to think through the forms that they could integrate and use that to inform their approach to the problem.

Some topics that are often very popular, such as remainder and factor theorems (**Question 3**) and numerical methods (**Question 6**), were affected by slips and lack of precision. Candidates are reminded to read the demands of the question carefully; regardless of what the question actually asked for, many candidates attempted to find the point of intersection of two lines in **Question 10**.

Comments on specific questions

Where numerical and other answers are given after the following comments on individual questions it should be understood that alternative forms are often acceptable and that the form given is not necessarily the only 'correct' answer.

Question 1

(a) The response to this task was mixed. There were some good clear sketches consisting of two straight line segments in approximately the correct position with the points of intersection and the axes marked. However, many sketches were not composed of straight lines or comprised more than two straight line segments. Of those that were the correct shape, the vertex was frequently on the vertical axis, in the middle of a quadrant or in the wrong place on the x axis. Some 'correct' sketches did not exist for $x < 0$ or for $y > 2$.

(b) Most candidates had a correct method for finding the critical values. Many candidates used a pair of linear equations. The most common error in this approach was a sign slip resulting in $1+3x = 4x+2$ or $1+3x = -4x-2$. For those candidates who started by forming a quadratic equation or inequality, there were many slips in the algebra. Some candidates were seemingly confused by having modulus signs on just one side of the equation, and only squared that side.

Having obtained two critical values, several candidates rejected one of them and only considered part of the solution. The incorrect answers '$x < \frac{1}{7}$ and $x > 3$' and '$\frac{1}{7} > x > 3$' were also common.

Question 2

The majority of candidates were aware of the need to use $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$ and there were several fully correct solutions. Errors in using the chain rule to differentiate *x* and *y* were common. The factor $\ln t$ was often missing from the derivative of *x*. The derivative of *y* often had the wrong sign or $-2t$ was missing. The question asked for the gradient where $t = e$ but the substitution was often missing or incomplete. Some correct solutions lost the minus sign in the final stages of the working.

A small number of candidates attempted to eliminate *t* before differentiating. Some demonstrated a correct method, but there were algebraic slips. Having obtained a function in *x* and *y* only, no candidate succeeded in finding the gradient.

Question 3

Many candidates gave a fully correct response to this question. Most of those who found $p\left(\frac{1}{2}\right)$ and $p(-1)$, and used these correctly, went on to obtain the correct answer. Those candidates who attempted to use long division had a more difficult task and frequently made slips in their working. There were several slips in simplifying correct equations as well as between the correct equations and the answer.

A minority of candidates overlooked the 12 and treated $(x+1)$ as a factor. A significant number of candidates set $x + 1 = 12$ and then p(11) = 0.

Question 4

(a) Many candidates produced fully correct diagrams. The majority of circles looked like circles, but a small number looked more like squares with rounded corners. Those candidates who used different scales on their axes did not always draw an ellipse. A minority of candidates confused $\text{Re}\, z = 3$ with $\text{Im}\, z = 3$. Having produced a correct diagram, most candidates selected the correct region. A small number of candidates had shading in multiple regions and did not indicate a clear conclusion.

(b) Many candidates appeared to be confused between modulus and argument. Several candidates were clearly attempting to find a distance. Only a minority of candidates identified the correct angle and split it into two correct parts. Many solutions involved only a single angle, and some of the two-part solutions used incorrect ratios.

A small number of candidates took the algebraic approach and tried to find the point where the tangent intersects the circle. Very few of these solutions were completed.

Some candidates attempted to read coordinates from their sketches. This method does not provide sufficient accuracy (other than by chance), so it was not accepted.

Question 5

It was common for candidates to make little progress with this question. Many did not realise that the improper algebraic fraction needed to be dealt with before integration. Some were able to gain partial marks by splitting the integrand into $\frac{x^2}{x^2+4} + \frac{x}{x^2+4}$ and successfully integrating the second fraction. A very small number of candidates did then go on to split the first fraction correctly.

Incorrect methods for splitting the fraction were often seen, with many candidates incorrectly assuming that the integral of a product is equal to the product of the integrals. Several candidates spent a long time on attempts at integration by parts. Some candidates realised that they needed to split the fraction but then assumed that $x^2 + 4 = (x-2)(x+2)$.

Some candidates split the fraction correctly and went no further. Some candidates rearranged the fraction correctly but then used incorrect methods such as claiming that $\int \frac{x-4}{x^2+4}\,dx = \frac{1}{2}\ln\left(x^2+4\right)$. Some candidates who were successful in splitting the fraction did go on to achieve full marks for the question.

The alternative method of using integration by substitution was rarely seen, although there were some very elegant and completely correct solutions using this method.

Question 6

(a) Candidates found this question challenging and many of the diagrams were unrecognisable. Candidates should be familiar with the graphs of $y = \cos x$ and $y = \tan x$ but few seemed to take an analytical approach to sketching the required curves in the given interval. Many candidates who did obtain a pair of acceptable sketches did not mention or indicate the root.

(b) The sign change method was the most popular approach here. A minority of candidates obtained correct values for the function(s) chosen but did not then complete the argument. A small number of candidates obtained incorrect values due to working in degrees.

(c) The majority of candidates were confident in applying the iterative formula, with most choosing an initial value within the given interval. Some were extremely cautious and produced many iterates despite the convergence being relatively rapid this time. Some candidates did not show sufficient cycles of the iteration to confirm the correct root, and some drew an incorrect conclusion from a correct list of iterates. Most candidates did work to the required degree of accuracy. A small number worked in degrees rather than radians.

Question 7

(a) The majority of candidates followed the instruction in the question and started with a correct expansion of $\cos(2\theta + \theta)$. A minority of candidates had a sign error in the expansion despite this being one of the formulae given in the booklet. Most candidates used the correct double angle formulae, and many obtained the given answer correctly. However, it was common to see slips in algebra, with signs appearing and disappearing and powers appearing incorrectly in some lines of working.

(b) Some candidates scored no marks because they started with $\cos(2\theta + \theta) = \cos 2\theta + \cos\theta$. The majority of candidates used the identity from **part (a)**. A common error was to overlook the term $\cos\theta\cos 2\theta$ or to 'simplify' this term to give $\cos^2 2\theta$.

Many candidates did obtain a correct cubic equation in $\cos\theta$, and went on to solve a correct quadratic in $\cos\theta$. The most common error occurred when candidates divided by $\cos\theta$ and overlooked the solution from this factor. Additional incorrect values were often included in the final answer.

Question 8

(a) The two alternative approaches shown in the mark scheme were both used. It was common for candidates to score the first two marks and make no further progress. Relatively few candidates recognised the need to equate real and imaginary parts. Those that did often made the algebra more complicated than necessary (frequently multiplying through by $a^2 + 4$), so they were awarded the third M1 without managing to go on to obtain the given result. Fully correct solutions were quite rare.

(b) Candidates who had already obtained a correct expression for λ in terms of a in **part (a)** found it straightforward to score all three marks.

It was possible to score full marks in this part of the question without completing **part (a)** – having solved the quadratic to obtain the values of a, these could be substituted into the original equation.

Several candidates obtained correct values for a but then made no further progress.

Question 9

(a) Many candidates made a confident start to this question. Most started by using the product rule. The derivative of $\cos 2x$ caused several errors, with sign slips and the 2 being lost (both within the function and as a factor). Those candidates who had demonstrated their intended method were still able to score the method mark. The 2 also caused difficulties later when $2\sin x \sin 2x$ often became $2\sin^2 x \cos x$. Several candidates who obtained a correct equation in one trig function then gave their final answer to more than 2 decimal places or in degrees.

Of particular concern in this question was the large number of candidates who rewrote $\cos x \cos 2x - 2\sin x \sin 2x$ as $\cos^2 2x - 2\sin^2 2x$ or $\cos^2 x - 2\sin^2 x$.

(b) Many candidates invested a lot time and space to find the limits for the integral. This was something that earned no marks without some correct integration. It was also something that should have been a simple deduction from consideration of $\cos 2x = 0$.

Very few candidates made any progress with the integration, and over a quarter of all candidates offered no solution. Several started by using the double angle formula. Very few of those who obtained $\int 2\sin x \cos^2 x - \sin x \, dx$ recognised this as something that they could integrate directly. Several used the incorrect result $\int \cos^3 x \, dx = \frac{1}{4}\cos^4 x$. Stronger candidates went on to use a substitution, which was often successful.

Many candidates attempted to use integration by parts, but very few applied the method twice to obtain a correct expression for $\int \sin x \cos 2x \, dx$. This was not surprising as this approach would likely only be familiar to students of Further Mathematics. A small number of correct solutions using the factor formulae were seen, but again this is a topic from Further Mathematics.

Question 10

(a) Many candidates made an incorrect start by attempting to find the point of intersection of the two lines.

The majority of those who attempted to use the scalar product did select the correct vectors to work with and there were several fully correct solutions. The most common errors were due to algebraic errors in simplifying the working. For example, $\sqrt{20+c^2}$ often became $\sqrt{20+c}$ or $\sqrt{20}+c$ or $\sqrt{20}c$. Some candidates formed an equation using 60, rather than $\cos 60$. Some candidates obtained a correct quadratic in c but did not solve it.

(b) Many candidates did not know how to start this question, with nearly 30 per cent offering no response.

Some were able to write down the vector from the given point to a general point on the line but then were not able to proceed further. Others stopped after finding just the vector $\begin{pmatrix} -3 \\ 1 \\ -5 \end{pmatrix}$.

Candidates who went on to form the relevant scalar product usually went to gain full marks. A few candidates did not set out all of their methods clearly, for example, missing out the calculation of the magnitude of the vector. This meant the final two marks could not be awarded because in a question with a given answer, all the steps in the working must be shown.

Question 11

(a) This question proved a real challenge for the majority of candidates. Candidates made errors from the outset, with a significant minority not attempting to separate the variables. In separating the variables, the minus sign was sometimes lost and many candidates obtained an incorrect form such as $-\int \frac{1}{x^2}\,dx = \int y^2 + y\,dy$. After separating the variables correctly, the common error was to write $\frac{1}{y^2+y} = \frac{1}{y^2} + \frac{1}{y}$. Candidates who recognised the need to use partial fractions usually completed this correctly. Some of the minority of candidates who completed the integration correctly, and found the constant of integration, went no further than an expression for $\frac{y}{y+1}$.

(b) A small number of correct answers were seen. Many incorrect answers were still given in terms of x. Almost half of all candidates did not offer a solution.

MATHEMATICS

Paper 9709/33
Pure Mathematics 3 (33)

Key messages

Candidates need to:

(i) be able to include the extra detail required in an **Answer Given** question, such as **Question 7a**, where steps of working needs to be justified. For example, crossing the 4 out because it does not appear in the answer without any indication of differentiation, is not sufficient.

(ii) think about the method prior to commencing the question. For example, whether to opt for solving as a quadratic equation, using the formula or equating real and imaginary parts, as in **Question 4**

(iii) ensure that they do not present two solutions as an answer, as in **Question 7b**, with the tangent parallel to the *x*-axis being taken as both the numerator and denominator needing to be zero.

General comments

The standard of work produced was of a high quality, with numerous candidates scoring in excess of 65 marks. Unfortunately, the earlier short questions appeared to be as challenging for some as the later questions.

However, it was good to see candidates appeared to have read recent reports and were trying to improve their presentation skills. However care needs to be taken when presenting an Argand Diagram (see later).

Most candidates demonstrated sufficient syllabus coverage and were able to tackle most of the questions. Candidates scored highly in **Question 6a** to **Question 9a**, however **Question 5**, **Question 9c**, **Question 11a** and **Question 11c** caused serious issues. In addition the actual concept of what was been requested in **Question 11a** did not appear to be understood.

Comments on specific questions

Question 1

Here squaring was not the best approach, since the correct solution of a quadratic equation was required for something that could be immediately written down from either two linear inequalities or two linear equations. Some candidates believed $|2^{x+1} - 2| < 0.5$ meant $2^{x+1} - 2 < \pm 0.5$, others $2^{x+1} - 2 < 0.5$ or $2^{x+1} - 2 > -0.5$, although this final result did not prevent some candidates from correcting to obtain the correct solution once the critical values were found. In addition a large number of candidates either used the incorrect relationship $\log(a + b) = \log a + \log b$ or failed to combine the critical values into the single region $-0.415 < x < 0.322$, and when they did it was sometimes not given to 3 significant figures as requested in the question. To gain full marks it was necessary to see $2^{x+1} = \frac{5}{2}$ result in $x + 1 = \log_2 \frac{5}{2}$, or some equivalent statement, and not just a value from the calculator with no working.

Question 2

This question was done reasonably well. However, an appreciable number of candidates seemed to be very unconfident with this topic and it was not uncommon to see no attempt at a diagram. In drawing an Argand diagram candidates should use a ruler and pencil, and they are also welcome to use other geometric equipment. They should also use a reasonable scale so that the diagram is not ridiculously small, use the same scale on both axes, label the scale on the axes, label geometric features such as right angles, equal distances, circle radius, etc., draw as accurately as possible and if using multiple shading, clearly indicate by labelling the final answer. Candidates who find equations of the boundaries of the appropriate regions need to state the coordinates where these boundaries cut the axes. Another error was to believe, mistakenly, that the locus associated with the first inequality was a circle, candidates should be encouraged to be familiar with the relevant different loci. Some candidates mistakenly shaded the wrong side of the perpendicular; candidates should be encouraged to understand the meaning of the associated inequalities (e.g. distance from z to $1 - 2i$ is less than the distance from z to O).

Question 3

Candidates who used the remainder theorem to do this question generally found the going easy and usually wrote out, and simplified, the two required equations and solved them simultaneously. Small arithmetic errors (for example, in cubing $\frac{1}{2}$) or algebraic slips were not uncommon. Candidates should be encouraged to make the algebra easier by, for example, clearing the fractions or using signs of terms to help in cancellation. Some candidates knew that the remainder theorem was the correct methodology but did not apply it correctly; for example, setting the first equation to 38 rather than −38 or attempting to modify the theorem before using it. Candidates should be encouraged to learn a rule and then apply it in that form before rearranging. Many candidates attempted to use long division instead. This is certainly not a recommended approach, especially when there are unknowns in either the divisor or dividend. Some candidates did manage to carry out both divisions successfully and correctly, equate the remainders and hence found the required values but most made some sign error, or similar error, in their division. Candidates should be encouraged to think about the methodology they utilise; most questions are designed to test a single methodology and, generally speaking, the quantity of work is not huge if the correct methodology is applied and the result sensibly dealt with. There were a small number of candidates who did not appear to understand what is meant by 'remainder.'

Question 4

Again, there was a great premium here for selecting the most appropriate method: the quadratic equation formula. Candidates who did so generally managed to find the correct solution in some form. Candidates should be encouraged to write down the formula and apply it with the first step being pure substitution and then to show each step, simplifying along the way. This should help avoid more basic arithmetic errors. In particular, many candidates used $-i$, rather than $3 - i$, as c; writing parameters out explicitly might well prevent this. An appreciable number of candidates also thought that $-(-2)$ equalled -4 rather than 2. However, most candidates then managed to deal with the complex denominator by multiplying both top and bottom by the correct conjugate. Candidates are strongly advised to show this step explicitly and correctly before multiplying both out carefully; sign errors here were common (for example $-i + 9i = -8i$ even before squaring the i's). Candidates are advised to carry out basic calculations in the next step after they arise. For example, collecting like terms or replacing $\sqrt{36}$ with 6 or i^2 with -1 (explicitly, with brackets if necessary). Generally, candidates should just do one step at a time to avoid the possibility of errors; sign errors, for example, seem to be very common in this kind of question.

Candidates who replaced w with $x + iy$ and then expanded generally managed to derive two equations, with or without errors, but were usually then unable to make further progress. A small number realised that they needed to eliminate x and y but only a very small number of candidates managed to do this successfully to obtain the correct solution. A small number of candidates multiplied by $3 - i$ (or divided by $3 + i$). This did produce a slightly simpler equation, since the denominator in the quadratic formula was now real instead of being complex. However, since the original equation is not actually that complicated, little was achieved other than the likelihood of making an error somewhere along the line. Finally a small number of candidates did not present the final answer in the correct form (i.e. they wrote $\frac{3+4i}{5}$). Candidates should be encouraged to re-read the demand of the question before they move onto the next question.

Question 5

Many candidates approached this question quite well. The use of the quotient procedure was more common than the product rule. It was used correctly in many cases although some forgot to square the denominator and a small number had the signs reversed. Most commonly errors arose when differentiating $\exp(3x^2 - 1)$. Others tried to alter the power or divide by $6x$. The next stage required the differential to be equated to zero and solved. For those with the correct numerators the main issues were cancelling out the x and hence losing a solution or combining the $6x$ and the $2x$ to get $4x$ instead of $8x$. Some errors were made in evaluating the y values whilst correct decimal values were common, although the latter failed to score due to the demand of the question.

Question 6

(a) This question was very well done, with a variety of methods seen; some more concise than others. However, the majority logically and accurately progressed from one line to the next with no large jumps in their mathematics.

(b) The majority of candidates reached $\sin^2 x = 0.25$ and correctly rejected $\sin^2 x = -1$. Some did this via the formula, some by factorisation and some on the calculator. At this stage, the most common error was the omission of the negative root, $\sin x = -\frac{1}{2}$. Many candidates did not obtain all four values stopping at 30° and 150° or sometimes rejecting −30° but not replacing it with 330°.

Question 7

(a) The majority of candidates recognised this to be implicit differentiation and succeeded in progressing to the given answer. Some weaker candidates tried to work backwards from the given answer. It was also common to see a line of working with the presence of the constant 4 that should not have been there, or to see this term crossed out to meet the given answer, with no clear reasoning why it was being removed. With a given answer candidates should have clearly shown $\frac{\mathrm{d}}{\mathrm{d}x}(4) = 0$, not $4\frac{\mathrm{d}}{\mathrm{d}x} = 0$ or 4 cancelled and replaced by 0.

(b) Most candidates knew it was the numerator which needed to be set to zero, but here, as elsewhere in the paper, marks were not awarded when the $x = 0$ solution was ignored. Another common error was to have sign errors in the equation $3x^2 + 6x = 0$. However, many did succeed in acquiring the values of 0 and –2 and went on to complete the question successfully. Unfortunately, some decided that one or more of their correct values of $x = 0$ or $y = 0$ or the negative values of $y = -3$ or -4 should be rejected.

Question 8

Most candidates recognised this as a differential equation and made an effort to separate variables. The candidates who did well recognised $\frac{1}{\cos^2 x}$ was equal to $\sec^2 x$. If they did not go down this path they were often limited to a maximum of 2 marks for dealing correctly with the exponential term. Those candidates who scored the first 4 marks often went on to get full marks. Some coefficient errors were made on both sides (e.g. omitting $\frac{1}{3}$ or using -4 instead of $-\frac{1}{4}$) and these resulted in accuracy marks not being awarded. A few candidates made the error of using $\tan y$ instead of $\tan 3y$. The vast majority of candidates included a constant and were able to gain a method mark for its evaluation provided they had both an $e^{\pm x}$ term and a $\tan by$ term.

Question 9

(a) The majority of candidates scored full marks. A few used A instead of $Ax + B$, while a number used $Ax + B$ and C as numerators but paired them with the wrong denominators.

(b) Candidates were less successful in this part – there were a number of candidates who wrote nothing or were awarded no marks. Accuracy in particular was a major problem. Common errors were using x instead of $\frac{x}{2}$, using $\frac{x}{2}$ instead of $-\frac{x}{2}$ or making a sign error in the x^3 term even when $\left(-\frac{x}{2}\right)^3$ appeared in the working, extracting 2 instead of $\frac{1}{2}$ or losing the 2 altogether when combining expansions with their $Ax + B$ and C or introducing a sign error in $Ax + B$ when combining with their expansion.

(c) This part proved very challenging. Some of those who took the correct approach included $|x| < 2$ and failed to select the single inequality required.

Question 10

(a) The product rule was usually correctly applied but a few instances were seen of $y = x\cos 2x$ being clearly treated as $y = uv'$ instead of $y = uv$, so the appearance of $\frac{1}{2}\sin 2x$ for v instead of $-2\sin 2x$ for v' was definitely from wrong work. Some candidates only found the gradient of the tangent instead of its equation and a few attempted the equation of the normal.

(b) Many good solutions were seen and most used the correct limits. A small number of candidates failed to show any working for the integration by parts and simply wrote down $\frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x$ followed by the answer. A few gave a decimal answer instead of the exact value as demanded by the question.

Question 11

(a) This was a simple question that many seemed to find difficult. Often little or no attempt was made or candidates simply wrote down a general point and proceeded no further. Some of those who found $\sqrt{6}$ did nothing further with it or used it incorrectly.

(b) A correct method was often seen but many used **m** = ... or ***AB*** = ... instead of **r** = ... so failed to achieve the accuracy mark. A few used **r** = **a** + λ**b** instead of **r** = **a** + λ**d**, and **r** = **d** + λ**a** was also seen.

(c) Candidates seemed to be on more familiar ground in this part with marks for expressing **l** or **m** in component form and solving for λ or μ being gained. However, the possibility of the lines being parallel was frequently ignored or the justification given incorrect. There were often transposition errors in the vectors being used (e.g. minus signs often went missing) so the value(s) of λ and/or μ were not always correct.

MATHEMATICS

Paper 9709/41
Mechanics

Key messages

- When answering questions involving any system of forces, a well annotated force diagram could help candidates to make sure they include all relevant terms when forming either an equilibrium situation or a Newton's second law equation. Such a diagram would have been particularly useful here in **Questions 2** and **5**.

- Non-exact numerical answers are required correct to 3 significant figures (or correct to 1 decimal places for angles in degrees), as stated on the question paper. Candidates would be advised to carry out all working to at least 4 significant figures if a final answer is required to 3 significant figures.

- In questions such as **Question 7** in this paper, where velocity is given as a non-linear function of time, then calculus must be used as it is not possible to apply the equations of constant acceleration.

General comments

The requests were well answered by many candidates. Candidates at all levels were able to show their knowledge of the subject. **Questions 2**, **3(a)** and **6(a)** were found to be the most accessible questions, whilst **Questions 5**, **6(b)** and **7** proved to be the most challenging.

In **Questions 1** and **6(b)**, the angles were given exactly as $\tan\alpha = \frac{3}{4}$ and $\sin^{-1} 0.08$ respectively. There is no need to evaluate the angle in these cases and in doing so can often lead to inexact answers. Any approximation of the angle can lead to a loss of accuracy.

One of the rubric points on the front cover of the question paper was to take $g = 10$ and it was noted that almost all candidates followed this instruction.

Comments on specific questions

Question 1

This question was answered well by most candidates. The most common error was to assume that the gravitational potential energy was given by $1.6\times10\times x$ (where x was the required distance the particle moves up the plane before coming to instantaneous rest), rather than the correct $1.6\times10\times x\times\frac{3}{5}$.

Question 2

The most common error in this question was to assume that the tensions were equal in both strings. There were also signs and trigonometric errors seen when forming the two required simultaneous equations for the two different tensions. Many responses correctly resolved horizontally and vertically to obtain the two equations $T_1\cos35 = T_2\cos40$ and $T_1\sin35 + T_2\sin40 = 2.4g$, but then struggled to then solve for T_1 and T_2 accurately. Many responses gave answers which were not accurate to the required 3 significant figures.

Question 3

(a) Almost all responses gave the correct answer for the distance covered by the bus in the first 8 seconds.

(b) It is important that candidates read the question carefully, as many in this part correctly calculated the acceleration of the bus as -0.42 (using the constant acceleration formulae for the two different stages of the bus's motion) but did not state the value of *a* correctly as 0.42. As the question had explicitly defined the value of *a* as a deceleration, only 0.42 could be accepted for full credit.

(c) Responses to this part were varied. Several responses left this part blank or did not appreciate that the area of the four distinct regions below the four-line segments of the velocity-time graph needed to be found. Those candidates who had correctly calculated the acceleration (or deceleration) in part **(b)** were usually successful in this part. Those candidates who could recall and correctly apply the formula for the area of a trapezium were generally more successful than those who needed to split this area into a rectangle and a triangle.

Question 4

(a) This part was answered extremely well, with most responses deriving the given speed using either a Newton's second law/constant acceleration approach or an energy approach. Utilising the former of these two led to generally more successful responses as, in the case of the energy approach, several candidates either did not include the initial kinetic energy or did not correctly calculate the work done by friction.

(b) Most responses appreciated the need to apply the conservation of linear momentum to find the velocity of the combined particle *R* after collision to then be able to calculate the loss of kinetic energy due to the collision. It was quite common to see the initial projection speed of *P* used instead of the given answer in **part (a)**. Furthermore, it was also common to see responses using an incorrect mass for the total momentum after the collision (with the most common values being either 6 or 2 rather than the correct 8). Of those responses which found the speed of *R* after collision as $7.5\sqrt{3}$, most then went on to correctly calculate the required loss of kinetic energy.

(c) Very few responses scored full marks here and many candidates left this part blank. Not all appreciated that the acceleration of *R* was simply $-0.4 \times g$ and that the initial speed of *R* had been found in the previous part. It was slightly more common to see an energy approach being taken when it could be argued that applying the constant acceleration formula $v^2 = u^2 + 2as$ yielded the correct distance with minimal required working.

Question 5

This was a question many candidates found challenging, and it was clear that many were unsure where to begin with this type of extended response. It was frequently unclear in candidate's responses both how, and to what, different mechanical principles were being applied. It would be extremely beneficial if candidates stated which principle (e.g. resolving parallel to the plane) and to which particle (e.g. *A*) they were applying said principle.

Common errors included assuming that the friction was the same for both particles, sign/trigonometric errors and assuming that the system was in motion (even though the question clearly stated the system was in limiting equilibrium). It is preferable to apply the required mechanical principles on each particle individually by setting up two equations (e.g. $1.6g\sin 50 - T - F_B = 0$ and $T - F_A = 1.2g\sin 40 = 0$) rather than trying to write down a single system equation. For many candidates, the only marks achieved in this question were from correctly calculating the two expressions for the frictional forces acting on *A* and *B*.

Finally, candidates are reminded that in questions such as this that it is advisable to be working with exact trigonometric expressions (e.g. $\cos 40$) until the end of the problem. Candidates should only use their calculators at the final stage to work out the numerical value of μ from an exact expression (so in this case, finding μ as $\frac{1.6g\sin 50 - 1.2g\sin 40}{1.2g\cos 40 + 1.6g\cos 50}$ before then evaluating this as 0.233).

Question 6

(a) (i) This part was answered extremely well. The most common error was to give the power developed by the engine of the car in W rather than the required kW.

(ii) This part was also answered well, with many responses correctly using their value from **part (a)(i)** to find the required driving force of the car, before correctly applying Newton's second law to find the required acceleration. The most common errors were either to misinterpret that power had increased by 9 kW and assume instead that the power was now 9 kW, or to not include the resistive force in their application of Newton's second law.

(b) Many responses appreciated the need to apply Newton's second law, but a number did not include the correct weight component as $1300 \times g \times 0.08$. Some incorrectly included the acceleration from **part (a)(ii)**. Many responses which stated a correct equation for the required constant speed were unable to solve this equation algebraically.

Question 7

The responses to this final question were mixed. Examiners reported seeing several perfect responses, as well as responses which were either blank or where little in the way of progress was made. Most responses appreciated the need to use calculus rather than the constant acceleration formulae, so many were able to achieve credit for correctly integrating the three given expressions for the velocity. It was surprising to note how many responses incorrectly believed that these expressions needed differentiating as well.

It was relatively common to see candidates correctly interacting with the fact there was no instantaneous change in velocity at $t = 8$ to correctly calculate k. However, it was only the strongest responses which set the final velocity expression equal to zero (once k had been found) to work out the time when the particle comes to rest at X. Of those that did work out this time correctly, most went on to find the distance OX as 262.2.

MATHEMATICS

Paper 9709/42
Mechanics (42)

Key messages

- In previous reports, it has been mentioned that when answering questions involving any system of forces, a well annotated force diagram could help candidates to include all relevant terms when forming either an equilibrium situation or a Newton's Law equation. Force diagrams are now being seen more often than in previous occasions when this examination has been sat.
- In questions such as **Question 7** on this paper, where velocity is given as a non-linear function of time, then calculus must be used and it is not possible to apply the equations of constant acceleration.
- Non-exact numerical answers are required correct to 3 significant figures or angles correct to 1 decimal place as stated on the front of the question paper. Candidates are strongly advised to carry out all working to at least 4 significant figures if a final answer is required to 3 significant figures.

General comments

The requests were well answered by many candidates. Candidates at all levels were able to show their knowledge of the subject. **Questions 1, 4(a)** and **5(a)** were found to be the most accessible questions, whilst **Questions 5(c), 6(b)** and **7(c)** proved to be the most challenging.

In **Question 6(a)**, the angle α was given exactly as $\sin \alpha = 0.02$. There is no need to evaluate the angle in situations like this as it can lead to a loss of accuracy.

One of the rubric points on the front cover of the question paper was to take $g = 10$ and it was noted that almost all candidates followed this instruction.

Comments on specific questions

Question 1

This was a well answered question by the majority of candidates. All but a small minority of candidates used an energy approach to answer the request, finding the two kinetic energies and the loss in potential energy. The only errors occurring with this method was for the wrong signs being used when forming a work-energy equation. Of those who attempted a constant acceleration approach, the majority wrongly used 1.6 m as a vertical height, for which very little credit could be awarded.

Question 2

The majority of candidates resolved vertically and horizontally to form one equation in T and another equation in both T and m. The horizontal equation in T could then be solved directly to find T, however a significant number of candidates eliminated T between the two equations to solve for m first.

Question 3

Two distinct approaches were made in answering this question, with both being equally successful. The energy method required four terms in a single equation, consisting of the change in kinetic energy, the loss in potential energy, the work done against the 120 N force and the given 200 J. The errors seen included omitting one of the required energies, having sign errors when forming the work-energy equation, or the work done against the 120 N force not using a component of the force. The Newton's second law method required four terms to find the acceleration of the block and then using this in a constant acceleration formula. The common errors seen with this method arose from the omission of the 40 N force resisting the motion or using

200 J as the resisting force. The majority of those who had a correct method to find an acceleration went on to use the constant acceleration correctly.

Question 4

(a) This proved to be a straightforward question for well-prepared candidates.

(b) Very few candidates were awarded full marks for this question. Most candidates were able to use Newton's second law correctly to find the acceleration. A minority of candidates had a resultant force comprising of friction only, so arrived at a seemingly correct acceleration but from incorrect work. Once an acceleration was found from a correct method, the majority only found the distance travelled in the first 3 seconds and did not consider the difference of this and the distance travelled in the first 2 seconds.

Question 5

(a) This question was answered well by the majority of candidates from either using constant acceleration or an energy approach. The only minor errors seen was the acceleration due to gravity being used as $-10\,\text{m}\,\text{s}^{-2}$ rather than $10\,\text{m}\,\text{s}^{-2}$.

(b) The request was to show that the velocity of A is $4.5\,\text{m}\,\text{s}^{-1}$ downwards immediately after the collision. Many responses which used the principle of conservation of momentum concluded with $4.5\,\text{m}\,\text{s}^{-1}$ or $-4.5\,\text{m}\,\text{s}^{-1}$ but either did not specify a direction or did not give the speed as $4.5\,\text{m}\,\text{s}^{-1}$.

(c) Finding the time for B to reach the ground was achieved by a significant number of candidates. The time required for A to reach the ground involved solving the correct three term quadratic by using $s = ut + \frac{1}{2}at^2$. Errors arose from inconsistent signs being used with u and a. Additionally, some candidates used an initial speed of $15\,\text{m}\,\text{s}^{-1}$ rather than the speed of $4.5\,\text{m}\,\text{s}^{-1}$.

Question 6

(a) This was a typical connected particles question. To answer the request, two Newton's second law equations are needed from the three possible equations. There were a significant number of good responses with the main errors coming from omitting the acceleration due to gravity in the weight components, or not including either the resistances or weight components.

(b) Only a minority of candidates made a valid attempt at this question. Most found the driving force as $\frac{4500000}{30} = 150000$ N. Then, the method was to use Newton's second law on either the system or on both the engine and the coach. Many just used an equation on the engine, assuming that the tension in the coupling was the same as in **part (a)**, with candidates not appreciating that the two parts are different situations, so that earlier values cannot be used in this new situation.

Question 7

(a) Many candidates differentiated the given expression to get an acceleration function, then equated the acceleration to 0 and solved for t. Errors commonly seen included assigning p and q with the wrong values, incorrectly factorising the quadratic expression, or an incorrect use of the quadratic formula.

(b) Those who found p and q correctly in **part (a)** usually found the corresponding velocities. The main difficulty candidates experienced was in the sketching of the graph. It was uncommon to see a correct cubic graph drawn, with many candidates having straight line segments rather than a curve. Some graphs did not show the curve for $14 \leqslant t \leqslant 15$, and some graphs showed the velocity to be 0 when $t = 15$.

(c) This question required candidates to realise that the velocity is positive from $t = 0$ to $t = 14$ and negative from $t = 14$ to $t = 15$, as should have been indicated on their graph. Many candidates integrated the expression for v correctly. However, many found the distance travelled from $t = 0$ to $t = 14$ by splitting this into two time intervals and finding both the distance travelled from $t = 0$ to $t = 2$

and the distance travelled from $t = 2$ to $t = 14$, which was not needed. A significant number of candidates evaluated the integral between $t = 0$ to $t = 15$, which give a displacement rather than a distance. Where candidates had a graph in **part (b)** consisting of line segments, the majority used area of triangles and trapezia in an erroneous attempt to find the distance travelled.

MATHEMATICS

Paper 9709/43
Mechanics (43)

Key messages

- Non-exact numerical answers are required correct to three significant figures as stated on the question paper. Candidates would be advised to carry out all working to at least four significant figures if a final answer is required to three significant figures.
- When answering questions involving forces in equilibrium or Newton's second law or an energy approach, a complete force diagram can be helpful to ensure that all relevant terms are included in the equations formed, e.g. **Question 2**, **Question 3(b),** and **Question 7(a), (b)**.
- In questions with a given answer, where equations have to be solved in order to find that answer, candidates should be advised not to use an equation solver on their calculator since this does not explicitly show the given answer, e.g. **Question 7(a)**.

General comments

This paper provided the opportunity for candidates at all levels to show their knowledge of the subject, whilst providing challenge for the stronger candidates. Much work of a very high standard was seen. **Question 3, Question 4(a), Question 5(a), 5(b), Question 6(a),** and **Question 7(a)** were found to be the most accessible questions, whilst **Question 2(b)** and **Question 7(b)** were found to be the most challenging.

In **Question 4(b)**, the angle was given exactly as $\sin \alpha = 0.09$. There is no need to evaluate the angle in problems such as this as any approximation of the angle can lead to a loss of accuracy in the answer. On this occasion, most candidates did use 0.09 rather than evaluating the angle. In **Question 7**, a significant number of candidates showed all of their working in terms of θ, rather than 30° and 20° for **parts (a)** and **(b)** respectively. Since **part (a)** was a 'show that' question, candidates who did this could not score full marks, and if they did not give a more accurate answer than the required one, it was then not clear whether they have used the correct value of θ.

Comments on specific questions

Question 1

Please note that due to a series-specific issue with **Question 1**, full marks have been awarded to all candidates for this question to make sure that no candidates were disadvantaged.

Question 2

(a) Please note that due to a series-specific issue with **Question 2(a)**, full marks have been awarded to all candidates for this question to make sure that no candidates were disadvantaged.

(b) This question proved to be rather demanding, with relatively few fully correct responses. The majority tried to find the acceleration and then use Newton's second law, with rather less using an approach based on energy. Those who used this first approach often used the velocity which they had worked out in **part (a)**, not realising that this was the initial velocity rather than the final velocity. Some used a distance of 4 rather than 0.04, forgetting to convert the distance to metres. Having found the acceleration, many then used a mass either of 1.2 or of 0.004, rather than the total mass of 1.204 kg. Similar errors were seen by those using an energy method, but in addition some candidates tried to use the velocity before the impact and the velocity after the impact in the same equation, which was of course incorrect.

Question 3

(a) In this question, candidates had to draw a force diagram. Most diagrams were clear and correct, but a number of diagrams included an extra force down the plane, often labelled as '*D*' or '*F*' or 'Driving force'. Some had the normal reaction going vertically upward or missed it out altogether.

(b) This part was very well answered, with most candidates finding the correct value of the coefficient of friction. Almost all candidates did try to use $F = \mu R$, but a few had a sign error in their equation and a few omitted either the weight term or, more rarely, the acceleration term.

(c) Most candidates found the correct value of the speed of the block, although some gave an answer of $3.8\,\text{m s}^{-1}$ rather than $3.79\,\text{m s}^{-1}$, which could not be accepted.

Question 4

(a) Almost all candidates found the correct answer. A few candidates did not convert to kW and so were only awarded one mark.

(b) Although there were many correct responses, there was also a variety of errors. The most common of these was either to miss out the work done by the engine altogether or to multiply or divide the power by 24 or 32 (the initial and final speeds). Most candidates did find the correct values of the kinetic energy, although some found $\frac{1}{2}\times1600\times(32-24)^2$. Many also found the potential energy, although this caused more difficulty as some omitted either the distance, the acceleration due to gravity or 0.09. Some candidates tried to use a method based on Newton's second law, but there was no credit available for such a method as the question states that the resistance is not constant.

Question 5

(a) Over 90 per cent of the candidature gained full marks on this part, usually by resolving forces. Of those who did not, many found two correct equations $T\sin\theta = 32$ and $T\cos\theta = 80$, but then could not solve them. Others had $\sin\theta$ and $\cos\theta$ the wrong way around.

(b) As with **part (a)**, the majority of candidates gained full marks. The main error in this part was again to have $\sin\theta$ and $\cos\theta$ the wrong way around. In both parts, some candidates tried to use the sine and/or the cosine rule, usually successfully.

Question 6

(a) Almost all candidates realised that it was necessary to integrate in order to find the velocity. Most then correctly found the constant of integration, with only a few forgetting to try to find it or, more rarely, making an error in finding it.

(b) This part was found to be more challenging than **part (a)**, although again almost all candidates integrated the velocity to find the displacement. A large minority simply integrated between 0 and 12, despite in **part (a)** having found the correct times at which the particle is at instantaneous rest. Many did realise that it was necessary to use three sets of limits, but a few of these made errors in their calculations, or did not use the absolute values of the integrals.

Question 7

(a) Most candidates correctly showed that the value of the coefficient of friction was 1.01. Using the system equation was slightly more popular than using the separate equations for particles *A* and *B*, although either approach was perfectly valid. Of those few candidates who did not gain full credit for this part, an incorrect sign in one of their equations was the most common error. Some candidates thought that they had to include acceleration, although if they later stated that this was zero, then they could gain full credit. Some candidates had all their working in terms of $\theta°$ rather than 30°, and sometimes m_A and m_B instead of 2.4 kg and 3.3 kg. In all questions, particularly those with a given answer, candidates should be advised that if values are given (such as in this case θ

= 30, m_A = 2.4 kg and m_B = 3.3 kg) they should use these values rather than working with the variables.

(b) This part was found more demanding than **part (a)**. It could either be solved using Newton's second law, or an energy method or a combination of both. For those using Newton's second law to solve the problem, this part was much more difficult than **part (a)** because there was a change in acceleration when particle *B* hit the ground. This required a new calculation using Newton's law, and some candidates were confused about how to allow for this. The initial equations also caused problems for those who thought that the tension in this part was, like the tension in **part (a)**, simply 33 N. This led to an incorrect system equation where the candidates had 2.4 for the acceleration instead of 5.7. A small minority of candidates solved the question using an energy method for either the first or the second part of the question. Of those who used an energy method for one part of the question, most used Newton's second law for the other part. Very few candidates used an energy method for both parts of the question.

Some candidates incorrectly wrote that θ was 30 degrees (from **part (a)**). If they completed everything else correctly, they were able to score the majority of marks using any non-zero value for friction. Many made sign errors using Newton's second law for particle *B*. If this was the only error they made, again they could score the majority of marks available.

To find the displacement of particle *A* after particle *B* had reached the ground, many candidates either used their acceleration from the first part of the question, or they used acceleration equal to gravity. If candidates did not use Newton's second law with the correct number of terms to find the acceleration of particle *A* after particle *B* had reached the ground, they could not score any of the final marks available. Many candidates made sign errors when using Newton's second law and a common response was making one sign error to give $a = -6.07$.

Candidates who used an energy method for the first part of the question to find the velocity often made errors by only considering the potential energy lost by either particle *A* or particle *B*. Most candidates who used an energy method for the second part of the question (after *B* had reached the ground), successfully found the kinetic energy of particle *A* after particle *B* had reached the ground. When attempting to write the energy equation to find displacement, many candidates did not multiply the friction by the distance.

MATHEMATICS

Paper 9709/51
Probability and Statistics 1 (51)

Key messages

Candidates should be aware of the need to communicate their method clearly. Simply stating values often does not provide sufficient evidence of the calculation undertaken, especially when there are errors earlier in the solution. The use of algebra to communicate processes is anticipated at this level and enables candidates to review their method effectively. When errors are corrected, candidates would be well advised to cross through and replace the incorrect working. It is extremely difficult to interpret accurately terms that are overwritten.

Candidates should state only non-exact answers to 3 significant figures. Exact answers should be stated exactly. It is important that candidates realise the need to work to at least 4 significant figures throughout to justify a 3 significant figures value. Many candidates rounded prematurely in calculations and lost accuracy in their solutions. It is an inefficient use of time to convert an exact fractional value to an inexact decimal equivalent, as there is no requirement for probabilities to be stated as a decimal.

The interpretation of success criteria is an essential skill for this component. Candidates would be well advised to include this within their preparation.

General comments

Although many well-structured responses were seen, some candidates made it difficult to follow their thinking within their solution by not using the response space in a clear manner. The best solutions often included some simple notation to clarify the process that was being used.

Candidates demonstrated a lower accuracy in constructing or reading from a statistical diagram than anticipated. The use of simple sketches and diagrams can help to clarify both context and information provided. These were often seen in successful solutions.

Some questions, for example **Question 2**, require candidates to calculate an initial value which is then used to complete the rest of the question. Candidates should be reminded of the requirement to show supporting work, as if their initial value is inaccurate, credit can be given for clear methods presented. It is good practice to read the question again after completion to ensure all demands of the task have been fulfilled and the answer is reasonable.

Sufficient time seems to have been available for candidates to complete all the work they were able to, although some candidates may not have managed their time effectively. A few candidates did not appear to have prepared well for some topics, in particular applying the normal approximation in different contexts. Many good solutions were seen for **Questions 4** and **6**. The context in **Questions 2** and **3** was found to be challenging for many.

Comments on specific questions

Question 1

Candidates were expected to use the cumulative frequency graph provided accurately. At this level, some evidence of use, such as drawing an appropriate line or indicating the required value on the axis, is required.

(a) The majority of candidates appreciated the need to consider the 30th and 90th values, but not all candidates read the graph with sufficient accuracy to obtain an acceptable difference for the interquartile range. A significant number of candidates calculated 90 – 30 and stated an interquartile range of 60, or simply stated the 60th value.

(b) Many good solutions recognised that 35% taking longer is equivalent to 65% taking less than the required time. Some good candidates calculated 35% of 120 children and then subtracted 42 from the total. Weaker solutions did not interpret 'took longer than *T* seconds' and simply used the graph to find the value for 42 candidates.

Some candidates found the vertical scale challenging to use accurately, assuming that each small grid square represented 1 child.

Question 2

The context of this question was found challenging by many candidates due to the necessity to determine the probability of obtaining a score of 8 or more to use in each question part. Solutions with a possibility or outcome space initially, or a simple listing of possible successful outcomes, were generally more successful. Candidates who did not find the required initial probability could gain significant credit if they showed their working clearly. As mentioned in previous reports, there was uncertainty whether a score of 8 should be included in the success criteria 'a score of 8 or more'.

(a) Many candidates recognised that a 'geometric distribution' style calculation was required and calculated $p(1 - p)^4$. A common misconception was assuming that the required score could be on any throw, and the binomial distribution used. A surprising number of solutions did not calculate the complement of their probability accurately.

(b) Nearly 25% of candidates made no attempt at this routine geometric approximation question, but many excellent solutions were seen. Most successful candidates used the simple approach of calculating the probability of success on the first, second, third and fourth throw and then adding their answers. Good candidates used the more efficient approach of finding the complement of the probability for 'not succeeding for 4 throws'. Again, there was inconsistency in interpreting the success criteria 'no more than 4 throws'.

Some weaker solutions showed an attempt to use the binominal approximation without success. Several candidates mixed approaches and introduced the binomial coefficient while using a geometric distribution method.

(c) This standard binomial distribution question was not attempted by a significant number of candidates. Good solutions stated clearly the required terms and then evaluated accurately without intermediate values being shown. A number of solutions were seen with intermediate values stated to less than 4 significant figures, which produced an inaccurate final probability. Some candidates seemed to work to just 2 significant figures only.

Again, the success criteria was not interpreted accurately, with the inclusion of P(3) seen regularly.

Question 3

Although **3(a)** and **3(c)** were relatively standard normal distribution questions, many candidates did not appear to have prepared well for this topic. Good solutions included simple sketches of the normal distribution, often with shading to identify the required probability area. The identification of key words in the stem of the question was noted on some successful solutions. This technique can help clarify the information that is presented.

(a) The standardisation required was carried out successfully by many candidates. Weaker solutions used a continuity correction, despite the data being continuous. The normal distribution tables were not used accurately in some solutions. Most candidates who obtained a probability that an egg was classified as small did not convert their answer to a percentage as required by the question. It is good practice to read the question again once a solution has been completed to ensure that all requirements have been fulfilled.

(b) Many candidates found this question challenging and no attempt was seen from many. Again, good solutions included a diagram, which clarified that the probability for a large egg was 1 – P(small or medium egg). Good solutions equated the standardisation formula with the *z*-value for their probability and gave a clear algebraic solution. Weaker solutions equated the standardisation formula with the probability.

(c) A significant number of candidates made little or no progress towards a solution. The best solutions showed clear calculations for the estimates of the mean and variance, used a diagram to help clarify the continuity correction required in the standardisation formula as the number of eggs is discrete, and used tables accurately to determine the probability required. Again, diagrams helped clarify the required probability area. Weaker solutions assumed that the mean and standard deviation were the same as in **3(a)** and **3(b)** or attempted to use the binomial distribution to calculate a probability. As the question required an approximation to be used, no credit could be awarded for the use of a calculator generated binomial distribution probability.

Question 4

(a) The best solutions included clear calculations to obtain the frequency density, scales that were appropriate to enable accurate plotting of values as required and axes that were clearly labelled. Candidates should be aware that units are required in labels when stated in the data. A significant number of candidates simply drew a frequency graph, which may have been a misinterpretation of the requirements of a histogram. As in **Question 1**, many candidates did not use appropriate accuracy when drawing the histogram, which should include the use of a ruler to draw straight lines. A common misconception was that the upper boundary of each class should be plotted at the higher value stated.

(b) Many candidates made a good start to this question, identifying the mid-values of each class and then using the frequencies to calculate the mean. Better responses often indicated that they were including the accurate upper and lower boundaries of the classes rather than simply the values provided in the data table. Weaker responses used the class width rather than the mid-value, divided by the number of classes rather than the total number of athletes, or added the frequencies given inaccurately and obtaining a total that was not 150.

Finding the standard deviation was considerably more challenging, with few fully accurate solutions seen. Although the variance formula was seen used, common errors were squaring the frequency rather than the mid-value, squaring their *mf* values from the mean calculation, not subtracting the square of the mean or not subtracting the mean at all.

Candidates who presented their work in a table were often successful, as there was a clear structure to their thinking and the layout could reduce errors when transferring values.

The weakest solutions simply found the mean of the class widths or ignored the frequencies and found the mean of a value within each class.

Question 5

Many candidates found this probability question challenging. Successful solutions often contained clear lists of scenarios that fulfilled success criteria for **5(b)** and **5(c)**. At this level, clear communication of the logic being applied in a solution is often required to support the values being used by the candidates, which by themselves may not convey that an appropriate method has been used.

(a) Good solutions showed the formation of an equation from the probability distribution table using the property that the probabilities would sum to 1, and showed a clear algebraic approach to solving to obtain the given answer. Weaker solutions simply substituted the given value into the table and found that the total was 1. This is not sufficient at this level for a 'show' question, where mathematical rigour is required.

(b) Good solutions often presented a list of possible scenarios which fulfilled the given condition in a clearly labelled table. The unevaluated probability calculations were linked with each scenario and a final total obtained. Weaker solutions assumed that the red spinner was unbiased and did not use the probabilities from **5(a)**. Common errors were not calculating the probability for a total of 3 or finding the probability for one way of scoring a total of 4 and multiplying by 3.

(c) Candidates found this question the most challenging on the paper. A small number of candidates appreciated the significance of the phrase 'given that' in the question and used conditional probability techniques. Again, candidates who initially listed possible scenarios were generally more successful. The recognition that the red spinner needed to be odd was a feature of many good solutions. Again, an organised list of scenarios assisted candidates to identify all possible outcomes. Some candidates did not respond to the change of context and continued with the 'sum' values from **5(b)**. Weaker candidates again assumed that the red spinner was unbiased.

Question 6

The context of this question appeared accessible for many candidates. Candidates who used simple diagrams to illustrate the logic of their solution were often successful.

(a) Many good candidates considered Rajid (*R*) and Sue (*S*) sitting at one table and filling the remaining 2 seats at that table from the remaining 5 friends as Tan (*T*) had to be seated on the other table. This total was then doubled as *R* and *S* could sit on either Table *X* or Table *Y*. Weaker solutions did not take the seating restrictions into account and either simply sat the 8 friends anywhere or chose 2 of the original 8 to sit with *R* and *S*.

(b) Most candidates realised that 'arrangements' in the question indicated that the actual seating position of the friends now needed to be considered, unlike **6(a)**. Good solutions often started by working out the number of ways that *R* and *S* could sit on one side of Table *X*, then multiplying by 2 to allow for seating on the other side before either arranging how the remaining 6 friends could sit, or choosing 2 friends to sit with *R* and *S* and then arranging the remainder on Table *Y*. Weaker solutions often failed to consider that *R* and *S* could sit in different orders or on the different sides of the table.

(c) Successful solutions appreciated that in the new context, *R* and *S* could stand next to each other in 2 ways but that then they should be considered as a single unit. A simple diagram with the 6 remaining friends in a line with an indication of the 5 places that *R* and *S* could stand often supported correct answers, which multiplied the 6! ways the 6 remaining friends could be arranged by the 5 places that *R* and *S* could stand by 2 for the ways that *R* and *S* could arrange themselves. A significant number of solutions omitted this factor 2.

MATHEMATICS

Paper 9709/52
Probability and Statistics 1 (52)

Key messages

Candidates should be aware of the need to communicate their method clearly. Simply stating values often does not provide sufficient evidence of the calculation undertaken, especially when there are errors earlier in the solution. The use of algebra to communicate processes is anticipated at this level and enables candidates to review their method effectively. When errors are corrected, candidates would be well advised to cross through and replace the incorrect working. It is extremely difficult to interpret accurately terms that are overwritten.

Candidates should state only non-exact answers to 3 significant figures. Exact answers should be stated exactly. It is important that candidates realise the need to work to at least 4 significant figures throughout to justify a 3 significant figures value. Many candidates rounded prematurely in calculations and lost accuracy in their solutions. It is an inefficient use of time to convert an exact fractional value to an inexact decimal equivalent, as there is no requirement for probabilities to be stated as a decimal.

The interpretation of success criteria is an essential skill for this component. Candidates would be well advised to include this within their preparation.

General comments

Although many well-structured responses were seen, some candidates made it difficult to follow their thinking within their solution by not using the response space in a clear manner. The best solutions often included some simple notation to clarify the process that was being used.

The use of simple sketches and diagrams can help to clarify both context and information provided. These were often seen in successful solutions. Candidates should be aware that cumulative frequency graphs are constructed with a curve, and that this needs to be reasonably accurately drawn. It was encouraging that the labelling of the statistical diagram has improved.

Sufficient time seems to have been available for candidates to complete all the work they were able to, although some candidates may not have managed their time effectively. A few candidates did not appear to have prepared well for some topics, in particular when more than one technique was required within a solution. Many good solutions were seen for **Questions 1** and **3**. The context in **Questions 5** and **6** was found to be challenging for many.

Comments on specific questions

Question 1

(a) This question was answered well by most candidates. The best solutions clearly stated the probability condition that was being used to form the two equations and presented a clear algebraic method for solving the simultaneous equations. Weaker solutions usually only formed the equation for $E(X) = 1.1$ and then attempted to solve.

It was unexpected at this level that several candidates found values for p or r which were negative or greater than 1, which were then used in **part (b)**. A small number of candidates simply stated values for p and r with no supporting work.

(b) Candidates who had obtained values for *p* and *r* usually answered this question well. The best solutions stated the variance formula with values substituted and then evaluated accurately. Some candidates calculated E(X) even though it was stated in **Question 1(a)**. Weaker candidates failed to subtract $(E(X))^2$ or simply subtracted E(X). A small number of candidates used the variance formula incorrectly and calculated either p^2x or $(px)^2$ for each term.

Because of the context, the final answer should not be presented as an improper fraction.

Question 2

This relatively straightforward question using the geometric and binomial distribution was found challenging by many candidates. Several candidates did not apply the context accurately and assumed that the probability of scoring a 5 on the 5-sided spinner was $\frac{1}{6}$.

The failure to accurately interpret the success criteria ‘fewer than’ and ‘more than’ was seen in a significant number of solutions.

(a) Most candidates recognised that the geometric distribution was appropriate to use in this part. The best solutions used the $1 - p^6$ approach and stated the final answer accurately, as it was exact. The longer alternative approach with P(1, 2, 3, 4, 5, 6) was often used by weaker candidates successfully, however arithmetical errors were more frequent. The weakest solutions simply calculated the probability that a 5 was scored on the 6th or 7th spin. A common misinterpretation was to include 7 spins in the success criteria.

A small number of candidates attempted the binomial distribution method but were often unsuccessful due to the omission of at least one term.

(b) Most candidates recognised that the binomial distribution was appropriate, however many candidates continued to use the geometric distribution. Good solutions identified the outcomes that fulfilled the success criteria, stated each required binomial term and evaluated accurately. There is no expectation that the probability of individual terms is stated in a solution. The more successful approach was to calculate P(5, 6, 7) as candidates who attempted 1 – P(0, 1, 2, 3, 4) – P(8, 9, 10) often omitted P(0) or found the difference between 1 – P(0, 1, 2, 3, 4) and 1 – P(8, 9, 10).

A common misconception was to give the final answer to 3 decimal places rather than 3 significant figures. Again, many candidates struggled to identify which terms fulfilled the criteria ‘more than 4 times but fewer than 8 times’.

Question 3

This standard question was well answered by many candidates. The best solutions provided clear calculations for the mean and variance, substituted accurately into the standardisation formula recognising that a continuity correction was required as the data was discrete and used a simple diagram of the normal distribution to clarify the probability area required. Weaker responses either omitted the continuity correction or misinterpreted the boundary that was required.

There were several candidates who attempted to use a binomial distribution which, although could be evaluated, would gain no credit as this is not a suitable approximation to use.

As in **Question 2(b)**, stating the final answer to 3 decimal places rather than 3 significant figures was a common misconception.

Question 4

There was an improvement in the overall quality of the stem-and-leaf diagrams. Candidates should be aware that at this level graphical statistical representations are expected to be accurate, so rulers should be used appropriately.

(a) The best solutions reordered the data before attempting to draw the diagram, had consistent spacing between terms and good vertical alignment, included the team as a title for the leaves and had a key which used different values for Aces and Jets while including the units (cm) in the descriptor. Weaker solutions had poor vertical alignment, especially between values in rows 16 and 18. The omission of the key totally, or the units from the key were the most common errors. A small number of candidates used separate keys for each team, which will gain no credit.

The failure to place the Aces on the left-hand side of the diagram was more common than anticipated.

Candidates should be aware that if an error is made in entering a data value, the correction will need to be within the accuracy expected, so it is good practice to complete the diagram in pencil so errors can be erased rather than crossed through.

(b) Many good solutions were noted. These clearly identified the median, stated the upper and lower quartile values before finding the difference accurately. The standard approach anticipated for the quartiles is to find mid-values between the upper/lower value and the median, rather than using a formulaic process. Some weak solutions recognised that the 3rd and 9th terms were the quartiles, but then calculated 9 – 3 = 6th term and restated the median value.

(c) This part proved to be challenging for many. The question specified that the comment needed to compare the spread of heights between the two teams, so any response that included comparisons of central tendency gained no credit as it was unclear as to whether the candidate could differentiate between spread and central tendency.

At this level, comments need to be in context, so simply comparing data values is insufficient as reference to 'heights' was an expectation. Candidates would be well advised to reread their answer, as some good responses were spoiled by errors with the team names, e.g., 'the heights of Jets players are more varied than the heights of players in Jets'. If data values are used to support the comment, for example using the ranges to show that the heights of the Jets are more varied, then the values do need to be correct.

Question 5

This question involved several different normal distribution processes of increasing challenge. **Question 5(c)** was often omitted and little progress was made in many solutions. Again, solutions which included simple diagrams were often more successful.

(a) (i) This question was answered well by the majority of candidates. The best solutions showed the standardisation formula with the values substituted, without a continuity correction as the data was continuous, evaluated accurately and used the tables appropriately to find the z-value. Many good solutions included a simple diagram to clarify the probability area required and justified the decision to subtract the table value from 1. Errors noted in weaker solutions included a continuity correction, not identifying the correct probability area and either giving a final answer of 0.345 or simply not converting the calculated z-value to a probability.

(a) (ii) Candidates found this question more challenging. Good solutions recognised that the 40% in the question was equivalent to the probability of having the height greater than h cm, so used the tables appropriately to find the linked z-value, formed an equation using the standardisation formula and showed clear algebraic processes to solve the equation. Weaker solutions used the 'negative' z-value. Here, a simple diagram would have helped clarify the probability area and so the required z-value. Many solutions simply equated the given probability or treated 0.4 as a z-value and used a linked probability value. Neither of these approaches could gain any credit.

A significant number of candidates did not follow the question demand that the final answer be correct to 2 decimal places and so spoiled a good solution. Candidates are well advised to read the question again when they have completed their solution to ensure their answer is reasonable within the context and meets the requirements of the question.

(b) Many found this part challenging and either omitted it or made little progress. The most successful solutions used clear and accurate notation for the normal distribution throughout, recognising that an equation in one variable was required. In these solutions, the statement that 'X is positive' was correctly interpreted that 0 was the value being standardised and substituted appropriately, usually substituting $\frac{2}{3}\mu$ for the denominator of the formula. A clear process for cancelling μ in the expression was shown to find a *z*-value, which was then accurately converted to a probability using tables. A common error was to show the variable *X* in the standardisation formula, which inhibited progress. Weaker candidates often used some of the information from **part (a)** here, and again made limited progress.

Question 6

This probability question was found challenging by many candidates. A significant number of solutions failed to identify that the first probability required in the context was the choice of bags, and this was omitted in both the tree diagram and the calculations for the remainder of the question. There was less consistency than previously seen regarding interpreting the context, with many solutions either assuming that the marbles were replaced or failed to add the red marble after the first selection.

(a) Some very good tree diagrams were seen. These often used the full answer space to ensure clarity, had the outcomes at each stage clearly identified and vertically aligned, used a ruler to draw the branches and wrote the probabilities clearly on the relevant branch. Weaker solutions often had a third, or even more, marble selection. Several tree diagrams were seen where not all the branches were labelled or the probabilities were not included for each branch.

Many candidates produced separate tree diagrams for each bag, with more able candidates using the probability of selecting the bag and marble for the first selection.

Some candidates included an 'amendment' branch between the first and second selection where they added the red marble to the bag.

One misconception was to link the probability of selecting a bag with the contents and used $\frac{10}{15}$ and $\frac{5}{15}$.

(b) Candidates who had a complete tree diagram in **part (a)** were usually able to find the required probability. A number of candidates who had omitted the initial 'bag selection' on their tree diagram used the probability appropriately in their calculations. Good solutions clearly identified the scenarios that were being considered for each probability, stated the necessary calculation and evaluated accurately. Weaker solutions often omitted one of the possible scenarios, or incorrectly calculated P(Y, B, B) as $\frac{1}{2}\times\frac{1}{5}$ rather than $\frac{1}{2}\times\frac{1}{5}\times 0$.

(c) Very few correct solutions were presented here, and the part was not attempted by nearly a fifth of candidates. The majority of candidates did not seem to identify that a conditional probability was required, with many solutions simply calculating the probability of selecting different colour marbles from bag *Y*. Again, those with a complete tree diagram in **part (a)** were often successful, with the required outcomes identified at the end of the diagram. The best solutions recognised that the required denominator was the complement of **part (b)**, but many attempted to calculate the value from first principles. A common error was to calculate $\frac{P(Y)\times P(B)}{P(B)}$ with an answer of $\frac{1}{2}$.

Question 7

Most candidates used an appropriate selection or arrangement approach to this question. The context was often found challenging in **parts (b)** and **(c)**, and solutions with simple 'diagrams' illustrating possible scenarios were often more successful.

(a) Solutions which considered the arrangement of the consonants first were often more successful. Good solutions had a simple diagram to support the logic of the approach used. Most recognised the need to divide by 2! × 2! to remove the effect of the two As and Ds. Weaker solutions often omitted one of these 2!.

A number of candidates, having found the expected number of arrangements, subtracted this value from the total number of arrangements which limited the marks awarded. Here, the final answer is accepted as the full solution provided by the candidate, even if the anticipated value is obtained within their working.

A significant number of candidates attempted the alternative approach of calculating the number of arrangements that the letters could be arranged in and then subtracting the arrangements which did not meet the success criteria. In the given context, this was a much more complex and involved process and few candidates were able to gain credit.

(b) This question was often found to be more accessible to candidates as the given conditions are more standard. Again, solutions with simple diagrams to support the logic of the process used were often more successful. Most candidates used the two standard approaches with equal success. Almost all solutions recognised that the As being placed at the ends reduced the number of letters that were effectively being arranged. The common error in the difference approach was not determining appropriately when the effect of the 2 Ds needed to be removed, either not dividing the total number of arrangements for the 7 remaining letters or dividing the arrangements when the Ds are together by 2!. Common errors in the alternative approach of arranging the 5 letters available and then inserting the Ds in different spaces was to multiply by 2!, for the arrangements that the 2 As could be placed, or failing to divide by 2! to remove the effect of the 2 Ds being identical.

Other, more complex, approaches were seen successfully completed and these all had simple diagrams to explain the logic of the process being used.

(c) This question was found challenging by most candidates. Good solutions started by clearly identifying all the possible scenarios which fulfilled the criteria, often in a table, and then calculating the number of arrangements for each. The total number of possible selections was then identified and calculated and used to form the required probability.

Many candidates did not interpret the criteria accurately, with scenarios with either 0 or 2 As being included frequently.

A common misconception was to assume that the selection of As and Ds did not affect the total number of possible selections so, for example, did not multiply by $^5C_1$ for the number of ways of selecting 1 A.

Few candidates attempted to produce a probability, and a common error was to use the total number of arrangements for the 9 letters rather than the number of selections of 4 letters possible. Candidates should be aware that it is good practice to reread the question once they have completed their solution to confirm that it is reasonable within the context and all the requirements of the question have been fulfilled.

MATHEMATICS

Paper 9709/53
Probability and Statistics 1

Key messages

Candidates need to understand that when there are several stages to a solution, such as in **Questions 3(b)** and **6(b)**, they should make their reasoning and working clear to the reader. In both of these probability questions, candidates were expected to make clear what the values of the numerator and denominator in the probability fraction represented. In **Question 6(b)**, candidates needed to demonstrate that they were adding values for the three separate scenarios of Jai and Kaz being in a group of 3, 4 or 5.

Candidates also need to understand that they need to show their working even when they use a calculator. Where a standardisation is involved, such as in **Questions 2(a)**, **2(b)** and **5(c)**, it is a requirement that they show the standardisation formula with the correct values substituted. It is clearly stated at the front of the paper that 'no marks will be given for unsupported answers from a calculator'.

General comments

More students are becoming familiar with the geometric distribution but a number still need to learn the formulae for $P(X \leqslant x)$ and $E(X)$.

Most candidates knew to give exact answers in full as required in **Questions 6(a)(i)** and **6(a)(ii)** but a significant number were not awarded full marks in other questions either for premature approximation, which resulted in inaccurate final answers, or for giving final answers to only two significant figures.

Comments on specific questions

Question 1

Please note that due to a series-specific issue with **Question 1**, full marks have been awarded to all candidates for this question to make sure that no candidates were disadvantaged.

Question 2

(a) This question was answered well, with most candidates appreciating that they should show use of the standardisation formula and that they will not gain full credit for unsupported answers from a calculator. Having found the z-values of -1.6 and 0.4, strong candidates showed clearly how they arrived at the correct final answer by adding the phi of 1.6 and of 0.4 before subtracting 1. Those who chose to draw a sketch of the distribution usually arrived at an answer in the appropriate probability area.

(b) Most candidates were able to use the tables the correct way round, although a significant number either rounded the z-value to 2 significant figures (0.47) or arrived at an inaccurate value, with 0.462 and 0.467 being commonly seen. As in the previous question, it was a requirement that we saw the standardisation expression equated to a z-value for the method mark to be awarded.

Question 3

(a) The majority of candidates answered this question using the anticipated method of looking separately at bags A and B, multiplying the fraction probabilities of the first and second marbles being white and then adding. Most candidates remembered that there was a probability of 2/6 that the marbles would be from bag A and a probability of 4/6 that the marbles would come from bag B.

Use of a tree diagram was helpful although it did not necessarily ensure that candidates remembered to include the probabilities of selecting from bag *A* or *B*. Only a few incorrectly dealt with the problem as though the first marble was replaced before selecting the second.

A significant number chose to use combinations and correctly added $\frac{2}{6} \times \frac{{}^{8}C_2}{{}^{15}C_2}$ to $\frac{4}{6} \times \frac{{}^{6}C_2}{{}^{15}C_2}$.

(b) This proved to be a challenging question. Strong candidates who understood conditional probability and used the expected method made it clear that they were dividing the probability of choosing a red and a white marble from bag *B* by the total probability of choosing a red and a white marble. They also remembered that the marbles could be in either order (RW or WR). Those who did not include both possibilities usually obtained the expected final answer from incorrect probabilities.

As in **part (a)**, a significant number used combinations rather than products of fraction probabilities, the most common correct method being to divide $\frac{4}{6} \times \frac{{}^{6}C_1 \times {}^{7}C_1}{{}^{15}C_2}$ (4/15) by

$\frac{4}{6} \times \frac{{}^{6}C_1 \times {}^{7}C_1}{{}^{15}C_2} + \frac{2}{6} \times \frac{{}^{8}C_1 \times {}^{4}C_1}{{}^{15}C_2}$ (116/315).

Unfortunately, many did not explain their working and some misunderstood the question entirely, only considering the probability of choosing a red and a white marble from bag *B*.

Question 4

Please note that due to a series-specific issue with **Question 4**, full marks have been awarded to all candidates for this question to make sure that no candidates were disadvantaged.

Question 5

(a) (i) Most candidates recognised the geometric distribution. Those who opted to consider each value of *X* in turn and sum the probabilities were generally successful. Those who attempted $P(X \leqslant 6) - P(X \leqslant 1)$ were more likely to make an error, often incorrectly subtracting the $P(X \leqslant 2)$ from $P(X \leqslant 6)$. A minority incorrectly tried to apply a binomial distribution.

(a) (ii) A growing number of candidates now seem to have learned the formula for the expected value of *X* in a geometric distribution. Some spoiled their answer by thinking it should be rounded to the nearest integer, perhaps because of the discrete nature of the distribution. Those who did not know the correct result often attempted lengthy calculations mirroring ones that would be seen to calculate E(*X*) from a distribution table, while others tried to use the answer to **part (a)(i)**.

(b) This was a familiar type of question to most and was generally answered well, with most candidates successfully summing the probabilities of 3, 4 or 5 of the friends passing the test. A small number chose to use the more difficult method of subtracting the probabilities of 0, 1 or 2 passing from 1. The words 'at least three' was incorrectly interpreted by some candidates and they either forgot to include 3 or only calculated the probability of 3 passing. Most candidates now realise the importance of explaining their working and know that they should show the complete binomial terms and not just their decimal values. Very few omitted the ${}^{n}C_r$ coefficients.

(c) This was recognised by most as a standard question involving a normal approximation to the binomial and was very well answered by most candidates. As in **Question 2(a)**, candidates who did not demonstrate the full standardisation expression, or who stated unsupported answers from their calculators, were not awarded full credit. Most candidates remembered to use a continuity correction and the majority applied it in the correct direction. The biggest challenge was calculating the appropriate probability area. Candidates are encouraged, as in **Question 2(a)**, to use a sketch to support their work. Once they have calculated the required *z*-value, they should use their sketch and establish whether the required area is more or less than 0.5.

Question 6

(a) (i) Most candidates recognised that there were 6! ways of arranging the six couples. Some presented this as the final answer and did not consider that each couple could be either way round. Others knew that they needed to deal with the two possible positions of each couple but only multiplied by 2 or by 6×2. Confident students knew to multiply by 2 raised to the power of 6.

(a) (ii) There were similar issues with this part of the question. Many knew to permutate the friends and their wives separately and multiplied 5! by 5!. However, only a minority of those recognised that they needed to consider the two ways of positioning Jai and Kaz and the fact that the friends and their wives could be either side of Jai and Kaz, meaning that they had to multiply $5! \times 5!$ by $2! \times 2!$.

(b) This proved to be a challenging question. The majority of those who were successful found the number of ways of grouping Jai and Kaz together in a group of 3, 4 or 5 and dividing the total by the number of possible ways of grouping the 12 people. For this question, candidates needed to make it clear that they were considering the three different scenarios of Jai and Kaz being in a group of 3, 4 or 5. Calculating the total number of possible ways was well done, with the method seen most often being to multiply the number of ways of choosing 5 from 12 by the number of ways of choosing 4 from the remaining 7. There were other ways to find this total, e.g. choosing 3 from 12 first etc. However, many were less successful in finding the numbers of ways of grouping Jai and Kaz together in the individual groups.

Some strong candidates considered the probability of Jai and Kaz being in each of the 3 groups and adding, either by multiplying the fraction probabilities $\left(\frac{5}{12}\times\frac{4}{11}+\frac{4}{12}\times\frac{3}{11}+\frac{3}{12}\times\frac{2}{11}\right)$ or using combinations $\left(\frac{^{10}C_3}{^{12}C_5}+\frac{^{10}C_2}{^{12}C_4}+\frac{^{10}C_1}{^{12}C_3}\right)$.

A few candidates unnecessarily complicated the question by trying to subtract the probability of Jai and Kaz not being together in a group from 1 and very few of these were successful.

MATHEMATICS

Paper 9709/61
Probability and Statistics 2 (61)

There were too few candidates for a meaningful report to be produced.

MATHEMATICS

Paper 9709/62
Probability and Statistics 2 (62)

Key messages

- Candidates are reminded of the need to present work clearly with just one solution offered.
- Full working must be shown; a final answer with no justification may not be sufficient for full credit.
- When the question requires an answer to be shown, it is particularly important that all relevant working is presented with no steps in the solution omitted.
- It is important that candidates refer to the context given in the question when requested to do so as general (textbook) statements will not be sufficient.
- Candidates need to keep to at least 4 significant figures of accuracy during working to ensure a final answer correct to 3 significant figures.
- If questions are continued on additional pages, the question number must be identified clearly.

General comments

There were a few questions on this paper that caused problems for candidates, in particular **Question 3**. However, **Questions 1** and **5** were well attempted. There was no indication of candidates having time pressures and presentation was generally good. However, there were times when full working was not shown; candidates need to be aware that, for example, when calculating cumulative Poisson or Binomial probabilities all individual terms must be shown in the working and they should not just give a final unsupported answer. Similarly, it is expected when using a normal distribution that the standardising equation is clearly shown. In ‘show that’ questions, all steps in the working must be clear.

The following comments on specific questions highlight many of the common errors seen, but it should be noted there were also many cases of fully correct, complete solutions too.

Comments on specific questions

Question 1

(a) This question was well attempted. Most solutions used the correct Normal distribution; the major errors noted were an incorrect or missing continuity correction or calculating the wrong tail probability.

(b) Again, this was reasonably well attempted with many candidates making a good attempt at a correct expression. However, some solutions omitted brackets and therefore failed to reach a correct final expression due to use of $100p^2$ rather than $(100p)^2$. Other solutions made the error of including P(3) in their Poisson expression.

Question 2

(a) The correct value for z was usually found, but errors were made in finding an expression of the correct form for the confidence interval. A common error was to centre the confidence interval around 4590 rather than $\frac{4590}{85}$.

(b) Candidates made a slightly better attempt at this part of the question. The correct z value was usually used, and many solutions used a correct expression for the width of the confidence interval. However, an occasional factor of 2 error was noted and some used $\sqrt{s}$ rather than s in their expression.

Question 3

(a) (i) In general, this was not well attempted. Most solutions did not appreciate that the comparison should not be made with a point probability but with a tail probability (in this case $P(X \geqslant 8)$). Whilst some candidates realised it should have been a tail probability, not all gave the correct tail. Some solutions incorrectly thought the comparison should have been with 0.025 rather than 0.05 and others thought the null hypothesis should have been accepted.

(ii) Again, this was not well attempted. Many solutions used $p = 0.8$ (or occasionally $p = \frac{1}{3}$) rather than $p = 0.5$, point probabilities were compared rather than tail probabilities, and two tail tests were incorrectly applied. For candidates who successfully got to the point of drawing a conclusion, not all remembered that this conclusion should be in context and not definite (i.e. with an appropriate level of uncertainty in the language used).

(b) Candidates made a better attempt at this part, with many realising that a Type I error could not occur because H_0 had not been rejected. Some solutions merely quoted a textbook definition which, on its own, is not acceptable as the explanation must be related to the question.

(c) This was not well attempted. Candidates used 0.8 rather than 0.5, did not find P(9,10) or thought the required probability was 0.05.

Question 4

(a) Many candidates were able to set up correct null and alternate hypotheses. Errors primarily included using an incorrect parameter (e.g. 'p').

(b) There were some good attempts made in forming an equation by standardising and equating to 1.645. Errors included omitting $\sqrt{200}$, using an incorrect z, using an incorrect sign for z or using 0.95 or 0.05 instead of a z value. Many candidates left their answer as 4.31 rather than greater than (or equal to) 4.31.

(c) Candidates did not always realise that the Central Limit theorem was not necessary as the information given in the question stated that the population of H was normally distributed. Many solutions did not describe the required distribution fully (for example, "'it' is Normal" would not be sufficient) and many thought that the value of 'n' was relevant.

Question 5

(a) Candidates were required to integrate $x\mathrm{f}(x)$ from a to b and put this equal to $\ln 2$. Most solutions made a reasonable attempt at this, though there were errors and misconceptions seen when simplifying logarithms. For example, some candidates wrote incorrect statements such as $\ln b - \ln a = \frac{\ln b}{\ln a}$. As the answer was given, all necessary steps in the working were required. On questions such as these, candidates are advised to ensure all working is shown in full.

(b) Again, this was reasonably well attempted, with many solutions following the correct method of integrating $\mathrm{f}(x)$ from a to b and equating to 1, then using **part (a)** to show $a = \frac{1}{2}$. Sign errors when integrating were occasionally seen. Again, all necessary steps in the working were required.

(c) Finding the median of X was reasonably well attempted. Many candidates attempted to integrate $\mathrm{f}(x)$ with the correct limits and equate to 0.5. Errors included incorrect limits and sign errors when integrating.

Question 6

Some candidates were able to make a valid start to this question, but few were fully successful. Finding the mean and variance of the cost of dried yeast and flour was attempted by many, but errors were made when finding variances; $0.02^2 \times 13.50^2$ and $3.0^2 \times 0.90^2$ were required but errors were made, usually by not squaring 13.50 or 0.90. Combining the costs was also not well attempted, again with many errors seen. This was particularly the case with answers for the variance. There was confusion between costs, selling price and profit and which Normal distribution to use; candidates did not always use the given information correctly. Standardising was usually attempted, but needed to be consistent with their method and the distribution that they were using (i.e. N(99.45, 7.3629), N(154.45, 7.3629) or N(45.55, 7.3629)).

Question 7

(a) This was a particularly well attempted part of the question. Most solutions used Po(2.4) and found the correct expression for P(2,3). The main error noted was to calculate P(2,3,4)

(b) Many candidates used Po(2.4) and found 1 – P(0,1) but then failed to square this probability, sometimes doubling instead. Another common error was to use Po(4.8) to find 1 – P(0,1). There was an occasional final answer of 0.479 given where candidates had rounded too early and squared 0.692 instead of squaring the more accurate figure of 0.6916, thus illustrating the need to keep to at least 4 significant figures of accuracy during working to ensure a final answer correct to 3 significant figures.

(c) (i) Finding the correct expressions for both $P(X = r)$ and $P(X = r + 1)$ was done well. However, simplifying the subsequent inequality was not; many candidates made errors simplifying factorials and powers. Some solutions correctly reached the simplified inequality of $r + 1 < 2.4$, but then did not realise that r had to be an integer so that the set of values required was just 0 and 1. Alternatively, some valid attempts were made by finding P(0), P(1), P(2), and P(3) in a trial and improvement approach.

(ii) Very few candidates realised that the greatest value of r was 2. An incorrect answer of 1 was common.

MATHEMATICS

Paper 9709/63
Probability and Statistics 2 (63)

Key messages

Showing working is always required in the solutions which candidates give for their answers. In particular, it is necessary to show complete sets of terms when a series expansion is being used. One such case is when several terms of a Poisson distribution are involved, such as in **Question 3(a)(i)** and in **Question 5**.

General comments

Calculations are required to be given to three significant figures. In some questions, this needs intermediate working to have more significant figures.

Comments on specific questions

Question 1

This question required the use of the Normal distribution of means of samples. It was necessary to select the appropriate area for the probability value, which many candidates did successfully. Some candidates omitted the $\sqrt{50}$. Other candidates incorrectly gave the large probability.

Question 2

For this confidence interval question it was necessary first to set up a correct expression of the form $p + z\sqrt{\frac{pq}{n}} = 0.487$ to find the value of z. Secondly, this value of z could be used to find phi and then the confidence interval percentage. Some candidates omitted the $\sqrt{50}$ or the division by n. Other candidates found only the value of $\Phi(0.9199)$ and did not proceed to alpha by using the calculation $2\times\Phi^{-1}$ or an equivalent conversion.

Question 3

(a) (i) First it was necessary to find the new Poisson parameter for a 10-minute period ($\lambda = 3$) from the given 0.3 hits per minute. Secondly to find the probability of at least 4 hits, the method was to find 1 – P($\leqslant$ 3). It was essential to give the complete set of Poisson terms for this expression.

(ii) For a 3-hour period the Poisson parameter would be 54, so the suitable approximating distribution would be N(54, 54). The necessary continuity correction factor required standardising with 39.5. A sketch could help with choosing this value and with selecting the appropriate area for the probability. Some candidates did not use a factor or chose an incorrect factor, such as 40.5.

(b) (i) The observation by the friend that the mean number of hits during the daytime was about twice the value during the night showed that over a whole day the mean value was not constant. This indicated that for a whole day a Poisson distribution was not appropriate. This observation could be expressed in other ways, such as stating that the number of hits did not occur at a constant rate. Some candidates referred to other possible properties such as randomness or independence, but these were not relevant in this situation.

(ii) In order to use the friend's observation about different mean numbers of hits during day-time and night-time and to use the overall rate of 0.3 hits per minute, it was helpful to set up an equation of the form $2\lambda + \lambda = 2 \times 0.3$. It was acceptable to work with time spans other than 'per minute'. For example, work with 12-hour shifts was acceptable.

Question 4

(a) This part referred only to chemical *A*. The answer for the mean income from *A* was in dollars. Either 25.75 or 25.8 was accepted.

(b) In order to compare the incomes from chemical *A* and chemical *B,* it was necessary to create a new variable $A - B$ and to find the mean (– 11.3) and variance (137.5056) of this new variable. To find the probability that the income generated by chemical *A* was greater than the income generated by chemical *B*, these values could be used to find the probability that $A - B > 0$.

A sketch could be useful to decide on the relevant area. Some candidates produced very clear, correct answers showing all these steps. Other candidates made various errors, particularly when attempting to find the variance of $A - B$.

Question 5

For a 5-minute period, the Poisson parameter was 1.55. To carry out the significance test the probability $P(X \geqslant 5)$ was required, which could be found from $1 - P(X \leqslant 4)$. Some candidates found 0.0210 correctly. Other candidates made one end error. It was essential to give the complete set of Poisson terms for this expression. Other candidates found only $P(X = 5)$, which did not lead to a valid comparison.

It was necessary to write down the comparison with 0.025 and then make a valid conclusion. The conclusion needed to be in context, not definite and with no contradictions. A phrase such as '*there is some evidence to suggest that…'* could be appropriate here.

Question 6

(a) To demonstrate the inequality, it was necessary to use the property of the pdf that the total area was 1 and hence that the area from 0 to 3 was $\frac{1}{2}$ (using the symmetry of the pdf). Hence, the sum of the given probabilities *p* and $\frac{13p}{10}$ was less than or equal to $\frac{1}{2}$. Some candidates incorrectly chose to approach this question using integration.

(b) A sketch of a possible curve for the pdf was helpful here to show the positions of *p* and 1.3*p*, as well as the symmetrical positions of equal values on the right-hand side of the pdf. Many different approaches to find the relevant areas were possible. One direct way was to sum the three adjacent areas of 1.3*p*, 1.3p and *p* from the sketch.

(c) In this part, while the use of limits from *a* to 2 was expected, other limits and values could be used. The integration and substitution led to the cubic equation $a^3 - 9a^2 + 8 = 0$ or an equivalent form. A complete solution could be found most simply by factorising to $(a - 1)(a^2 - 8a - 8)$. The two solutions from the quadratic were outside the range of the pdf and so not relevant, leaving the sole answer as $a = 1$. Candidates who did not factorise but found $a = 1$ by inspection would gain credit. Some candidates attempted to use incorrect algebra, which was not worthy of full credit.

Question 7

(a) For this hypothesis test the hypotheses were given. Candidates did need to calculate the unbiased estimates of the mean and variance, which most candidates did successfully.

To standardise the sample mean required the variance of sample means and hence the divisor 50. The resulting *z* value then needed to be compared to the critical value – 1.645. Alternatively, probabilities could be compared (0.102 or 0.103 > 0.05). In either case, the comparison needed to be shown. The conclusion had to be stated in context, be not definite and have no contradictions. Many candidates made reasonable attempts at these steps.

(b) The first step required finding the critical value for accepting/rejecting the null hypothesis when μ = 0.5. This required the use of 0.5, 50 and –1.645 to obtain the critical value 0.448. The second step involved the use of 0.448, 0.4 and 50 and selecting the relevant area.

A range of answers was allowed for the probability to allow for the many significant figures involved. While some candidates followed this process correctly, many other candidates omitted the first step in this process.