CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary and Advanced Level

MARK SCHEME for the November 2004 question papers

9709 MATHEMATICS

9709/01 – Paper 1, maximum raw mark 75

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which Examiners were initially instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began. Any substantial changes to the mark scheme that arose from these discussions will be recorded in the published *Report on the Examination*.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the *Report on the Examination*.

- CIE will not enter into discussion or correspondence in connection with these mark schemes.

CIE is publishing the mark schemes for the November 2004 question papers for most IGCSE and GCE Advanced Level syllabuses.

UNIVERSITY of CAMBRIDGE
International Examinations

Grade thresholds taken for Syllabus 9709 (Mathematics) in the November 2004 examination.

	maximum mark available	minimum mark required for grade:		
		A	B	E
Component 1	75	65	58	33

The thresholds (minimum marks) for Grades C and D are normally set by dividing the mark range between the B and the E thresholds into three. For example, if the difference between the B and the E threshold is 24 marks, the C threshold is set 8 marks below the B threshold and the D threshold is set another 8 marks down. If dividing the interval by three results in a fraction of a mark, then the threshold is normally rounded down.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of

10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR -1 A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

November 2004

GCE A AND AS LEVEL

MARK SCHEME

MAXIMUM MARK: 75

SYLLABUS/COMPONENT: 9709/01

MATHEMATICS

UNIVERSITY of CAMBRIDGE
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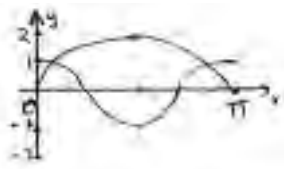
1 $(3x–2/x)^5$ Required term has ${}_5C_2$ or ${}_5C_3$ = 10 Also has 3^3 and 2^2 → 1080	B1 B1 B1 B1 **[4]**	Needs 10 or implied by answers. Can be implied or in the expansion Co. If all expansion given, gets ¾ unless the required term is isolated from the expansion – or ringed etc.
2 (i) 81,54,36 r = 54/81 or 36/54 S_{10} = 81 $(1– ⅔^{10}) ÷ (1– ⅔)$ → 239 **(ii)** n = (180 – 25) ÷ 5 + 1 = 32 Use of any S_n formula → 3280	B1 M1 A1 **[3]** B1 M1 A1 **[3]**	Value of r – unsimplified – allow 0.66 Correct formula – power 10 and used Co. More than 3 s.f. ok, but needs 238.8 31 gets M0 Correct formula – not for n = 25,5,180 Co
3 tan 60 = BC ÷ 6 BC = $6\sqrt{3}$ Area = ½ × 6 × "BC" – ½ × 6^2 × π/3  → $18\sqrt{3} – 6\pi$	M1 A1 M1 M1 A1 **[5]**	Use of tan = opp ÷ adj In this form somewhere with $\sqrt{3}$ Area of triangle as ½bh or ½absinC Area of Sector. Co. (Must be in this form, not decimals). No $\sqrt{3}$, max 3 out of 5.
4 (i)  **(ii)** → 2 points of intersection.	B1 B1 B1 B1 **[4]** B1√ **[1]**	Mark two graphs independently. Half a cycle – all above axis for 0 to π. 2 shown as the max with ½ cycle only. One whole cycle for 0 to π –1 to 1 shown with one cycle only. Providing 2 trig graphs used. (ignore other half if 0 to 2π used)
5 (i) $x^2 – 4x + 7 = 9 – 3x \rightarrow x^2 – x – 2 = 0$ Solution of this x = 2 or –1 → (2, 3) and (–1, 12) Mid point is M (½, 7½) **(ii)** dy/dx = 2x – 4 Equate to m of line (–3) + solution → (½, 5¼) **(iii)** Distance = 2¼	M1 DM1 A1 A1 **[4]** B1 M1 A1 **[3]** B1√ **[1]**	Complete elimination of y (or x) Correct solution of eqn = 0. All 4 values needed. Beware fortuitous ans. Answer given. Co Equates dy/dx to constant m, m ≠ 0. Must have calculus – not for perp m. Co For distance between "his" points.

6 (i) $5s^2 + 3c^2 = 5s^2 + 3(1 - s^2)$ $\rightarrow 3 + 2\sin^2 x$ a = 3, b = 2	M1 A1 **[2]**	Use of $s^2 + c^2 = 1$ $3 + 2\sin^2 x$ gets both marks.
(ii) $3 + 2s^2 = 7s$ Sets to 0 and solves. s = ½ or s = 3 Only values are $\pi/6$ and $5\pi/6$	M1 A1A1√ **[3]**	Sets to 0 + correct method of soln. Co for one value. Other π = "1st" (If degrees, give A0,A1√ for 180 –)
(iii) Minimum value = "a" = 3 Maximum value is "a + b" = 5 Range $3 \leq f(x) \leq 5$	B1√B1√ **[2]**	For his "a" and "a+b". Condone <. Allow 3 and 5 on their own.
7 $dy/dx = 6/\sqrt{(4x - 3)}$ P(3, 3)		
(i) x = 3, m = 2. Perpendicular m = –½	M1	Use of $m_1 m_2 = -1$ even if algebraic
$\rightarrow y - 3 = -½ (x - 3) \rightarrow x + 2y = 9$	M1 A1 **[3]**	Correct form of line eqn or $y = mx + c$ Needs putting as $x + 2y = 9$ for A mark. (tangent gets 0/3).
(ii) $\int \rightarrow 6(4x - 3)^{½} \div ½ \div 4$	M1 A1	M1 for $(4x - 3)^k \div k$. A1 for k = ½ and ÷ 4
$y = 3(4x - 3) + c$	M1	Using (3, 3) to find c only after attempt at integration.
Uses (3, 3) $\rightarrow c = -6$	A1 **[4]**	Allow full marks once –6 obtained.
8 (i) (**i** + 7**j** + 2**k**).(–5**i** + 5**j** + 6**k**) $\rightarrow -5 + 35 + 12 = 42$ $42 = \sqrt{54}\ \sqrt{86} \cos\theta$ $\rightarrow$ angle AOB = 0.907	M1 M1 M1 A1 **[4]**	Use of $\rightarrow x_1x_2 + y_1y_2 + z_1z_2$ Modulus used in dot product Everything linked correctly Accept if more accuracy given. Must be radians. Any combination of **OA/AO OB/BO** is ok for the three M1 marks. If **AB** used with **OA/OB** max M1 M1
(ii) **BC** = ½ (**b** – **a**) = – 3**i** – **j** + 2**k**		
OC = **OB** + **BC** = –5**i** + 5**j** + 6**k** – 3**i** – **j** + 2**k** = –8**i** + 4**j** + 8**k**	M1 A1	Could be from **OA** + **AC** Correct only.
Unit Vector = (– 8**i** + 4**j** + 8**k**) ÷ 12	M1A1√ **[4]**	Knowing to divide by length of vector. (leaving as √ is acceptable for both marks)

9 f : $x \to 2x - a$		M1 for putting "x" as "$2x - 3$"
(i) ff(x) = 11, $2(2x - 3) - 3 = 11$	M1 DM1	Everything completed to give answer. (if –3 omitted → 4¼, allow M1 only)
[or backwards $2x - 3 = 11$, $x = 7$, $2x - 3 = 7$ (M1), (M1)]		n.b. $2(2x - 3) = ff(x)$ gets M1 – not DM1 co
$\to x = 5$	A1 **[3]**	
(ii) $2x - a = x^2 - 6x \to x^2 - 8x + a = 0$	M1	Setting up a 3-term quadratic equation in x
Use of $b^2 - 4ac = 0$	M1	Using $b^2 - 4ac$ on quadratic = 0 or ≥ 0)
$\to$ a = 16 (or inspection)	A1 **[3]**	Co. Can be stated from the ($-8x$).
(iii) $x^2 - 6x = (x - 3)^2 - 9$		
$\to$ p = 3, q = 9	B1 B1 **[2]**	Allow if $(x - 3)^2 - 9$ without p or q stated
(iv) $y = (x - 3)^2 - 9$		
$x = \pm\sqrt{(y + 9)} + 3$	M1	Attempt to make x the subject, but only from completing square expression
$y = h^{-1}(x) = \sqrt{(x + 9)} + 3$	DM1A1	Replace y by x – sign lost for A. Special case "ans = $\sqrt{(y + 9)} + 3$" allow 2/3.
Domain of $h^{-1} = \{x: x \geq -9\}$	B1 **[4]**	Co. (allow ≥ -9 or $y \geq -9$ etc.)
10 (i) $dy/dx = 2x - 2/x^2$	B1 B1	For $-2/x^2$ or for $-2x^{-2}$ For "$2x$" and for "2"
$d^2y/dx^2 = 2 + 4/x^3$	B1√ **[3]**	For $+4/x^3$ or $4x^{-3}$ or for diff. his dy/dx as long as it is a negative power of x
(ii) $dy/dx = 0$ $2x - 2/x^2 = 0$	M1	Putting his $dy/dx = 0$ and solving for x
$\to x^3 = 1 \to x = 1$, $y = 3$	A1	Co (± 1 gets M1A0 but can get next M1A1)
If $x = 1$, $d^2y/dx^2 > 0$, Minimum	M1A1√ **[4]**	Looking at sign of d^2y/dx^2 or other. √ for his x into his d^2y/dx^2
(iii) Vol = $\pi\int y^2 dx = \pi\int(x^4 + 4/x^2 + 4x)\, dx$	M1	Attempt at squaring + integration Still gets M1 if $(a+b)^2 = a^2+b^2$
$= \pi [x^5/5 - 4/x + 2x^2]$	3× A1	For each term and π. Can get A1A1 for above error.
$[\ ]_2 - [\ ]_1 =$ 71π/5 or 44.6	DM1A1 **[6]**	Use of limits, "–" needed for M1. co. (no π – loses last A1 and one of first A marks)

DM1 for quadratic. Quadratic must be set to 0.
Factors. Attempt at two brackets. Each bracket set to 0 and solved.
Formula. Correct formula. Correct use, but allow for numerical slips in b^2 and $-4ac$.