UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2011**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **3** printed pages and **1** blank page.

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1 Using the substitution $u = e^x$, or otherwise, solve the equation

$$e^x = 1 + 6e^{-x},$$

giving your answer correct to 3 significant figures. [4]

2 The parametric equations of a curve are

$$x = 3(1 + \sin^2 t), \quad y = 2\cos^3 t.$$

Find $\frac{dy}{dx}$ in terms of t, simplifying your answer as far as possible. [5]

3 The polynomial $x^4 + 3x^3 + ax + 3$ is denoted by $p(x)$. It is given that $p(x)$ is divisible by $x^2 - x + 1$.

(i) Find the value of a. [4]

(ii) When a has this value, find the real roots of the equation $p(x) = 0$. [2]

4 The variables x and θ are related by the differential equation

$$\sin 2\theta \frac{dx}{d\theta} = (x + 1)\cos 2\theta,$$

where $0 < \theta < \frac{1}{2}\pi$. When $\theta = \frac{1}{12}\pi$, $x = 0$. Solve the differential equation, obtaining an expression for x in terms of θ, and simplifying your answer as far as possible. [7]

5 **(i)** By sketching a suitable pair of graphs, show that the equation

$$\sec x = 3 - \tfrac{1}{2}x^2,$$

where x is in radians, has a root in the interval $0 < x < \frac{1}{2}\pi$. [2]

(ii) Verify by calculation that this root lies between 1 and 1.4. [2]

(iii) Show that this root also satisfies the equation

$$x = \cos^{-1}\left(\frac{2}{6 - x^2}\right).$$ [1]

(iv) Use an iterative formula based on the equation in part **(iii)** to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

6 **(i)** Express $\cos x + 3\sin x$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$, giving the exact value of R and the value of α correct to 2 decimal places. [3]

(ii) Hence solve the equation $\cos 2\theta + 3\sin 2\theta = 2$, for $0^\circ < \theta < 90^\circ$. [5]

7 With respect to the origin O, the position vectors of two points A and B are given by $\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$ and $\overrightarrow{OB} = 3\mathbf{i} + 4\mathbf{j}$. The point P lies on the line through A and B, and $\overrightarrow{AP} = \lambda\overrightarrow{AB}$.

(i) Show that $\overrightarrow{OP} = (1 + 2\lambda)\mathbf{i} + (2 + 2\lambda)\mathbf{j} + (2 - 2\lambda)\mathbf{k}$. [2]

(ii) By equating expressions for $\cos AOP$ and $\cos BOP$ in terms of λ, find the value of λ for which OP bisects the angle AOB. [5]

(iii) When λ has this value, verify that $AP : PB = OA : OB$. [1]

8 Let $\mathrm{f}(x) = \dfrac{12 + 8x - x^2}{(2 - x)(4 + x^2)}$.

(i) Express $\mathrm{f}(x)$ in the form $\dfrac{A}{2 - x} + \dfrac{Bx + C}{4 + x^2}$. [4]

(ii) Show that $\displaystyle\int_0^1 \mathrm{f}(x)\,\mathrm{d}x = \ln\left(\tfrac{25}{2}\right)$. [5]

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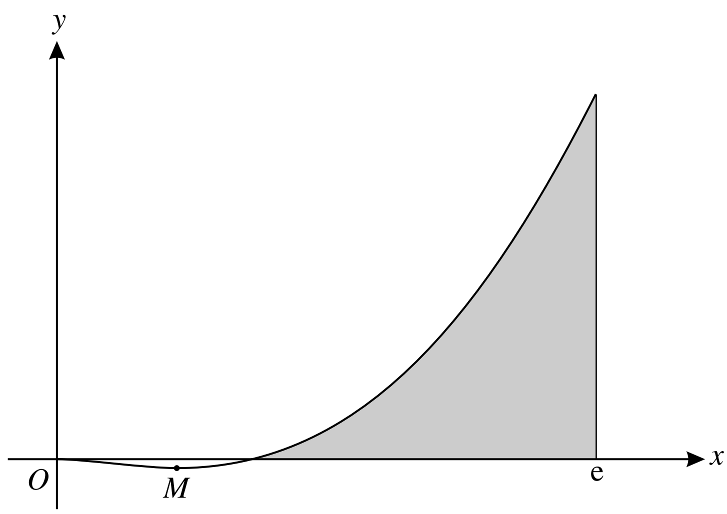

The diagram shows the curve $y = x^2 \ln x$ and its minimum point M.

(i) Find the exact values of the coordinates of M. [5]

(ii) Find the exact value of the area of the shaded region bounded by the curve, the x-axis and the line $x = \mathrm{e}$. [5]

10 **(a)** Showing your working, find the two square roots of the complex number $1 - (2\sqrt{6})\mathrm{i}$. Give your answers in the form $x + \mathrm{i}y$, where x and y are exact. [5]

(b) On a sketch of an Argand diagram, shade the region whose points represent the complex numbers z which satisfy the inequality $|z - 3\mathrm{i}| \leqslant 2$. Find the greatest value of $\arg z$ for points in this region. [5]

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