



# Cambridge International AS & A Level

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**MATHEMATICS**

**9709/33**

Paper 3 Pure Mathematics 3

**October/November 2020**

**1 hour 50 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

## INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

## INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

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- 2 On a sketch of an Argand diagram, shade the region whose points represent complex numbers  $z$  satisfying the inequalities  $|z| \geq 2$  and  $|z - 1 + i| \leq 1$ . [4]

3 The parametric equations of a curve are

$$x = 3 - \cos 2\theta, \quad y = 2\theta + \sin 2\theta,$$

for  $0 < \theta < \frac{1}{2}\pi$ .

Show that  $\frac{dy}{dx} = \cot \theta$ .

[5]

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4 Solve the equation

$$\log_{10}(2x + 1) = 2 \log_{10}(x + 1) - 1.$$

Give your answers correct to 3 decimal places.

[6]

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- 5 (a) By sketching a suitable pair of graphs, show that the equation  $\operatorname{cosec} x = 1 + e^{-\frac{1}{2}x}$  has exactly two roots in the interval  $0 < x < \pi$ . [2]

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- (b) The sequence of values given by the iterative formula

$$x_{n+1} = \pi - \sin^{-1} \left( \frac{1}{e^{-\frac{1}{2}x_n} + 1} \right),$$

with initial value  $x_1 = 2$ , converges to one of these roots.

Use the formula to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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6 (a) Express  $\sqrt{6} \cos \theta + 3 \sin \theta$  in the form  $R \cos(\theta - \alpha)$ , where  $R > 0$  and  $0^\circ < \alpha < 90^\circ$ . State the exact value of  $R$  and give  $\alpha$  correct to 2 decimal places. [3]

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(b) Find the other roots of this equation.

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8 The coordinates (x, y) of a general point of a curve satisfy the differential equation

x dy/dx = (1 - 2x^2)y,

for x > 0. It is given that y = 1 when x = 1.

Solve the differential equation, obtaining an expression for y in terms of x. [6]

Dotted lines for writing the solution.

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(b) Hence obtain the expansion of  $f(x)$  in ascending powers of  $x$ , up to and including the term in  $x^2$ . [5]

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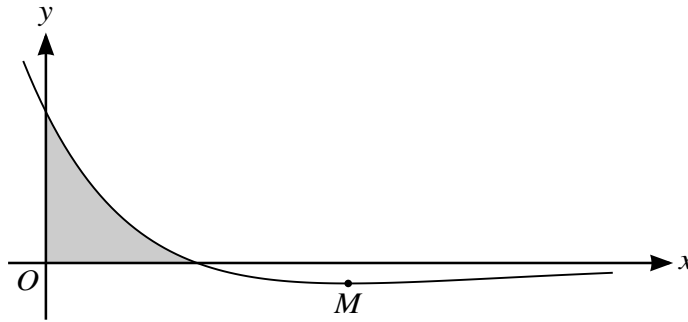
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The diagram shows the curve  $y = (2 - x)e^{-\frac{1}{2}x}$ , and its minimum point  $M$ .

(a) Find the exact coordinates of  $M$ .

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11 Two lines have equations  $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + \mathbf{k} + \lambda(a\mathbf{i} + 2\mathbf{j} - \mathbf{k})$  and  $\mathbf{r} = 2\mathbf{i} + \mathbf{j} - \mathbf{k} + \mu(2\mathbf{i} - \mathbf{j} + \mathbf{k})$ , where  $a$  is a constant.

(a) Given that the two lines intersect, find the value of  $a$  and the position vector of the point of intersection. [5]

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(b) Given instead that the acute angle between the directions of the two lines is  $\cos^{-1}(\frac{1}{6})$ , find the two possible values of  $a$ . [6]

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**Additional Page**

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