



# Cambridge International AS & A Level

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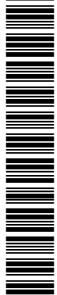
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**MATHEMATICS**

**9709/32**

Paper 3 Pure Mathematics 3

**October/November 2020**

**1 hour 50 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

## INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

## INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Blank pages are indicated.

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1 Solve the equation

$$\ln(1 + e^{-3x}) = 2.$$

Give the answer correct to 3 decimal places.

[3]

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- 2 (a) Expand  $\sqrt[3]{1 + 6x}$  in ascending powers of  $x$ , up to and including the term in  $x^3$ , simplifying the coefficients. [4]

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- (b) State the set of values of  $x$  for which the expansion is valid. [1]

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3 The variables  $x$  and  $y$  satisfy the relation  $2^y = 3^{1-2x}$ .

(a) By taking logarithms, show that the graph of  $y$  against  $x$  is a straight line. State the exact value of the gradient of this line. [3]

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(b) Find the exact  $x$ -coordinate of the point of intersection of this line with the line  $y = 3x$ . Give your answer in the form  $\frac{\ln a}{\ln b}$ , where  $a$  and  $b$  are integers. [2]

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4 (a) Show that the equation  $\tan(\theta + 60^\circ) = 2 \cot \theta$  can be written in the form

$$\tan^2 \theta + 3\sqrt{3} \tan \theta - 2 = 0. \quad [3]$$

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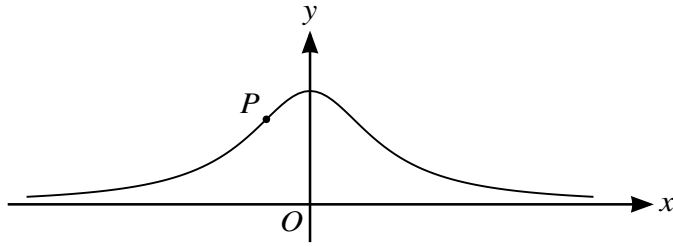
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**(b)** Hence solve the equation  $\tan(\theta + 60^\circ) = 2 \cot \theta$ , for  $0^\circ < \theta < 180^\circ$ . [3]

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The diagram shows the curve with parametric equations

$$x = \tan \theta, \quad y = \cos^2 \theta,$$

for  $-\frac{1}{2}\pi < \theta < \frac{1}{2}\pi$ .

- (a) Show that the gradient of the curve at the point with parameter  $\theta$  is  $-2 \sin \theta \cos^3 \theta$ . [3]

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The gradient of the curve has its maximum value at the point *P*.

**(b)** Find the exact value of the *x*-coordinate of *P*. [4]

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6 The complex number  $u$  is defined by

$$u = \frac{7+i}{1-i}.$$

(a) Express  $u$  in the form  $x + iy$ , where  $x$  and  $y$  are real. [3]

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(b) Show on a sketch of an Argand diagram the points  $A$ ,  $B$  and  $C$  representing  $u$ ,  $7 + i$  and  $1 - i$  respectively. [2]



7 The variables  $x$  and  $t$  satisfy the differential equation

$$e^{3t} \frac{dx}{dt} = \cos^2 2x,$$

for  $t \geq 0$ . It is given that  $x = 0$  when  $t = 0$ .

(a) Solve the differential equation and obtain an expression for  $x$  in terms of  $t$ . [7]

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(b) State what happens to the value of  $x$  when  $t$  tends to infinity. [1]

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- 8 With respect to the origin  $O$ , the position vectors of the points  $A$ ,  $B$ ,  $C$  and  $D$  are given by

$$\vec{OA} = \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}, \quad \vec{OB} = \begin{pmatrix} 4 \\ -1 \\ 1 \end{pmatrix}, \quad \vec{OC} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \quad \text{and} \quad \vec{OD} = \begin{pmatrix} 3 \\ 2 \\ 3 \end{pmatrix}.$$

- (a) Show that  $AB = 2CD$ . [3]

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- (b) Find the angle between the directions of  $\vec{AB}$  and  $\vec{CD}$ . [3]

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(c) Show that the line through  $A$  and  $B$  does not intersect the line through  $C$  and  $D$ . [4]

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9 Let  $f(x) = \frac{7x + 18}{(3x + 2)(x^2 + 4)}$ .

(a) Express  $f(x)$  in partial fractions.

[5]

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(b) Hence find the exact value of  $\int_0^2 f(x) dx$ .

[6]

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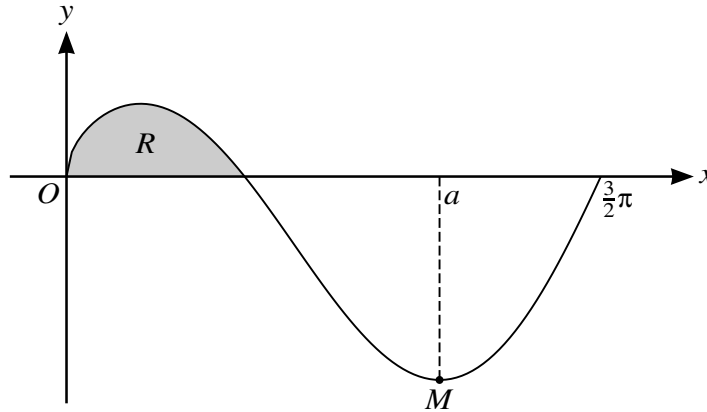
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The diagram shows the curve  $y = \sqrt{x} \cos x$ , for  $0 \leq x \leq \frac{3}{2}\pi$ , and its minimum point  $M$ , where  $x = a$ . The shaded region between the curve and the  $x$ -axis is denoted by  $R$ .

- (a) Show that  $a$  satisfies the equation  $\tan a = \frac{1}{2a}$ . [3]

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- (b) The sequence of values given by the iterative formula  $a_{n+1} = \pi + \tan^{-1}\left(\frac{1}{2a_n}\right)$ , with initial value  $x_1 = 3$ , converges to  $a$ .

Use this formula to determine  $a$  correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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**Additional Page**

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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