Cambridge International AS & A Level

Cambridge International Examinations
Cambridge International Advanced Subsidiary and Advanced Level

CANDIDATE NAME

CENTRE NUMBER | | | | | |

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9945076812

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **(P1)** **October/November 2018**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

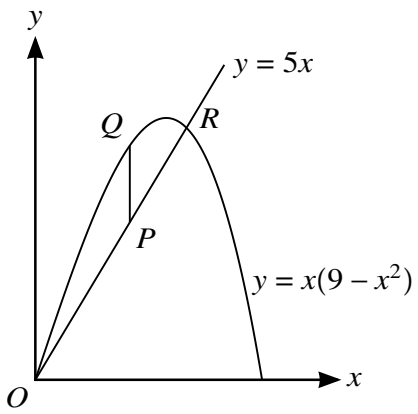
This document consists of **20** printed pages.

CAMBRIDGE International Examinations

1 Find the coefficient of $\frac{1}{x^2}$ in the expansion of $\left(3x + \frac{2}{3x^2}\right)^7$. [4]

2 Showing all necessary working, find $\int_1^4 \left(\sqrt{x} + \frac{2}{\sqrt{x}}\right) \mathrm{d}x$. [4]

3



The diagram shows part of the curve $y = x(9 - x^2)$ and the line $y = 5x$, intersecting at the origin O and the point R. Point P lies on the line $y = 5x$ between O and R and the x-coordinate of P is t. Point Q lies on the curve and PQ is parallel to the y-axis.

(i) Express the length of PQ in terms of t, simplifying your answer. [2]

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(ii) Given that t can vary, find the maximum value of the length of PQ. [3]

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4 Functions f and g are defined by

$$\mathrm{f} : x \mapsto 2 - 3\cos x \quad \text{for } 0 \leqslant x \leqslant 2\pi,$$

$$\mathrm{g} : x \mapsto \tfrac{1}{2}x \quad \text{for } 0 \leqslant x \leqslant 2\pi.$$

(i) Solve the equation $\mathrm{fg}(x) = 1$. [3]

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(ii) Sketch the graph of $y = \mathrm{f}(x)$. [3]

5 The first three terms of an arithmetic progression are 4, x and y respectively. The first three terms of a geometric progression are x, y and 18 respectively. It is given that both x and y are positive.

(i) Find the value of x and the value of y. [4]

(ii) Find the fourth term of each progression. [3]

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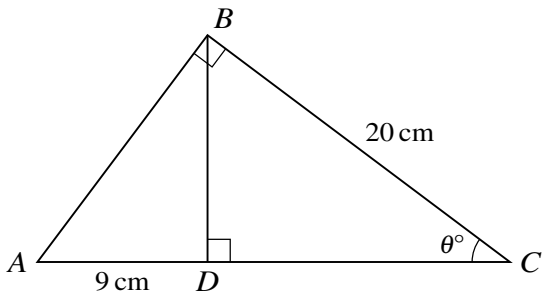

The diagram shows a triangle ABC in which $BC = 20$ cm and angle $ABC = 90°$. The perpendicular from B to AC meets AC at D and $AD = 9$ cm. Angle $BCA = \theta°$.

(i) By expressing the length of BD in terms of θ in each of the triangles ABD and DBC, show that $20 \sin^2 \theta = 9 \cos \theta$. [4]

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(ii) Hence, showing all necessary working, calculate θ. [3]

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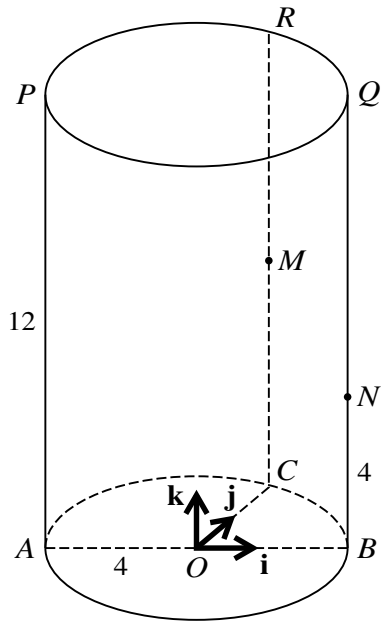

The diagram shows a solid cylinder standing on a horizontal circular base with centre O and radius 4 units. Points A, B and C lie on the circumference of the base such that AB is a diameter and angle $BOC = 90°$. Points P, Q and R lie on the upper surface of the cylinder vertically above A, B and C respectively. The height of the cylinder is 12 units. The mid-point of CR is M and N lies on BQ with $BN = 4$ units.

Unit vectors **i** and **j** are parallel to OB and OC respectively and the unit vector **k** is vertically upwards.

Evaluate $\overrightarrow{PN}.\overrightarrow{PM}$ and hence find angle MPN. [7]

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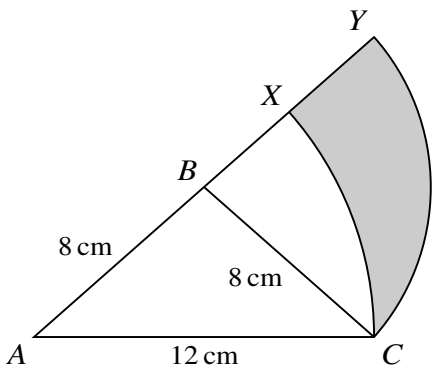

The diagram shows an isosceles triangle ACB in which $AB = BC = 8$ cm and $AC = 12$ cm. The arc XC is part of a circle with centre A and radius 12 cm, and the arc YC is part of a circle with centre B and radius 8 cm. The points A, B, X and Y lie on a straight line.

(i) Show that angle $CBY = 1.445$ radians, correct to 4 significant figures. [3]

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(ii) Find the perimeter of the shaded region. [4]

9 The function f is defined by $f : x \mapsto 2x^2 - 12x + 7$ for $x \in \mathbb{R}$.

(i) Express $2x^2 - 12x + 7$ in the form $2(x + a)^2 + b$, where a and b are constants. [2]

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(ii) State the range of f. [1]

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The function g is defined by $g : x \mapsto 2x^2 - 12x + 7$ for $x \leqslant k$.

(iii) State the largest value of k for which g has an inverse. [1]

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(iv) Given that g has an inverse, find an expression for $g^{-1}(x)$. [3]

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10 The equation of a curve is $y = 2x + \frac{12}{x}$ and the equation of a line is $y + x = k$, where k is a constant.

(i) Find the set of values of k for which the line does not meet the curve. [3]

In the case where $k = 15$, the curve intersects the line at points A and B.

(ii) Find the coordinates of A and B. [3]

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(iii) Find the equation of the perpendicular bisector of the line joining A and B. [3]

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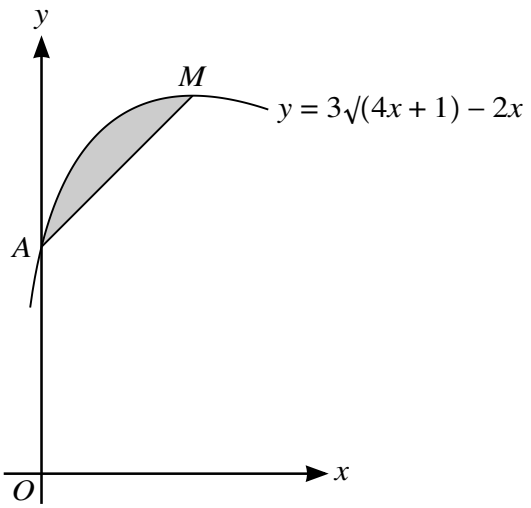

The diagram shows part of the curve $y = 3\sqrt{(4x + 1)} - 2x$. The curve crosses the y-axis at A and the stationary point on the curve is M.

(i) Obtain expressions for $\frac{dy}{dx}$ and $\int y\,dx$. [5]

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(ii) Find the coordinates of M. [3]

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(iii) Find, showing all necessary working, the area of the shaded region. [4]

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Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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