Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME	
CENTRE NUMBER	
CANDIDATE NUMBER	

8697473988

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **October/November 2020**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

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[Turn over

1 **(a)** Express $x^2 + 6x + 5$ in the form $(x + a)^2 + b$, where a and b are constants. [2]

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(b) The curve with equation $y = x^2$ is transformed to the curve with equation $y = x^2 + 6x + 5$.

Describe fully the transformation(s) involved. [2]

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2 The function f is defined by $\mathrm{f}(x) = \dfrac{2}{(x+2)^2}$ for $x > -2$.

(a) Find $\displaystyle\int_1^\infty \mathrm{f}(x)\,\mathrm{d}x$. [3]

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(b) The equation of a curve is such that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x)$. It is given that the point $(-1, -1)$ lies on the curve.

Find the equation of the curve. [2]

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3 Solve the equation $3\tan^2\theta + 1 = \dfrac{2}{\tan^2\theta}$ for $0° < \theta < 180°$. [5]

4 A curve has equation $y = 3x^2 - 4x + 4$ and a straight line has equation $y = mx + m - 1$, where m is a constant.

Find the set of values of m for which the curve and the line have two distinct points of intersection. [5]

5 In the expansion of $(a + bx)^7$, where a and b are non-zero constants, the coefficients of x, x^2 and x^4 are the first, second and third terms respectively of a geometric progression.

Find the value of $\frac{a}{b}$. [5]

6 The function f is defined by $\mathrm{f}(x) = \dfrac{2x}{3x-1}$ for $x > \frac{1}{3}$.

(a) Find an expression for $\mathrm{f}^{-1}(x)$. [3]

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(b) Show that $\dfrac{2}{3} + \dfrac{2}{3(3x-1)}$ can be expressed as $\dfrac{2x}{3x-1}$. [2]

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(c) State the range of f. [1]

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7 The first and second terms of an arithmetic progression are $\frac{1}{\cos^2\theta}$ and $-\frac{\tan^2\theta}{\cos^2\theta}$, respectively, where $0 < \theta < \frac{1}{2}\pi$.

(a) Show that the common difference is $-\frac{1}{\cos^4\theta}$. [4]

(b) Find the exact value of the 13th term when $\theta = \frac{1}{6}\pi$. [3]

8 The equation of a curve is $y = 2x + 1 + \dfrac{1}{2x+1}$ for $x > -\frac{1}{2}$.

(a) Find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$. [3]

(b) Find the coordinates of the stationary point and determine the nature of the stationary point. [5]

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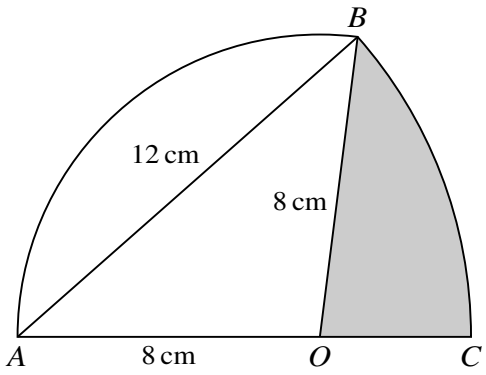

In the diagram, arc AB is part of a circle with centre O and radius 8 cm. Arc BC is part of a circle with centre A and radius 12 cm, where AOC is a straight line.

(a) Find angle BAO in radians. [2]

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(b) Find the area of the shaded region. [4]

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(c) Find the perimeter of the shaded region. [3]

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10 A curve has equation $y = \frac{1}{k}x^{\frac{1}{2}} + x^{-\frac{1}{2}} + \frac{1}{k^2}$ where $x > 0$ and k is a positive constant.

(a) It is given that when $x = \frac{1}{4}$, the gradient of the curve is 3.

Find the value of k. [4]

(b) It is given instead that $\int_{\frac{1}{4}k^2}^{k^2} \left(\frac{1}{k} x^{\frac{1}{2}} + x^{-\frac{1}{2}} + \frac{1}{k^2} \right) \mathrm{d}x = \frac{13}{12}$.

Find the value of k. [5]

11 A circle with centre C has equation $(x - 8)^2 + (y - 4)^2 = 100$.

(a) Show that the point $T\,(-6, 6)$ is outside the circle. [3]

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Two tangents from T to the circle are drawn.

(b) Show that the angle between one of the tangents and CT is exactly 45°. [2]

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The two tangents touch the circle at A and B.

(c) Find the equation of the line AB, giving your answer in the form $y = mx + c$. [4]

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(d) Find the x-coordinates of A and B. [3]

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Additional Page

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