UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Subsidiary Level

MATHEMATICS **9709/02**

Paper 2 Pure Mathematics 2 **(P2)**

October/November 2006

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
At the end of the examination, fasten all your work securely together.

This document consists of **3** printed pages and **1** blank page.

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1 Solve the inequality $|2x - 1| > |x|$. [4]

2 **(i)** Express 4^x in terms of y, where $y = 2^x$. [1]

(ii) Hence find the values of x that satisfy the equation

$$3(4^x) - 10(2^x) + 3 = 0,$$

giving your answers correct to 2 decimal places. [5]

3 The polynomial $4x^3 - 7x + a$, where a is a constant, is denoted by $p(x)$. It is given that $(2x - 3)$ is a factor of $p(x)$.

(i) Show that $a = -3$. [2]

(ii) Hence, or otherwise, solve the equation $p(x) = 0$. [4]

4 **(i)** Prove the identity

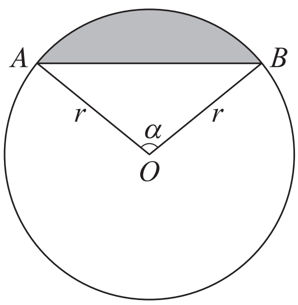
$$\tan(x + 45^\circ) - \tan(45^\circ - x) \equiv 2\tan 2x. \quad [4]$$

(ii) Hence solve the equation

$$\tan(x + 45^\circ) - \tan(45^\circ - x) = 2,$$

for $0^\circ \leqslant x \leqslant 180^\circ$. [3]

5



The diagram shows a chord joining two points, A and B, on the circumference of a circle with centre O and radius r. The angle AOB is α radians, where $0 < \alpha < \pi$. The area of the shaded segment is one sixth of the area of the circle.

(i) Show that α satisfies the equation

$$x = \tfrac{1}{3}\pi + \sin x. \quad [3]$$

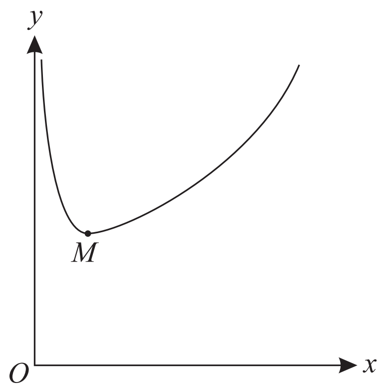
(ii) Verify by calculation that α lies between $\tfrac{1}{2}\pi$ and $\tfrac{2}{3}\pi$. [2]

(iii) Use the iterative formula

$$x_{n+1} = \tfrac{1}{3}\pi + \sin x_n,$$

with initial value $x_1 = 2$, to determine α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

6



The diagram shows the part of the curve $y = \dfrac{e^{2x}}{x}$ for $x > 0$, and its minimum point M.

(i) Find the coordinates of M. [5]

(ii) Use the trapezium rule with 2 intervals to estimate the value of

$$\int_1^2 \frac{e^{2x}}{x}\,dx,$$

giving your answer correct to 1 decimal place. [3]

(iii) State, with a reason, whether the trapezium rule gives an under-estimate or an over-estimate of the true value of the integral in part **(ii)**. [1]

7 **(i)** Given that $y = \tan 2x$, find $\dfrac{dy}{dx}$. [2]

(ii) Hence, or otherwise, show that

$$\int_0^{\frac{1}{6}\pi} \sec^2 2x\,dx = \tfrac{1}{2}\sqrt{3},$$

and, by using an appropriate trigonometrical identity, find the exact value of $\displaystyle\int_0^{\frac{1}{6}\pi} \tan^2 2x\,dx$. [6]

(iii) Use the identity $\cos 4x \equiv 2\cos^2 2x - 1$ to find the exact value of

$$\int_0^{\frac{1}{6}\pi} \frac{1}{1 + \cos 4x}\,dx.$$ [2]

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