



Cambridge International AS & A Level

CANDIDATE
NAME

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CENTRE
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MATHEMATICS

9709/31

Paper 3 Pure Mathematics 3

October/November 2020

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Blank pages are indicated.

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- 2 On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z| \geq 2$ and $|z - 1 + i| \leq 1$. [4]

- 5 (a) By sketching a suitable pair of graphs, show that the equation $\operatorname{cosec} x = 1 + e^{-\frac{1}{2}x}$ has exactly two roots in the interval $0 < x < \pi$. [2]

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- (b) The sequence of values given by the iterative formula

$$x_{n+1} = \pi - \sin^{-1} \left(\frac{1}{e^{-\frac{1}{2}x_n} + 1} \right),$$

with initial value $x_1 = 2$, converges to one of these roots.

Use the formula to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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7 (a) Verify that $-1 + \sqrt{5}i$ is a root of the equation $2x^3 + x^2 + 6x - 18 = 0$. [3]

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8 The coordinates (x, y) of a general point of a curve satisfy the differential equation

$$x \frac{dy}{dx} = (1 - 2x^2)y,$$

for $x > 0$. It is given that $y = 1$ when $x = 1$.

Solve the differential equation, obtaining an expression for y in terms of x . [6]

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A series of 25 horizontal dotted lines for writing.

9 Let $f(x) = \frac{8 + 5x + 12x^2}{(1 - x)(2 + 3x)^2}$.

(a) Express $f(x)$ in partial fractions. [5]

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(b) Hence obtain the expansion of $f(x)$ in ascending powers of x , up to and including the term in x^2 .
[5]

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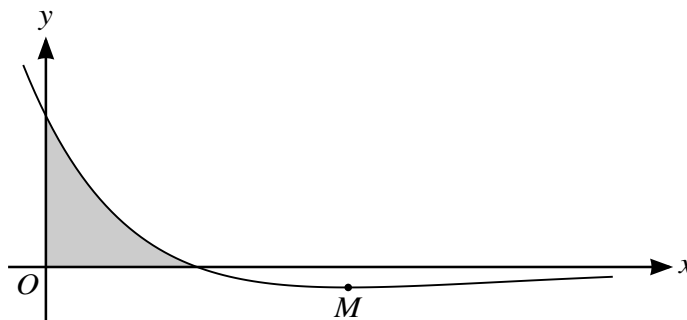
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The diagram shows the curve $y = (2 - x)e^{-\frac{1}{2}x}$, and its minimum point M .

(a) Find the exact coordinates of M .

[5]

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- (b) Find the area of the shaded region bounded by the curve and the axes. Give your answer in terms of e . [5]

11 Two lines have equations $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + \mathbf{k} + \lambda(a\mathbf{i} + 2\mathbf{j} - \mathbf{k})$ and $\mathbf{r} = 2\mathbf{i} + \mathbf{j} - \mathbf{k} + \mu(2\mathbf{i} - \mathbf{j} + \mathbf{k})$, where a is a constant.

(a) Given that the two lines intersect, find the value of a and the position vector of the point of intersection. [5]

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(b) Given instead that the acute angle between the directions of the two lines is $\cos^{-1}\left(\frac{1}{6}\right)$, find the two possible values of a . [6]

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