Cambridge Assessment International Education

Cambridge International A Level

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **October/November 2022**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2022 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

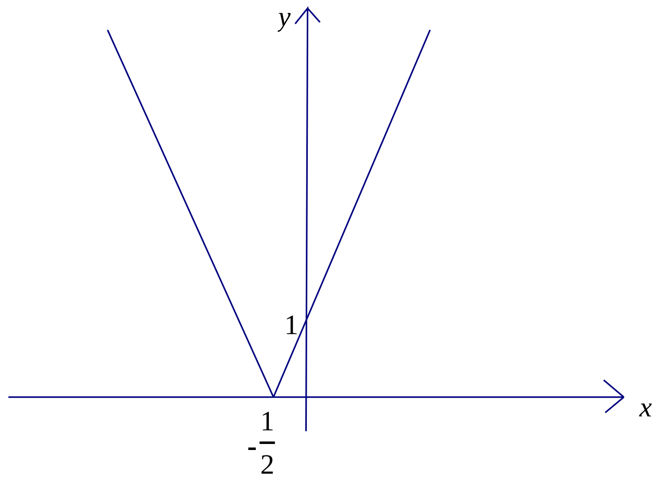
DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

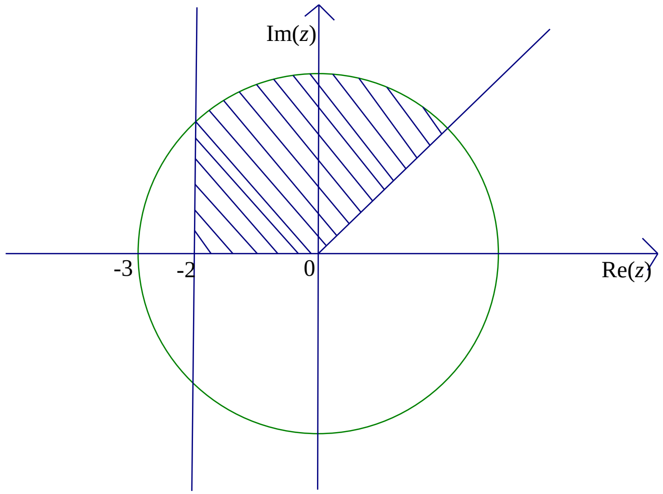
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	Show a recognisable sketch graph of $y = \|2x+1\|$	B1	 Ignore $y = 3x + 5$ if also drawn on the sketch.
		1	

Question	Answer	Marks	Guidance
1(b)	Find x-coordinate of intersection with $y = 3x + 5$	**M1**	
	Obtain $x = -\frac{6}{5}$	**A1**	
	State final answer $x < -\frac{6}{5}$ only	**A1**	Do not condone $\le$ for $<$ in the final answer.
	Alternative method 1 for question 1(b)		
	Solve the linear inequality $3x + 5 < -(2x + 1)$, or corresponding equation	**M1**	Must solve the relevant equation.
	Obtain critical value $x = -\frac{6}{5}$	**A1**	Ignore -4 if seen.
	State final answer $x < -\frac{6}{5}$ only	**A1**	
	Alternative method 2 for question 1(b)		
	Solve the quadratic inequality $(3x+5)^2 < (2x+1)^2$, or corresponding equation	**M1**	$5x^2 + 26x + 24 < 0$
	Obtain critical value $x = -\frac{6}{5}$	**A1**	Ignore -4 if seen.
	State final answer $x < -\frac{6}{5}$ only	**A1**	
		3	

Question	Answer	Marks	Guidance
2	Show a circle with radius 3 and centre the origin	**B1**	
	Show the line $x = -2$	**B1**	
	Show the correct half line for $\frac{\pi}{4}$	**B1**	
	Shade the correct region	**B1**	For the vertical line and the circle, allow the B1 marks if all you see is the relevant part.
		4	

Question	Answer	Marks	Guidance
3	Use law of logarithm of a product or power	M1	One correct application of a log law.
	Obtain a correct linear equation in any form, e.g. $(3x-1)\ln 2 = \ln 5 - x\ln 3$	A1	
	Solve for x	M1	As far as $x = \ldots$ with only minor slips in processing.
	Obtain answer $x = \dfrac{\ln 10}{\ln 24}$	A1	
	Alternative method for question 3		
	Use laws of indices to split at least one exponential term	M1	e.g. $\dfrac{2^{3x}}{2}$ or an arrangement with 8^x
	Obtain $24^x = 10$	A1	OE
	Solve for x	M1	
	Obtain answer $x = \dfrac{\ln 10}{\ln 24}$	A1	
		4	

Question	Answer	Marks	Guidance
4	Use correct $\tan(A+B)$ formula and obtain an equation in tan *x* or an equation in $\cos x$ and $\sin x$	**M1**	e.g. $\dfrac{\tan x+\tan 45°}{1-\tan x\tan 45°}=\dfrac{2}{\tan x}$ Allow if 2 in denominator or $\dfrac{\sin x\cos 45°+\cos x\sin 45°}{\cos x\cos 45°-\sin x\sin 45°}=\dfrac{2\cos x}{\sin x}$.
	Obtain correct 3 term equation $\tan^2 x+3\tan x-2=0$, or equivalent	**A1**	or $3\sin x\cos x=2\cos^2 x-\sin^2 x$
	Solve a 3-term quadratic in tan *x* and obtain a value for *x*	**M1**	
	Obtain answer, e.g. 29.3°	**A1**	29.316…
	Obtain second answer, e.g. 105.7° and no other	**A1**	105.583…. Ignore answers outside the given interval. Treat answers in radians as a misread.
		5	

Question	Answer	Marks	Guidance
5(a)	State or imply $u^2=4e^{\frac{1}{2}\pi i}$	**B1**	
	Obtain answer $v=\dfrac{4}{3}e^{\frac{1}{6}\pi i}$	**B1 + B1**	For the modulus and the argument.
		3	
5(b)	State $n = 6$	**B1**	
		1	

Question	Answer	Marks	Guidance
6(a)	Express $\cos 4\theta$ in terms of $\cos 2\theta$ and/or $\sin 2\theta$	**B1**	
	Express $\cos 2\theta$ in terms of $\cos\theta$ and/or $\sin\theta$	**B1**	Anywhere
	Expand to obtain a correct expression in terms of $\cos\theta$	**B1**	e.g. $2\left(2\cos^2\theta-1\right)^2-1+4\left(2\cos^2\theta-1\right)+3$
	Reduce correctly to $\cos 4\theta+4\cos 2\theta+3\equiv 8\cos^4\theta$	**B1**	AG
		4	
6(b)	Use the identity and carry out method to calculate a root	**M1**	$8\cos^4\theta-3=4$
	Obtain answer, e.g. $14.7°$	**A1**	
	Obtain second answer, e.g.$165.3°$, and no other in the given interval	**A1 FT**	Ignore answers outside the given interval. Treat answers in radians as a misread.
		3	

Question	Answer	Marks	Guidance
7(a)	Use correct product or quotient rule	**M1**	
	Obtain correct derivative in any form	**A1**	e.g. $\frac{dy}{dx}=\frac{\cos^2 x+2x\sin x\cos x}{\cos^4 x}$ or $\frac{dy}{dx}=\sec^2 x+2x\sec^2 x\tan x$
	Equate derivative at $x = a$ to 12 and obtain $a=\cos^{-1}\left(\sqrt[3]{\frac{\cos a+2a\sin a}{12}}\right)$	**A1**	AG
		3	
7(b)	Evaluate a relevant expression or pair of expressions at $a = 0.9$ and $a = 1$	**M1**	Must be calculated in radians.
	Complete the argument correctly with correct calculated values	**A1**	e.g. $\begin{matrix}\cos 0.9=0.622>0.553\\ \cos 1=0.540<0.570\end{matrix}$ or $\begin{matrix}0.9<0.985\\ 1>0.964\end{matrix}$ or $\begin{matrix}0.0846>0\\ -0.0358<0\end{matrix}$ or could be looking at values of the gradient 8.46 & 14.1
		2	
7(c)	Use the process $a_{n+1}=\cos^{-1}\left(\sqrt[3]{\frac{\cos a_n+2a_n\sin a_n}{12}}\right)$ correctly at least once	**M1**	Must be working in radians.
	Obtain final answer 0.97	**A1**	
	Show sufficient iterations to 4 d.p. to justify 0.97 to 2 d.p., or show there is a sign change in the interval (0.965, 0.975)	**A1**	e.g. 0.95, 0.9743, 0.9694, 0.9704
		3	

Question	Answer	Marks	Guidance
8(a)	Separate variables correctly	B1	$\int \frac{1}{x}\,\mathrm{d}x = \int k\mathrm{e}^{-0.1t}\,\mathrm{d}t$
	Obtain term ln x	B1	
	Obtain term $-10k\mathrm{e}^{-0.1t}$	B1	Not from $\int x\mathrm{e}^{-0.1t}\,\mathrm{d}t$
	Use x = 20, t = 0 to evaluate a constant or as limits in a solution containing terms $a\ln x, be^{-0.1t}$ where $ab \neq 0$	M1	
	Obtain $\ln x = 10k\left(1-\mathrm{e}^{-0.1t}\right)+\ln 20$	A1	or equivalent ISW
		5	
8(b)	Use x = 40, t = 10 to find k or 10k	M1	Available for their function of the correct structure even if they found no constant in **(a)**.
	Obtain 10k = 1.09654	A1	or equivalent e.g. $10k = \dfrac{\ln 2}{1-e^{-1}}$
	State that x tends to 59.9	A1	Need a number, not an expression for that value 3 sf or better 59.87595…..
		3	

Question	Answer	Marks	Guidance
9(a)	Use correct product or quotient rule	*M1	
	Obtain correct derivative in any form	A1	e.g. $\frac{dy}{dx} = -e^{-\frac{x}{3}} - \frac{1}{3}(3-x)e^{-\frac{x}{3}}$
	Equate their derivative to zero and solve for x	DM1	
	Obtain $x = 6$	A1	
	Obtain $y = -3e^{-2}$	A1	Or exact equivalent.
		5	
9(b)	Commence integration and reach $a(3-x)e^{-\frac{1}{3}x} + b\int e^{-\frac{1}{3}x}\,dx$, where $ab \neq 0$	*M1	
	Obtain $-3(3-x)e^{-\frac{1}{3}x} - 3\int e^{-\frac{1}{3}x}\,dx$, or equivalent	A1	
	Complete integration and obtain $3xe^{-\frac{1}{3}x}$, or equivalent	A1	$-3e^{-\frac{x}{3}}(3-x) + 9e^{-\frac{x}{3}}$
	Substitute limits $x = 0$ and $x = 3$, having integrated twice	DM1	
	Obtain answer $\frac{9}{e}$, or exact equivalent	A1	
		5	

Question	Answer	Marks	Guidance
10(a)	State or imply the form $\dfrac{A}{1+x}+\dfrac{B}{2+x}+\dfrac{C}{(2+x)^2}$	B1	
	Use a correct method to find a constant	M1	
	Obtain one of $A = 3$, $B = -1$ and $C = -2$	A1	SR after B0 can score M1A1 for one correct value
	Obtain a second value	A1	
	Obtain the third value	A1	$\dfrac{A}{1+x}$ + $\dfrac{Dx+E}{(2+x)^2}$, where $A = 3$, $D = -1$ and $E = -4$, is awarded B1 M1 A1 A1 A1 as above.
		5	
10(b)	Use a correct method to find the first two terms of the expansion of $(1+x)^{-1}$, $(2+x)^{-1}$, $\left(1+\frac{1}{2}x\right)^{-1}$, $(2+x)^{-2}$ or $\left(1+\frac{1}{2}x\right)^{-2}$	M1	For the A, D, E form of fractions, award M1 A1FT A1FT for the expanded partial fractions, then if $D \neq 0$, M1 for multiplying out fully, and A1 for the final answer.
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	A3 FT	$3\left(1-x+x^2.....\right)$ $-\frac{1}{2}\left(1-\frac{x}{2}+\frac{x^2}{4}.....\right)$ $-\frac{2}{4}\left(1-x+\frac{3}{4}x^2\right)$
	Obtain final answer $2-\dfrac{9}{4}x+\dfrac{5}{2}x^2$	A1	
		5	

Question	Answer	Marks	Guidance
11(a)	State $\overrightarrow{OM} = 2\mathbf{i} + 2\mathbf{j}$ or equivalent	B1	Can be implied by $\overrightarrow{MB} = -2\mathbf{i} + 2\mathbf{j}$ or $\overrightarrow{MA} = 2\mathbf{i} - 2\mathbf{j}$.
	Obtain $\overrightarrow{MD} = -2\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$	B1	
	Use a correct method to find $\overrightarrow{ON}$	M1	e.g. $\overrightarrow{OC} + \frac{2}{3}\overrightarrow{CB}$
	Obtain answer $3\mathbf{j} + \mathbf{k}$	A1	
		4	
11(b)	Use the correct process for evaluating the scalar product of $\overrightarrow{MD}$ and $\overrightarrow{ON}$	M1	
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and reach the inverse cosine of the result	M1	$\cos^{-1}\left(\frac{-6+3}{\sqrt{10}\sqrt{17}}\right)$
	Obtain final answer $103.3°$	A1	
		3	

Question	Answer	Marks	Guidance
11(c)	Taking a general point P of ON to have position vector $\lambda(3\mathbf{j}+\mathbf{k})$, form an equation in λ by *either* equating the scalar product of $\overrightarrow{ON}$ and $\overrightarrow{MP}$ to zero, *or* applying Pythagoras to triangle OMP, *or* equating the derivative of $\lvert\overrightarrow{MP}\rvert$ to zero	**M1**	e.g. $\begin{pmatrix} -2 \\ -2+3\lambda \\ \lambda \end{pmatrix} . \begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix} = 0$
	Solve and obtain $\lambda = \frac{3}{5}$	**A1**	
	Substitute for λ and calculate MP	**M1**	$\overrightarrow{MP} = -2\mathbf{i} - \frac{1}{5}\mathbf{j} + \frac{3}{5}\mathbf{k}$
	Obtain $\sqrt{\frac{22}{5}}$	**A1**	AG
	Alternative method for question 11(c)		
	Use a scalar product to find the projection OQ of OM on OM	**M1**	
	Obtain $OQ = \frac{6}{\sqrt{10}}$	**A1**	
	Use Pythagoras in triangle OMQ to find MQ	**M1**	
	Obtain $\sqrt{\frac{22}{5}}$	**A1**	AG
		4	