#### **CAMBRIDGE INTERNATIONAL EXAMINATIONS**

GCE Advanced Subsidiary Level and GCE Advanced Level

## MARK SCHEME for the October/November 2013 series

# 9709 MATHEMATICS

**9709/13** Paper 1, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2013 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.



Page 2	Mark Scheme	Syllabus	Paper
	GCE A LEVEL – October/November 2013	9709	13

### **Mark Scheme Notes**

Marks are of the following three types:

- M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B Mark for a correct result or statement independent of method marks.
- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep\*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

Page 3	Mark Scheme	Syllabus	Paper
	GCE A LEVEL – October/November 2013	9709	13

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)	
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)	
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)	
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)	
CWO	Correct Working Only – often written by a 'fortuitous' answer	
ISW	Ignore Subsequent Working	
MR	Misread	
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)	
SOS	See Other Solution (the candidate makes a better attempt at the same question)	

### **Penalties**

- MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through "marks. MR is not applied when the candidate misreads his own figures this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
- PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Page 4	Page 4 Mark Scheme		Paper
GCE A LEVEL – October/November 2013		9709	13

1	(x+1)(x-2) or other valid method -1, 2 x < -1, x > 2	M1 A1 A1	Attempt soln of eqn or other method  Penalise $\leq$ , $\geq$
		[3]	
2	$f(x) = 2x^{-\frac{1}{2}} + x (+c)$	M1A1	Attempt integ $x^{-\frac{1}{2}}$ or $+x$ needed for M
	$5 = -2 \times \frac{1}{2} + 4 + c$	M1	Sub (4, 5). c must be present
	c = 2	A1 [4]	
3	(i) gradient of perpendicular = $-\frac{1}{2}$ soi $y - 1 = -\frac{1}{2}(x - 3)$	B1 B1 [2]	
	(ii) $C = (-9, 6)$ $AC^2 = [3 - (-9)]^2 + [1 - 6]^2$ (ft on their C) AC = 13	B1 M1 A1 [3]	soi in (i) or (ii) <b>OR</b> $AB^2 = [3-(-21)]^2 + [1-11]^2$ M1 AB = 26 A1 AC = 13 A1
4	(i) $OD = 4i + 3j$ CD = 4i + 3j - 10k	B1 B1√ [2]	$\sqrt{}$ for $\mathbf{OD} - \mathbf{10k}$
	(ii) OD.CD = 9 + 16 = 25  OD  = $\sqrt{25}$ or  CD  = $\sqrt{125}$ $25 = \sqrt{25} \times \sqrt{125} \times \cos \theta$ oe ODC = 63.4° (or 1.11 rads)	M1 M1 M1 A1 [4]	Use of $x_1x_2 + y_1y_2 + z_1z_2$ Correct method for moduli All connected correctly
5	(a) $\frac{a}{1-r} = 8a \Rightarrow 1(a) = 8(a)(1-r)$ $r = \frac{7}{8}$ oe	B1 B1 [2]	
	<b>(b)</b> $a + 4d = 197$	B1	Or $2a + 9d = 408$
	$\frac{10}{2}[2a+9d] = 2040$ $d = 14$	B1 M1A1 [4]	Attempt to solve simultaneously
6	(i) sector areas are $\frac{1}{2}11^2\alpha, \frac{1}{2}5^2\alpha$	B1	Sight of 11 <sup>2</sup> , 5 <sup>2</sup>
	$k = \frac{\frac{1}{2} \times 11^2 \alpha - \frac{1}{2} \times 5^2 \alpha}{\frac{1}{2} \times 5^2 \alpha}$	M1	Or $\frac{11^2 - 5^2}{5^2}$
	$k = \frac{96}{25}$ or 3.84	A1 [3]	

Page 5	Mark Scheme	Syllabus	Paper
GCE A LEVEL – October/November 2013		9709	13

(ii) perimeter shaded region = $11a + 5a + 6 + 6 = 16a + 12 = 16a + 12 = 16a + 12 = 16a + 12 = 16a + 13 = 181 = 16a + 12 = 16a + 13 = 181$			1	<del>                                     </del>
perimeter unshaded region = $5\alpha + 5 + 5 = 5\alpha + 10$ $15\alpha + 12 = 2 (5\alpha + 10)$ $\alpha = 4/3 \text{ or } 1.33$ M1  A1A1  A1A1  (b) $2\theta + \frac{\pi}{3} = \frac{5\pi}{6} \left( \text{ or } \frac{13\pi}{6} \text{ or } \frac{\pi}{6} \right)$ $2\theta = \frac{\pi}{2} = \left( \text{ or } \frac{11\pi}{6} \right)$ $\theta = \frac{\pi}{4}, \frac{11\pi}{12}$ M1  A1A1  (ii) $10 \times 3^3 (x^8)$ soi leading to their answer $270 (x^8)$ B1  (iii) $k \times (i)$ $405 \sin i$ $+(iii)$ $675 (x^8)$ M1  A1C  M1  A1C  M1  B1  [1]  B1  B1  B1 for 10, 5C2 or 5C3. B1 for $3^3$ . But must be multiplied.  B1  [3]  M1  A1  A1  A1  A1  A1  A1  A1  A1  A1			B1	
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		perimeter unshaded region = $5\alpha + 5 + 5 =$	B1	
$ α = 4/3 \text{ or } 1.33 $ A1 [4]  7 (a) $x^2 - 1 = \sin \frac{\pi}{3}$ $x = \pm 1.366$ (b) $2\theta + \frac{\pi}{3} = \frac{5\pi}{6} \left( \text{ or } \frac{13\pi}{6} \text{ or } \frac{\pi}{6} \right)$ $2\theta = \frac{\pi}{2} = \left( \text{ or } \frac{11\pi}{6} \right)$ $\theta = \frac{\pi}{4}, \frac{11\pi}{12}$ A1A1 [4]  8 (i) 81 (x <sup>8</sup> ) (ii) 10 × 3 <sup>3</sup> (x <sup>8</sup> ) soi leading to their answer $270 (x^8)$ B1 [1] (iii) k × (i) $405 \text{ soi} \\ + \text{ (iii)} \\ 675 (x^8)$ A1A1 [4]  9 $\frac{dy}{dx} = -k^2(x + 2)^{-2} + 1 = 0$ $x + 2 = \pm k$ $x = -2 \pm k$ $\frac{d^2y}{dx^2} = 2k^2(x + 2)^{-3}$ M1 A1 Attempt to differentiation & set to zero A1 Attempt to differentiate again A1 Attempt to differentiate again A1 Attempt to differentiate again M1 Attempt to differentiate again M1 Sub their x value with k in it into $\frac{d^3y}{dx^2}$ When $x = -2 = k$ , $\frac{d^2y}{dx^2} = \left(\frac{2}{k}\right)$ which is (<0) A1 When $x = -2 - k$ , $\frac{d^2y}{dx^2} = \left(\frac{2}{k}\right)$ which is (<0) M1 A1 When $x = -2 - k$ , $\frac{d^2y}{dx^2} = \left(\frac{2}{k}\right)$ which is (<0) M1 A1 Only 1 of bracketed items needed for each but $\frac{d^2y}{dx^2}$ and x need to be correct.			M1	
[4]     7				
7 (a) $x^3 - 1 = \sin \frac{\pi}{3}$ $x = \pm 1.366$ AlAl\[ \begin{align*} \sqrt{1} & \text{ for negative of } 1^{st} \text{ answer} \]  (b) $2\theta + \frac{\pi}{3} = \frac{5\pi}{6} \left( \text{ or } \frac{13\pi}{6} \text{ or } \frac{\pi}{6} \right)$ $2\theta = \frac{\pi}{2} = \left( \text{ or } \frac{11\pi}{6} \right)$ $\theta = \frac{\pi}{4} \cdot \frac{11\pi}{12}$ AlAl\[ \text{ soc decimals } 0.785 & 2.88 \text{ scores M1B1} \]  8 (i) 81 ( $x^8$ )  (ii) $10 \times 3^3$ ( $x^8$ ) soi leading to their answer $270$ ( $x^8$ )  B1  (iii) $k \times (i)$ $40^5$ soi $41^5$ $10^5$				
7 (a) $x^3 - 1 = \sin \frac{\pi}{3}$ $x = \pm 1.366$ AlAl\[ \begin{align*} \sqrt{1} & \text{ for negative of } 1^{st} \text{ answer} \]  (b) $2\theta + \frac{\pi}{3} = \frac{5\pi}{6} \left( \text{ or } \frac{13\pi}{6} \text{ or } \frac{\pi}{6} \right)$ $2\theta = \frac{\pi}{2} = \left( \text{ or } \frac{11\pi}{6} \right)$ $\theta = \frac{\pi}{4} \cdot \frac{11\pi}{12}$ AlAl\[ \text{ soc decimals } 0.785 & 2.88 \text{ scores M1B1} \]  8 (i) 81 ( $x^8$ )  (ii) $10 \times 3^3$ ( $x^8$ ) soi leading to their answer $270$ ( $x^8$ )  B1  (iii) $k \times (i)$ $40^5$ soi $41^5$ $10^5$		$\pi$	M1	
(b) $2\theta + \frac{\pi}{3} = \frac{5\pi}{6} \left( \text{ or } \frac{13\pi}{6} \text{ or } \frac{\pi}{6} \right)$ $2\theta = \frac{\pi}{2} = \left( \text{ or } \frac{11\pi}{6} \right)$ $\theta = \frac{\pi}{4} \cdot \frac{11\pi}{2}$ M1 Isolating $2\theta$ $\theta = \frac{\pi}{4} \cdot \frac{11\pi}{2}$ SC decimals $0.785 \& 2.88 \text{ scores M1B1}$ 8 (i) $81 (x^8)$ B1 [1] (ii) $10 \times 3^3 (x^8)$ soi leading to their answer $270 (x^8)$ B1 [3] (iii) $k \times (i)$ $405 \text{ soi}$ $+ (ii)$ $675 (x^8)$ M1 M1 M1 M1 M1 M1 M2 M2 M3 M4	7	(a) $x^2 - 1 = \sin \frac{\pi}{3}$		
(b) $2\theta + \frac{\pi}{3} = \frac{5\pi}{6} \left( \text{or } \frac{13\pi}{6} \text{ or } \frac{\pi}{6} \right)$ $2\theta = \frac{\pi}{2} = \left( \text{or } \frac{11\pi}{6} \right)$ $\theta = \frac{\pi}{4} \cdot \frac{11\pi}{12}$ M1 Isolating $2\theta$ $\theta = \frac{\pi}{4} \cdot \frac{11\pi}{12}$ SC decimals $0.785 \& 2.88 \text{ scores M1B1}$ 8 (i) $81 (x^8)$ B1 [1] (ii) $10 \times 3^3 (x^8)$ soi leading to their answer $270 (x^8)$ B1 [1] (iii) $k \times (i)$ $405 \text{ soi}$ $+ (ii)$ $675 (x^8)$ M1 A1 DM1 A1 [4]  9 $\frac{dy}{dx} = -k^2 (x+2)^{-2} + 1 = 0$ $x+2 = \pm k$ $x = -2 \pm k$ A1 DM1 A1 A1 Attempt to differentiation & set to zero A1 Attempt to solve cao Attempt to solve cao Attempt to differentiate again M1 Attempt to differentiate again M2 Attempt to differentiate again M1 Sub their $x$ value with $k$ in it into $\frac{d^2y}{dx^2}$ When $x = -2 = k$ , $\frac{d^2y}{dx^2} = \left(\frac{2}{k}\right)$ which is $(>0)$ min When $x = -2 - k$ , $\frac{d^2y}{dx^2} = \left(\frac{2}{k}\right)$ which is $(<0)$ M1 A1 Dny Attempt to differentiate again M1 Only 1 of bracketed items needed for each When $x = -2 - k$ , $\frac{d^2y}{dx^2} = \left(\frac{2}{k}\right)$ which is $(<0)$ M1 Dny Attempt to be be correct.		$x = \pm 1.366$	A1A1 <sup>∧</sup>	for negative of 1 <sup>st</sup> answer
$2\theta = \frac{\pi}{2} = \left(\text{or } \frac{11\pi}{6}\right)$ $\theta = \frac{\pi}{4}, \frac{11\pi}{12}$ $8  \textbf{(i)}  81  (x^8)$ $(\textbf{ii)}  10 \times 3^3  (x^8) \text{ soi leading to their answer}$ $270  (x^8)$ $\theta = \frac{\pi}{4}, \frac{11\pi}{12}$ $\theta = \frac{\pi}{4}, \frac{11\pi}{4}$ $\theta = \frac{\pi}{4}, $			[3]	
$2\theta = \frac{\pi}{2} = \left(\text{or } \frac{11\pi}{6}\right)$ $\theta = \frac{\pi}{4}, \frac{11\pi}{12}$ $8  \textbf{(i)}  81  (x^8)$ $(\textbf{ii)}  10 \times 3^3  (x^8) \text{ soi leading to their answer}$ $270  (x^8)$ $\theta = \frac{\pi}{4}, \frac{11\pi}{12}$ $\theta = \frac{\pi}{4}, \frac{11\pi}{4}$ $\theta = \frac{\pi}{4}, $		π 5π ( 12π π)		
$2\theta = \frac{\pi}{2} = \left(\text{or } \frac{11\pi}{6}\right)$ $\theta = \frac{\pi}{4}, \frac{11\pi}{12}$ $8  \textbf{(i)}  81  (x^8)$ $(\textbf{ii)}  10 \times 3^3  (x^8) \text{ soi leading to their answer}$ $270  (x^8)$ $\theta = \frac{\pi}{4}, \frac{11\pi}{12}$ $\theta = \frac{\pi}{4}, \frac{11\pi}{4}$ $\theta = \frac{\pi}{4}, $		<b>(b)</b> $2\theta + \frac{\pi}{3} = \frac{3\pi}{6} \left( \text{ or } \frac{13\pi}{6} \text{ or } \frac{\pi}{6} \right)$	B1	1 correct angle on RHS is sufficient
$\theta = \frac{\pi}{4}, \frac{11\pi}{12}$ $A1A1 \begin{bmatrix} 4 \end{bmatrix}$ SC decimals 0.785 & 2.88 scores M1B1  8 (i) 81 ( $x^8$ ) $(ii) 10 \times 3^3 (x^8) \text{ soi leading to their answer}$ $270 (x^8) B1 \begin{bmatrix} 11 \end{bmatrix} (iii) k \times (i) \\ 405 \text{ soi} \\ + (ii) \\ 675 (x^8) A1 \\ DM1 \\ A1 \end{bmatrix} A1 \begin{bmatrix} 4 \end{bmatrix}$				
8 (i) $81 (x^8)$ (ii) $10 \times 3^3 (x^8)$ soi leading to their answer $270 (x^8)$ B1  (iii) $k \times (i)$ $405 \text{ soi}$ $+ (ii)$ $675 (x^8)$ M1  M1A1  A1  [4]  9 $\frac{dy}{dx} = -k^2(x+2)^{-2} + 1 = 0$ $x+2 = \pm k$ $x = -2 \pm k$ $\frac{d^2y}{dx^2} = 2k^2(x+2)^{-3}$ M1A1  M1A1  Attempt to solve cao  M1Atempt to differentiate again  M1  Sub their $x$ value with $k$ in it into $\frac{d^2y}{dx^2}$ When $x = -2 = k$ , $\frac{d^2y}{dx^2} = \left(\frac{2}{k}\right)$ which is $(<0)$ min  When $x = -2 - k$ , $\frac{d^2y}{dx^2} = \left(\frac{2}{-k}\right)$ which is $(<0)$ M1  M1  M1  M1  Attempt to differentiate again  M1  Sub their $x$ value with $k$ in it into $\frac{d^2y}{dx^2}$ Only 1 of bracketed items needed for each but $\frac{d^2y}{dx^2}$ and $x$ need to be correct.		$2\theta = \frac{\pi}{2} = \left( \text{ or } \frac{\pi}{6} \right)$	M1	Isolating $2\theta$
8 (i) $81 (x^8)$ (ii) $10 \times 3^3 (x^8)$ soi leading to their answer $270 (x^8)$ B1  (iii) $k \times (i)$ $405 \text{ soi}$ $+ (ii)$ $675 (x^8)$ M1  M1A1  A1  [4]  9 $\frac{dy}{dx} = -k^2(x+2)^{-2} + 1 = 0$ $x+2 = \pm k$ $x = -2 \pm k$ $\frac{d^2y}{dx^2} = 2k^2(x+2)^{-3}$ M1A1  M1A1  Attempt to solve cao  M1Atempt to differentiate again  M1  Sub their $x$ value with $k$ in it into $\frac{d^2y}{dx^2}$ When $x = -2 = k$ , $\frac{d^2y}{dx^2} = \left(\frac{2}{k}\right)$ which is $(<0)$ min  When $x = -2 - k$ , $\frac{d^2y}{dx^2} = \left(\frac{2}{-k}\right)$ which is $(<0)$ M1  M1  M1  M1  Attempt to differentiate again  M1  Sub their $x$ value with $k$ in it into $\frac{d^2y}{dx^2}$ Only 1 of bracketed items needed for each but $\frac{d^2y}{dx^2}$ and $x$ need to be correct.		$\theta - \frac{\pi}{2} \frac{11\pi}{2}$	Δ1Δ1	CC decimals 0.795 % 2.99 seems M1D1
(ii) $10 \times 3^3 (x^8)$ soi leading to their answer $270 (x^8)$ B1B1  B1B1  B1 for $10, 5C2$ or $5C3$ . B1 for $3^3$ . But must be multiplied.  (iii) $k \times (i)$ $405$ soi $+(ii)$ $675 (x^8)$ M1  A1  DM1  A1  [4]   9 $\frac{dy}{dx} = -k^2(x+2)^{-2} + 1 = 0$ $x+2=\pm k$ $x=-2\pm k$ $x=-2\pm k$ A1  DM1  A1  Attempt differentiation & set to zero  A1  Attempt to solve cao  M1  Attempt to differentiate again  M1  Attempt to differentiate again  M1  Sub their $x$ value with $k$ in it into $\frac{d^2y}{dx^2}$ When $x=-2=k$ , $\frac{d^2y}{dx^2} = \left(\frac{2}{k}\right)$ which is $(>0)$ min  When $x=-2-k$ , $\frac{d^2y}{dx^2} = \left(\frac{2}{-k}\right)$ which is $(<0)$ M1  A1  Only 1 of bracketed items needed for each  When $x=-2-k$ , $\frac{d^2y}{dx^2} = \left(\frac{2}{-k}\right)$ which is $(<0)$ M2  M1  A1  DM1  A1  Attempt to differentiate again  M1  Sub their $x$ value with $k$ in it into $\frac{d^2y}{dx^2}$ Only 1 of bracketed items needed for each  but $\frac{d^2y}{dx^2}$ and $x$ need to be correct.		4'12		SC decimals 0.763 & 2.00 scores WIDI
(ii) $10 \times 3^3 (x^8)$ soi leading to their answer $270 (x^8)$ B1B1  B1B1  B1 For $10, 5C2$ or $5C3$ . B1 for $3^3$ . But must be multiplied.  (iii) $k \times (i)$ $405$ soi $+(ii)$ $675 (x^8)$ M1  A1  DM1  A1  [4]   9 $\frac{dy}{dx} = -k^2(x+2)^{-2} + 1 = 0$ $x+2=\pm k$ $x=-2\pm k$ A1  Attempt to differentiate again  Attempt again agai	Q	(i) 81 (x <sup>8</sup> )	R1	
(ii) $10 \times 3^3$ ( $x^8$ ) soi leading to their answer $270$ ( $x^8$ )  B1B1 B1 [3]  (iii) $k \times (i)$ $405$ soi $+ (ii)$ $675$ ( $x^8$ )  M1 A1 DM1 A1 [4]  9 $\frac{dy}{dx} = -k^2(x+2)^{-2} + 1 = 0$ $x+2 = \pm k$ $x = -2 \pm k$ A1 $\frac{d^2y}{dx^2} = 2k^2(x+2)^{-3}$ M1 A1 Attempt differentiation & set to zero Attempt to solve cao M1 Attempt to differentiate again M1 Sub their $x$ value with $k$ in it into $\frac{d^2y}{dx^2}$ When $x = -2 = k$ , $\frac{d^2y}{dx^2} = \left(\frac{2}{k}\right)$ which is (>0) min When $x = -2 - k$ , $\frac{d^2y}{dx^2} = \left(\frac{2}{-k}\right)$ which is (<0)  M2  M1 Attempt to differentiate again M1 Sub their $x$ value with $k$ in it into $\frac{d^2y}{dx^2}$ Only 1 of bracketed items needed for each When $x = -2 - k$ , $\frac{d^2y}{dx^2} = \left(\frac{2}{-k}\right)$ which is (<0) M2  M3  M4  M4  M5  M6  M7  M6  M7  M9  M1  M1  M1  M1  M1  M1  M1  M1  M1	0	(i) or (x)		
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$		2 . 9		_
270 ( $x^8$ )  B1  [3]  (iii) $k \times (i)$ 405 soi + (ii) 675 ( $x^8$ )  M1  A1  DM1  A1  [4]  9 $\frac{dy}{dx} = -k^2(x+2)^{-2} + 1 = 0$ $x+2 = \pm k$ $x = -2 \pm k$ A1 $\frac{d^2y}{dx^2} = 2k^2(x+2)^{-3}$ M1  Attempt to differentiate again  M1  Attempt to differentiate again  M1  Sub their $x$ value with $k$ in it into $\frac{d^2y}{dx^2}$ When $x = -2 = k$ , $\frac{d^2y}{dx^2} = \left(\frac{2}{k}\right)$ which is (>0) min  When $x = -2 - k$ , $\frac{d^2y}{dx^2} = \left(\frac{2}{-k}\right)$ which is (<0)  M2  M1  Only 1 of bracketed items needed for each  When $x = -2 - k$ , $\frac{d^2y}{dx^2} = \left(\frac{2}{-k}\right)$ which is (<0)  M2  M1  DM1  Attempt to differentiate again  M1  Sub their $x$ value with $k$ in it into $\frac{d^2y}{dx^2}$ When $x = -2 - k$ , $\frac{d^2y}{dx^2} = \left(\frac{2}{-k}\right)$ which is (<0)  M1  DM1  Attempt to differentiate again  M1  Sub their $x$ value with $k$ in it into $\frac{d^2y}{dx^2}$ When $x = -2 - k$ , $\frac{d^2y}{dx^2} = \left(\frac{2}{-k}\right)$ which is (<0)  M1  DM1  Attempt to differentiate again  M1  Sub their $x$ value with $k$ in it into $\frac{d^2y}{dx^2}$ When $x = -2 - k$ , $\frac{d^2y}{dx^2} = \left(\frac{2}{-k}\right)$ which is (<0)  A1  DM2  Attempt to differentiate again  A1  DM3  A1  A2  DM4  A2  DM5  A2  DM7  A2  DM8  A2  DM9  A2  DM9  A3  DM9  A1  DM9  A2  DM9  A2  DM9  A2  DM9  A3  DM9  A4  A4  DM9  A4  A4  DM9  A4  A4  A4  A4  A4  A4  A4  A4  A4  A		(ii) $10 \times 3^{\circ} (x^{\circ})$ soi leading to their answer	B1B1	· · · · · · · · · · · · · · · · · · ·
(iii) $k \times (i)$ $405 \text{ soi}$ $+ (ii)$ $675 (x^8)$ M1 A1 DM1 A1 [4]  9 $\frac{dy}{dx} = -k^2(x+2)^{-2} + 1 = 0$ $x+2 = \pm k$ $x = -2 \pm k$ A1 A1 DM1 A1 Attempt differentiation & set to zero  M1A1 Attempt to solve cao M1 Attempt to differentiate again M1 Attempt to differentiate again M1 Sub their $x$ value with $k$ in it into $\frac{d^2y}{dx^2}$ When $x = -2 = k$ , $\frac{d^2y}{dx^2} = \left(\frac{2}{k}\right)$ which is $(>0)$ min When $x = -2 - k$ , $\frac{d^2y}{dx^2} = \left(\frac{2}{-k}\right)$ which is $(<0)$ M1 DM1 Attempt to differentiate again M1 Sub their $x$ value with $k$ in it into $\frac{d^2y}{dx^2}$ Unly 1 of bracketed items needed for each but $\frac{d^2y}{dx^2}$ and $x$ need to be correct.		$270 (x^8)$	R1	be multiplied.
(iii) $k \times (i)$ $405 \text{ soi}$ $+ (ii)$ $675 (x^8)$ M1 A1 DM1 A1 [4]  9 $\frac{dy}{dx} = -k^2(x+2)^{-2} + 1 = 0$ $x+2 = \pm k$ $x = -2 \pm k$ A1 $\frac{d^2y}{dx^2} = 2k^2(x+2)^{-3}$ M1  Attempt to solve A1  cao M1  Attempt to differentiate again M1  Sub their $x$ value with $k$ in it into $\frac{d^2y}{dx^2}$ When $x = -2 = k$ , $\frac{d^2y}{dx^2} = \left(\frac{2}{k}\right)$ which is (> 0) min When $x = -2 - k$ , $\frac{d^2y}{dx^2} = \left(\frac{2}{-k}\right)$ which is (< 0)  M2  Attempt to differentiate again M1  Sub their $x$ value with $k$ in it into $\frac{d^2y}{dx^2}$ When $x = -2 - k$ , $\frac{d^2y}{dx^2} = \left(\frac{2}{-k}\right)$ which is (< 0) M2  Only 1 of bracketed items needed for each When $x = -2 - k$ , $\frac{d^2y}{dx^2} = \left(\frac{2}{-k}\right)$ which is (< 0) M3  Attempt to differentiate again M1  Sub their $x$ value with $x$ in it into $x$ in the properties of $x$ and $x$ in th		-··		
405 soi + (ii) 675 ( $x^8$ )  A1 DM1 A1  [4]  9 $\frac{dy}{dx} = -k^2(x+2)^{-2} + 1 = 0$ M1A1 Attempt differentiation & set to zero $x+2=\pm k$ DM1 Attempt to solve cao $\frac{d^2y}{dx^2} = 2k^2(x+2)^{-3}$ M1 Attempt to differentiate again  M1 Sub their $x$ value with $k$ in it into $\frac{d^2y}{dx^2}$ When $x = -2 = k$ , $\frac{d^2y}{dx^2} = \left(\frac{2}{k}\right)$ which is (> 0) min When $x = -2 - k$ , $\frac{d^2y}{dx^2} = \left(\frac{2}{-k}\right)$ which is (< 0) A1 Double of bracketed items needed for each when $x = -2 - k$ , $\frac{d^2y}{dx^2} = \left(\frac{2}{-k}\right)$ which is (< 0) A1 but $\frac{d^2y}{dx^2}$ and $x$ need to be correct.				
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$				$k \neq 1,0$
$\begin{array}{cccccccccccccccccccccccccccccccccccc$				
9 $\frac{dy}{dx} = -k^2(x+2)^{-2} + 1 = 0$ M1A1 Attempt differentiation & set to zero $x+2=\pm k$ DM1 Attempt to solve $x=-2\pm k$ Cao $\frac{d^2y}{dx^2} = 2k^2(x+2)^{-3}$ M1 Attempt to differentiate again  M1 Sub their $x$ value with $k$ in it into $\frac{d^2y}{dx^2}$ When $x=-2=k$ , $\frac{d^2y}{dx^2} = \left(\frac{2}{k}\right)$ which is $(>0)$ min  When $x=-2-k$ , $\frac{d^2y}{dx^2} = \left(\frac{2}{-k}\right)$ which is $(<0)$ A1 but $\frac{d^2y}{dx^2}$ and $x$ need to be correct.				
9 $\frac{dy}{dx} = -k^2(x+2)^{-2} + 1 = 0$ M1A1 Attempt differentiation & set to zero DM1 Attempt to solve cao $\frac{d^2y}{dx^2} = 2k^2(x+2)^{-3}$ M1 Attempt to differentiate again M1 Sub their $x$ value with $k$ in it into $\frac{d^2y}{dx^2}$ When $x = -2 = k$ , $\frac{d^2y}{dx^2} = \left(\frac{2}{k}\right)$ which is (> 0) min When $x = -2 - k$ , $\frac{d^2y}{dx^2} = \left(\frac{2}{-k}\right)$ which is (< 0) A1 but $\frac{d^2y}{dx^2}$ and $x$ need to be correct.		0/3 (A )		
$x + 2 = \pm k$ $x = -2 \pm k$ $\frac{d^2 y}{dx^2} = 2k^2 (x + 2)^{-3}$ M1 Attempt to solve cao $M1$ Sub their $x$ value with $k$ in it into $\frac{d^2 y}{dx^2}$ When $x = -2 = k$ , $\frac{d^2 y}{dx^2} = \left(\frac{2}{k}\right)$ which is $(>0)$ min When $x = -2 - k$ , $\frac{d^2 y}{dx^2} = \left(\frac{2}{-k}\right)$ which is $(<0)$ M1 Only 1 of bracketed items needed for each $M1 = -2 - k$ When $x = -2 - k$ , $\frac{d^2 y}{dx^2} = \left(\frac{2}{-k}\right)$ which is $(<0)$ M1 but $\frac{d^2 y}{dx^2}$ and $x$ need to be correct.		,		
$x + 2 = \pm k$ $x = -2 \pm k$ $\frac{d^2 y}{dx^2} = 2k^2 (x + 2)^{-3}$ M1 Attempt to solve cao $M1$ Sub their $x$ value with $k$ in it into $\frac{d^2 y}{dx^2}$ When $x = -2 = k$ , $\frac{d^2 y}{dx^2} = \left(\frac{2}{k}\right)$ which is $(>0)$ min When $x = -2 - k$ , $\frac{d^2 y}{dx^2} = \left(\frac{2}{-k}\right)$ which is $(<0)$ M1 Only 1 of bracketed items needed for each $M1 = -2 - k$ When $x = -2 - k$ , $\frac{d^2 y}{dx^2} = \left(\frac{2}{-k}\right)$ which is $(<0)$ M1 Dut $\frac{d^2 y}{dx^2}$ and $x$ need to be correct.	9	$\frac{dy}{dx} = -k^2(x+2)^{-2} + 1 = 0$	M1A1	Attempt differentiation & set to zero
$\frac{d^2y}{dx^2} = 2k^2(x+2)^{-3}$ M1 Attempt to differentiate again  M1 Sub their x value with k in it into $\frac{d^2y}{dx^2}$ When $x = -2 = k$ , $\frac{d^2y}{dx^2} = \left(\frac{2}{k}\right)$ which is (> 0) min  When $x = -2 - k$ , $\frac{d^2y}{dx^2} = \left(\frac{2}{-k}\right)$ which is (< 0)  M1 Only 1 of bracketed items needed for each  but $\frac{d^2y}{dx^2}$ and x need to be correct.			DM1	Attempt to solve
When $x = -2 = k$ , $\frac{d^2y}{dx^2} = \left(\frac{2}{k}\right)$ which is (> 0) min When $x = -2 - k$ , $\frac{d^2y}{dx^2} = \left(\frac{2}{-k}\right)$ which is (< 0) A1 Sub their $x$ value with $k$ in it into $\frac{d^2y}{dx^2}$ Only 1 of bracketed items needed for each but $\frac{d^2y}{dx^2}$ and $x$ need to be correct.			A1	•
When $x = -2 = k$ , $\frac{d^2y}{dx^2} = \left(\frac{2}{k}\right)$ which is (> 0) min When $x = -2 - k$ , $\frac{d^2y}{dx^2} = \left(\frac{2}{-k}\right)$ which is (< 0) A1 Sub their $x$ value with $k$ in it into $\frac{d^2y}{dx^2}$ Only 1 of bracketed items needed for each but $\frac{d^2y}{dx^2}$ and $x$ need to be correct.		$\frac{d^2y}{dx^2} = 2k^2(x+2)^{-3}$	M1	Attempt to differentiate again
When $x = -2 = k$ , $\frac{d^2 y}{dx^2} = \left(\frac{2}{k}\right)$ which is (> 0) min A1 Only 1 of bracketed items needed for each  When $x = -2 - k$ , $\frac{d^2 y}{dx^2} = \left(\frac{2}{-k}\right)$ which is (< 0) A1 but $\frac{d^2 y}{dx^2}$ and $x$ need to be correct.		$dx^2 = 2\pi (x + 2)$		.2
When $x = -2 - k$ , $\frac{d^2 y}{dx^2} = \left(\frac{2}{-k}\right)$ which is (< 0) A1 but $\frac{d^2 y}{dx^2}$ and x need to be correct.			M1	Sub their x value with k in it into $\frac{d^2y}{dx^2}$
max		When $x = -2 = k$ , $\frac{d^2 y}{dx^2} = \left(\frac{2}{k}\right)$ which is (> 0) min	A1	Only 1 of bracketed items needed for each
max		When $x = -2 - k$ , $\frac{d^2 y}{dx^2} = \left(\frac{2}{-k}\right)$ which is $(< 0)$	A1	but $\frac{d^2y}{dx^2}$ and x need to be correct.
[8]				
			[8]	

Page 6	Mark Scheme	Mark Scheme Syllabus Pa			
	GCE A LEVEL – October/November 2013	9709	13		

10 (i)	Range is $(y) \ge c^2 + 4c$	B1	Allow >
	$x^2 + 4x = (x+2)^2 - 4$	M1	$\mathbf{OR} \ \frac{\mathrm{d}y}{\mathrm{d}x} = 2x + 4 = 0$
	(Smallest value of $c$ is) $-2$	A1 [3]	-2 with no (wrong) working gets B2
(ii)	$5a + b = 11$ $(a + b)^{2} + 4 (a + b) = 21$ $(11 - 5a + a)^{2} + 4 (11 - 5a + a) = 21$ $(8) (2a^{2} - 13a + 18) = (8) (2a - 9) (a - 2)$ $= 0$ $a = \frac{9}{2}, 2 \text{ OR } b = \left(-\frac{23}{2}\right), 1$	B1 B1 M1 M1	OR corresponding equation in $b$ OR (8) $(2b + 23) (b - 1) = 0$ A1 for either $a$ or $b$ correct. Condone $2^{nd}$
		A1 [6]	value. Spotted solution scores only B marks.
Alt.	(ii) Last 5 marks $f^{-1}(x) = \sqrt{x+4} - 2$ B1 $g(1) = f^{-1} = (21)$ used M1 $a+b = \sqrt{25} - 2 = 3$ A1 Solve $a+b=3$ , $5a+b=11$ M1 a=2, $b=1$ A1		Alt. (ii) Last 4 marks $(a+b+7) (a+b-3) = 0$ M1A1 (Ignore solution involving $a+b=-7$ ) Solve $a+b=3$ , $5a+b=11$ M1 $a=2$ , $b=1$ A1
11 (i)	$\frac{dy}{dx} = \left[\frac{1}{2}(x4 + 4x + 4)^{-\frac{1}{2}}\right] \times \left[4x^3 + 4\right]$ At $x = 0$ , $\frac{dy}{dx} = \frac{1}{2} \times \frac{1}{2} \times 4 = (1)$ Equation is $y - 2 = x$	B1B1 M1 A1 [4]	Sub $x = 0$ and attempt eqn of line following differentiation.
(ii)	$x + 2 = \sqrt{x^4 + 4x + 4} \Rightarrow (x + 2)^2$ = $x^4 + 4x + 4$ $x^2 - x^4 = 0$ oe $x = 0, \pm 1$	B1 B1 B2,1,0 [4]	AG www
(iii	$(\pi) \left[ \frac{x^5}{5} + 2x^2 + 4x \right]$	M1A1	Attempt to integrate $y^2$
	$(\pi)\bigg[0-\bigg(\frac{-1}{5}+2-4\bigg)\bigg]$	DM1	
	$\frac{11\pi}{5}$ (6.91) oe	A1 [4]	Apply limits $-1 \rightarrow 0$