UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/07**

Paper 7 Probability & Statistics 2 **(S2)**

October/November 2006

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles
in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger
numbers of marks later in the paper.
At the end of the examination, fasten all your work securely together.

This document consists of **3** printed pages and **1** blank page.

UNIVERSITY *of* CAMBRIDGE
International Examinations

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1 The time taken for Samuel to drive home from work is distributed with mean 46 minutes. Samuel discovers a different route and decides to test at the 5% level whether the mean time has changed. He tries this route on a large number of different days chosen randomly and calculates the mean time.

    (i) State the null and alternative hypotheses for this test. [1]

    (ii) Samuel calculates the value of his test statistic z to be -1.729. What conclusion can he draw? [2]

2 **(i)** Write down the mean and variance of the distribution of the means of random samples of size n taken from a very large population having mean μ and variance σ^2. [2]

    (ii) What, if anything, can you say about the distribution of sample means

        (a) if n is large, [1]

        (b) if n is small? [1]

3 A survey was conducted to find the proportion of people owning DVD players. It was found that 203 out of a random sample of 278 people owned a DVD player.

    (i) Calculate a 97% confidence interval for the true proportion of people who own a DVD player. [4]

A second survey to find the proportion of people owning DVD players was conducted at 10 o'clock on a Thursday morning in a shopping centre.

    (ii) Give one reason why this is not a satisfactory sample. [1]

4 In summer, wasps' nests occur randomly in the south of England at an average rate of 3 nests for every 500 houses.

    (i) Find the probability that two villages in the south of England, with 600 houses and 700 houses, have a total of exactly 3 wasps' nests. [3]

    (ii) Use a suitable approximation to estimate the probability of there being fewer than 369 wasps' nests in a town with 64 000 houses. [4]

5 Climbing ropes produced by a manufacturer have breaking strengths which are normally distributed with mean 160 kg and standard deviation 11.3 kg. A group of climbers have weights which are normally distributed with mean 66.3 kg and standard deviation 7.1 kg.

    (i) Find the probability that a rope chosen randomly will break under the combined weight of 2 climbers chosen randomly. [5]

Each climber carries, in a rucksack, equipment amounting to half his own weight.

    (ii) Find the mean and variance of the combined weight of a climber and his rucksack. [3]

    (iii) Find the probability that the combined weight of a climber and his rucksack is greater than 87 kg. [2]

6 Pieces of metal discovered by people using metal detectors are found randomly in fields in a certain area at an average rate of 0.8 pieces per hectare. People using metal detectors in this area have a theory that ploughing the fields increases the average number of pieces of metal found per hectare. After ploughing, they tested this theory and found that a randomly chosen field of area 3 hectares yielded 5 pieces of metal.

   (i) Carry out the test at the 10% level of significance. [6]

   (ii) What would your conclusion have been if you had tested at the 5% level of significance? [1]

Jack decides that he will reject the null hypothesis that the average number is 0.8 pieces per hectare if he finds 4 or more pieces of metal in another ploughed field of area 3 hectares.

   (iii) If the true mean after ploughing is 1.4 pieces per hectare, calculate the probability that Jack makes a Type II error. [3]

7 At a town centre car park the length of stay in hours is denoted by the random variable X, which has probability density function given by

$$f(x) = \begin{cases} kx^{-\frac{3}{2}} & 1 \leqslant x \leqslant 9, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

   (i) Interpret the inequalities $1 \leqslant x \leqslant 9$ in the definition of $f(x)$ in the context of the question. [1]

   (ii) Show that $k = \frac{3}{4}$. [2]

   (iii) Calculate the mean length of stay. [3]

The charge for a length of stay of x hours is $\left(1 - e^{-x}\right)$ dollars.

   (iv) Find the length of stay for the charge to be at least 0.75 dollars [3]

   (v) Find the probability of the charge being at least 0.75 dollars. [2]

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