UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the October/November 2009 question paper

for the guidance of teachers

9709 MATHEMATICS

9709/31 Paper 31, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1	*EITHER*:	State or imply non-modular inequality $(2-3x)^2 < (x-3)^2$, or corresponding equation, and make a reasonable solution attempt at a 3-term quadratic	M1	
		Obtain critical value $x = -\frac{1}{2}$	A1	
		Obtain $x > -\frac{1}{2}$	A1	
		Fully justify $x > -\frac{1}{2}$ as only answer	A1	
	*OR*1:	State the relevant critical linear equation, i.e. $2-3x = 3-x$	B1	
		Obtain critical value $x = -\frac{1}{2}$	B1	
		Obtain $x > -\frac{1}{2}$	B1	
		Fully justify $x > -\frac{1}{2}$ as only answer	B1	
	*OR*2:	Obtain the critical value $x = -\frac{1}{2}$ by inspection, or by solving a linear inequality	B2	
		Obtain $x > -\frac{1}{2}$	B1	
		Fully justify $x > -\frac{1}{2}$ as only answer	B1	
	*OR*3:	Make recognisable sketches of $y = 2-3x$ and $y = \lvert x-3 \rvert$ on a single diagram	B1	
		Obtain critical value $x = -\frac{1}{2}$	B1	
		Obtain $x > -\frac{1}{2}$	B1	
		Fully justify $x > -\frac{1}{2}$ as only answer	B1	[4]
		[Condone $\geqslant$ for $>$ in the third mark but not the fourth.]		

2	*EITHER*:	Use laws of indices correctly and solve a linear equation for 3^x, or for 3^{-x}	M1	
		Obtain 3^x, or 3^{-x} in any correct form, e.g. $3^x = \dfrac{3^2}{(3^2-1)}$	A1	
		Use correct method for solving $3^{\pm x} = a$ for x, where $a > 0$	M1	
		Obtain answer $x = 0.107$	A1	
	OR:	State an appropriate iterative formula, e.g. $x_{n+1} = \dfrac{\ln(3^{x_n}+9)}{\ln 3} - 2$	B1	
		Use the formula correctly at least once	M1	
		Obtain answer $x = 0.107$	A1	
		Show that the equation has no other root but 0.107	A1	[4]
		[For the solution 0.107 with no relevant working, award B1 and a further B1 if 0.107 is shown to be the only root.]		

3	**(i)**	Use the iterative formula correctly at least once	M1	
		State final answer 2.78	A1	
		Show sufficient iterations to at least 4 d.p. to justify its accuracy to 2 d.p., or show there is a sign change in an appropriate function in (2.775, 2.785)	A1	[3]
	(ii)	State a suitable equation, e.g. $x = \dfrac{3}{4}x + \dfrac{15}{x^3}$	B1	
		State that the exact value of α is $\sqrt[4]{60}$, or equivalent	B1	[2]

4		Use product or quotient rule	M1	
		Obtain derivative in any correct form	A1	
		Equate derivative to zero and obtain an equation of the form $a \sin 2x = b$, or a quadratic in $\tan x$, $\sin^2 x$, or $\cos^2 x$	M1*	
		Carry out correct method for finding one angle	M1(dep*)	
		Obtain answer, e.g. 0.365	A1	
		Obtain second answer 1.206 and no others in the range (allow 1.21)	A1	[6]
		[Ignore answers outside the given range.]		
		[Treat answers in degrees, 20.9° and 69.1°, as a misread.]		
5	**(i)**	*EITHER*: Use double angle formulae correctly to express LHS in terms of trig functions of 2θ	M1	
		Use trig formulae correctly to express LHS in terms of $\sin\theta$, converting at least two terms	M1	
		Obtain expression in any correct form in terms of $\sin\theta$	A1	
		Obtain given answer correctly	A1	
		OR: Use double angle formulae correctly to express RHS in terms of trig functions of 2θ	M1	
		Use trig formulae correctly to express RHS in terms of $\cos 4\theta$ and $\cos 2\theta$	M1	
		Obtain expression in any correct form in terms of $\cos 4\theta$ and $\cos 2\theta$	A1	
		Obtain given answer correctly	A1	[4]
	(ii)	State indefinite integral $\frac{1}{4}\sin 4\theta - \frac{4}{2}\sin 2\theta + 3\theta$, or equivalent	B2	
		(award B1 if there is just one incorrect term)		
		Use limits correctly, having attempted to use the identity	M1	
		Obtain answer $\frac{1}{32}(2\pi - \sqrt{3})$, or any simplified exact equivalent	A1	[4]
6	**(i)**	*EITHER*: State that the position vector of M is $2\mathbf{i} + \mathbf{j} - 2\mathbf{k}$, or equivalent	B1	
		Carry out a correct method for finding the position vector of N	M1	
		Obtain answer $3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$, or equivalent	A1	
		Obtain vector equation of MN in any correct form, e.g. $\mathbf{r} = 2\mathbf{i} + \mathbf{j} - 2\mathbf{k} + \lambda(\mathbf{i} - 3\mathbf{j} + 3\mathbf{k})$	A1	
		OR: State that the position vector of M is $2\mathbf{i} + \mathbf{j} - 2\mathbf{k}$, or equivalent	B1	
		Carry out a correct method for finding a direction vector for MN	M1	
		Obtain answer, e.g. $\mathbf{i} - 3\mathbf{j} + 3\mathbf{k}$, or equivalent	A1	
		Obtain vector equation of MN in any correct form, e.g. $\mathbf{r} = 2\mathbf{i} + \mathbf{j} - 2\mathbf{k} + \lambda(\mathbf{i} - 3\mathbf{j} + 3\mathbf{k})$	A1	[4]
		[SR: The use of $AN = AC/3$ can earn M1A0, but $AN = AC/2$ gets M0A0.]		
	(ii)	State equation of BC in any correct form, e.g. $\mathbf{r} = 3\mathbf{i} + 2\mathbf{j} - 3\mathbf{k} + \mu(\mathbf{i} - 5\mathbf{j} + 5\mathbf{k})$	B1	
		Solve for λ or for μ	M1	
		Obtain correct value of λ, or μ, e.g. $\lambda = 3$, or $\mu = 2$	A1	
		Obtain position vector $5\mathbf{i} - 8\mathbf{j} + 7\mathbf{k}$	A1	[4]
7	**(i)**	Substitute $x = -2 + \mathrm{i}$ in the equation and attempt expansion of $(-2 + \mathrm{i})^3$	M1	
		Use $\mathrm{i}^2 = -1$ correctly at least once and solve for k	M1	
		Obtain $k = 20$	A1	[3]
	(ii)	State that the other complex root is $-2 - \mathrm{i}$	B1	[1]

	(iii)	Obtain modulus $\sqrt{5}$	B1	
		Obtain argument 153.4° or 2.68 radians	B1	[2]
	(iv)	Show point representing u in relatively correct position in an Argand diagram	B1	
		Show vertical line through $z = 1$	B1	
		Show the correct half-lines from u of gradient zero and 1	B1	
		Shade the relevant region	B1	[4]
		[SR: For parts **(i)** and **(ii)** allow the following alternative method:		
		State that the other complex root is –2 – i	B1	
		State quadratic factor $x^2 + 4x + 5$	B1	
		Divide cubic by 3-term quadratic, equate remainder to zero and solve for k, or, using 3-term quadratic, factorise cubic and obtain k	M1	
		Obtain $k = 20$	A1]	
8	**(i)**	State or imply partial fractions are of the form $\dfrac{A}{x+1}+\dfrac{B}{(x+1)^2}+\dfrac{C}{3x+2}$	B1	
		Use any relevant method to obtain a constant	M1	
		Obtain one of the values $A = 1$, $B = 2$, $C = -3$	A1	
		Obtain a second value	A1	
		Obtain the third value	A1	[5]
	(ii)	Use correct method to obtain the first two terms of the expansion of $(x + 1)^{-1}$, $(x + 1)^{-2}$, $(3x + 2)^{-1}$ or $(1 + \frac{3}{2}x)^{-1}$	M1	
		Obtain correct unsimplified expansion up to the term in x^2 of each partial fraction	A1√ + A1√ + A1√	
		Obtain answer $\frac{3}{2}-\frac{11}{4}x+\frac{29}{8}x^2$, or equivalent	A1	[5]

[Symbolic binomial coefficients, e.g. $\begin{pmatrix}-1\\1\end{pmatrix}$, are not sufficient for the first M1. The f.t. is on A, B, C.]

[The form $\dfrac{Dx+E}{(x+1)^2}+\dfrac{C}{3x+2}$, where $D = 1$, $E = 3$, $C = -3$, is acceptable. In part **(i)** give B1M1A1A1A1.
In part **(ii)** give M1A1√A1√ for the expansions, and, if $DE \neq 0$, M1 for multiplying out fully and A1 for the final answer.]
[If B or C omitted from the form of fractions, give B0M1A0A0A0 in **(i)**; M1A1√A1√ in **(ii)**, max 4/10]
[If D or E omitted from the form of fractions, give B0M1A0A0A0 in **(i)**; M1A1√A1√ in **(ii)**, max 4/10]
[In the case of an attempt to expand $(5x + 3)(x + 1)^{-2}(3x + 2)^{-1}$, give M1A1A1 for the expansions, M1 for multiplying out fully, and A1 for the final answer.]
[Allow use of Maclaurin, giving M1A1√A1√ for differentiating and obtaining $f(0) = \frac{3}{2}$ and $f'(0) = -\frac{11}{4}$, A1√ for $f''(0) = \frac{29}{4}$, and A1 for the final answer (the f.t. is on A, B, C if used).]

9	**(i)**	State coordinates (1, 0)	B1	[1]
	(ii)	Use correct quotient or product rule	M1	
		Obtain derivative in any correct form	A1	
		Equate derivative to zero and solve for x	M1	
		Obtain $x = e^2$ correctly	A1	[4]

	(iii)	Attempt integration by parts reaching $a\sqrt{x}\ln x \pm a\int \sqrt{x}\,\frac{1}{x}\,\mathrm{d}x$	M1*	
		Obtain $2\sqrt{x}\ln x - 2\int \frac{1}{\sqrt{x}}\,\mathrm{d}x$	A1	
		Integrate and obtain $2\sqrt{x}\ln x - 4\sqrt{x}$	A1	
		Use limits $x = 1$ and $x = 4$ correctly, having integrated twice	M1(dep*)	
		Justify the given answer	A1	[5]
10	**(i)**	State or imply $\frac{\mathrm{d}A}{\mathrm{d}t} = kV$	M1*	
		Obtain equation in r and $\frac{\mathrm{d}r}{\mathrm{d}t}$, e.g. $8\pi r\frac{\mathrm{d}r}{\mathrm{d}t} = k\frac{4}{3}\pi r^3$	A1	
		Use $\frac{\mathrm{d}r}{\mathrm{d}t} = 2$, $r = 5$ to evaluate k	M1(dep*)	
		Obtain given answer	A1	[4]
	(ii)	Separate variables correctly and integrate both sides	M1	
		Obtain terms $-\frac{1}{r}$ and $0.08t$, or equivalent	A1 + A1	
		Evaluate a constant or use limits $t = 0$, $r = 5$ with a solution containing terms of the form $\frac{a}{r}$ and bt	M1	
		Obtain solution $r = \frac{5}{(1 - 0.4t)}$, or equivalent	A1	[5]
	(iii)	State the set of values $0 \leqslant t < 2.5$, or equivalent [Allow $t < 2.5$ and $0 < t < 2.5$ to earn B1.]	B1	[1]