Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/42**

Paper 4 Mechanics **May/June 2022**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2022 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	Conservation of momentum	**M1**	3 terms; allow M1 if speed of A after collision is $\frac{1}{4}\times 8.5$. Allow $5\times 8.5 = 5X + 3Y$ where $\lvert X\rvert$ and $\lvert Y\rvert$ are different which may be seen by later work. If $\lvert X\rvert$ and $\lvert Y\rvert$ are subsequently used as being equal then M0.
	$5\times 8.5 = 5\times 0.25v + 3v$	**A1**	OE e.g. $5\times 8.5 = 5V + 3\times 4V$
	Speed of $B = 10\,\text{ms}^{-1}$	**A1**	Do not award if 10 from using mgv, maximum 2/3 –10 is A0 as speed required not velocity
		3	
1(b)	KE before $= \frac{1}{2}\times 5\times 8.5^2 \left[= 180.625\right]$ KE after $= \frac{1}{2}\times 5\times 2.5^2 + \frac{1}{2}\times 3\times 10^2 \left[= 15.625 + 150 = 165.625\right]$	**1**	Attempt at any of the 3 terms for KE, using their $10\,\text{ms}^{-1}$ Not $\frac{1}{2}\times(5+3)\times 8.5^2$, not $\frac{1}{2}\times(5+3)\times 2.5^2$ not $\frac{1}{2}\times(5+3)\times 10^2$ unless $\lvert X\rvert = \lvert Y\rvert$ seen
	KE loss $\left[= 180.625 - 165.625\right] = 15$ J	**A1**	Accept –15, AWRT ±15.0
		2	

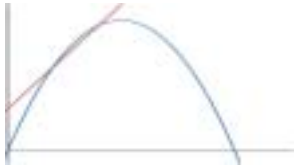
Question	Answer	Marks	Guidance
2	Resolving either direction	**M1**	3 terms; allow sign errors and allow sin/cos mix
	$(X =) \pm (20\cos 60 - 14 - 16\cos 50)$ $[= \mp 14.2846\ldots\ldots]$	**A1**	
	$(Y =) \pm (60 - 20\sin 60 - 16\sin 50)$ $[= \pm 30.42278\ldots\ldots]$	**A1**	
	$R = \sqrt{(14.2846\ldots)^2 + (30.42278\ldots)^2}$	**M1**	Attempt to solve for R; one missing term in total
	$\theta = \tan^{-1}\left(\frac{30.42278\ldots}{14.2846\ldots}\right)\left[= \tan^{-1}(2.1297\ldots\ldots)\right]$ OR $\alpha = \tan^{-1}\left(\frac{14.2846\ldots}{30.42278\ldots}\right)\left[= \tan^{-1}(0.4596\ldots\ldots)\right]$	**M1**	Attempt to solve for θ or α; one missing term in total
	R = 33.6 N Direction is 64.8° above the 14 N force or 25.2° above the negative x-axis or 25.2° left of the 60 N force or bearing 335° or 115° anticlockwise from the positive x-axis	**A1**	Both correct. OE; allow 64.9, 25.1 Giving an angle only is insufficient. Direction may be seen on a diagram, with minimum of arrow on resultant. Arrows on both components only is A0 as it doesn't show the direction of the resultant. However the direction is stated, it must be able to be drawn uniquely.
		6	

Question	Answer	Marks	Guidance
3(a)	$2.4g - T = 2.4a$ $T - 1.2g = 1.2a$ $2.4g - 1.2g = (2.4 + 1.2)a$	**M1**	Attempt at Newton's second law on either particle or the system with correct number of terms; allow sign errors.
		A1	Any 2 consistent and correct May have an a in opposite direction to our a
	Attempt to solve for a or T	**M1**	From equation(s) with correct number of relevant terms. If g missing then M0A0M1A0, maximum1/4. Must get $a =$ or $T =$ Must not assume $T = 16$. May attempt to verify a value of a using $T = 16$ in 2 equations
	$T = 16$ N and $a = \frac{10}{3}$ ms^{-2}	**A1**	Both correct; allow $a = 3.33$. **AG** for $T = 16$. Assuming $T = 16$ and only one equation is M1A0M0A0 maximum 1/4. Withhold A mark if $T = 15.9... \approx 16$, but condone $T = 1.2 \times 3.33 + 12 = 16$ or $T = 24 - 2.4 \times 3.33 = 16$
		4	

Question	Answer	Marks	Guidance
3(b)	$v^2 = 2\times\frac{10}{3}\times 2.1 \quad [=14] \quad \left[v=\sqrt{14}=3.741\ldots\ldots\right]$ OR $\frac{1}{2}\times 2.4\times v^2 = 2.4g\times 2.1 - 16\times 2.1$ OR $\frac{1}{2}\times 2.4\times v^2 + \frac{1}{2}\times 1.2\times v^2 = 2.4g\times 2.1 - 1.2g\times 2.1$	**M1**	Use of suvat or use energy to find v or v^2, using their $a \neq \pm g$ (unless 10 comes from their attempt at a) from **(a)**, $s = 2.1$
	$0 = 14 - 2\times g\times s \rightarrow s = \ldots$ or $\frac{1}{2}(1.2)\left(\sqrt{14}\right)^2 = (1.2)\times g\times h \rightarrow h = \ldots$	**M1**	Attempt to use $v^2 = u^2 + 2as$ (or other complete method), using $a = -g$, to find additional height after string slack, using their v or v^2.
	$s = [1.5 + 2.1 + 0.7 =]$ 4.3 m	**A1**	AWRT 4.3(0); Allow use of $a = 3.33$ to give $s = 4.2993 \approx 4.3(0)$ Allow use of $v = 3.74$ to give $s = 4.29938 \approx 4.3(0)$
	Alternative for question 3(b) - using energy on particle *B*		
	$16\times 2.1 = 1.2gH$	**M1**	Apply energy to *B*, 2 terms
	$H = 2.8$	**A1**	
	$s = [1.5 + 2.8 =]$ 4.3 m	**A1**	
		3	

Question	Answer	Marks	Guidance
4(a)	Use suvat to find expressions for s_A or s_B	**M1**	For s_A must be using $u = 8$ and time of $t \pm 4$ For s_B, using $s = ut + \frac{1}{2}at^2$ with $u = 20$ and $a = \pm 2$
	$(s_A =) 8(4 + t) [= 32 + 8t]$	**A1**	Any unsimplified expression; ISW
	$(s_B =) 20t - \frac{1}{2} \times 2 \times t^2$	**A1**	Any unsimplified expression; ISW If 0 marks scored then allow **SC**: B1 for $(s_A =) 8t$ and B1 for $(s_B =) 20(t-4) - \frac{1}{2} \times 2 \times (t-4)^2$ maximum 2/3
		3	

Question	Answer	Marks	Guidance
4(b)	$8(4+t)=20t-\frac{1}{2}\times 2\times t^2$	***M1**	Equating their expressions for s_A and s_B to form an equation in t where s_A is of the form $\pm 8t \pm 32$ and s_B is of the form $\pm 20t \pm \frac{1}{2}\times 2\times t^2$
	Attempt to solve a 3-term quadratic to find at least one t value	**DM1**	For reference $t^2-12t+32=0$ Allow if no working seen and have correct real solution(s) to *their* 3-term quadratic. If working shown and if using the formula, it must be using the correct formula. If factorising must have 3 of the 4 terms correct of $(t-4)(t-8)$
	$t=4$ and 8	**A1**	If 0 marks scored then allow **SC:** M1 for $\pm 8t = \pm 20(t\pm 4) \pm \frac{1}{2}\times 2\times (t\pm 4)^2$ and A1 for $t=8$ and 12 maximum 2/3.
		3	

Question	Answer	Marks	Guidance
4(c)	Straight line	**B1 FT**	Positive gradient, intersecting positive s axis. Full domain not required. FT if they get $s_A = 8t$ using the **SC** in **(a)**
	Inverted quadratic, passing through origin.	**B1 FT**	Full domain not required but must clearly go beyond the maximum. FT if they get $s_B = 20(t-4) - \frac{1}{2} \times 2 \times (t-4)^2$ using the SC in **(a)**, with curve though positive t axis before turning point.
	All correct, line through (0, 32), quadratic through (20, 0), intersections indicated at $t = 4$ and $t = 8$. 	**B1**	Intersections must occur before the turning point.
		3	

Question	Answer	Marks	Guidance
5	Attempt at resolving parallel to the plane	*M1	3 terms. Allow sign errors, sin/cos mix. Allow g missing, otherwise dimensionally correct.
	$65\cos 36 = 12g \times \sin 24 + F$	A1	$F = 3.777707\ldots\ldots$
	Attempt at resolving perpendicular to the plane	*M1	3 terms. Allow sign errors, sin/cos mix. Allow g missing, otherwise dimensionally correct.
	$12g \times \cos 24 = R + 65\sin 36$	A1	$R = 71.419\ldots\ldots$
	Use $F = \mu R$ $\left[\mu = \dfrac{65\cos 36 - 12g \times \sin 24}{12g \times \cos 24 - 65\sin 36} = \dfrac{52.586 - 48.808}{109.625 - 38.206} = \dfrac{3.777\ldots}{71.419\ldots}\right]$	DM1	To get an equation in μ only. Dependent on two previous M marks. Allow g missing
	$\mu = 0.0529$	A1	Allow AWRT 0.053 Do not accept fractional equivalent.
		6	

Question	Answer	Marks	Guidance
6(a)	KE change $= \pm\left(0.5\times900\times16^2 - 0.5\times900\times11^2\right)$ $\left[= \pm(115200 - 54450) = \pm60750\right]$	**B1**	
	PE $= 900g\times150\times0.12$ $\left[=162000\right]$	**B1**	Allow $900g\times150\times\sin6.89°$ or $900g\times150\times\sin6.9°$ Not from use of constant acceleration/Newton's second law.
	[Work done by car's engine =] 24000×12 $\left[=288000\right]$	**B1**	OE e.g. $24000 = \frac{\text{WD}}{12}$
	Work done against resistive forces $= 24000\times12 - 900g\times150\times0.12 - 0.5\times900\times16^2 + 0.5\times900\times11^2$ $= 288000 - 162000 - 115200 + 54450$	**M1**	Use of work-energy 5 terms; dimensionally correct. Work done by car's engine not from using one of the given speeds. Allow sign errors.
	Work done = 65 250 J	**A1**	or 65.25 kJ Allow AWRT 65300 J or 65.3 kJ from correct work
		5	

Question	Answer	Marks	Guidance
6(b)	Driving Force $= \frac{32000}{v}$	**B1**	OE e.g. $32000 = DF \times v$
	$\frac{32000}{v} = 1520 + 4v$	**M1**	Apply N2L to the car with $a = 0$ (3 terms) and attempt to solve a 3-term quadratic in v. For reference $4v^2 + 1520v - 32000 = 0$ Allow if no working seen and have correct real solution(s) to *their* 3-term quadratic. If working shown and if using the formula, it must be using the correct formula. If factorising must have 3 of the 4 terms correct of $(v - 20)(v + 400)$
	Speed $= 20\,\text{ms}^{-1}$	**A1**	Only.
		3	

Question	Answer	Marks	Guidance
7(a)	$\left(a=\right)\dfrac{\mathrm{d}v}{\mathrm{d}t}=0.5$	**B1**	For acceleration during the first 10 seconds
	Differentiate to get $\left(a=\right)\dfrac{\mathrm{d}v}{\mathrm{d}t}=2\times0.25t-8 \quad \left[=0.5t-8\right]$	**B1**	Allow unsimplified
	$a\left[=0.5\times10-8\right]=-3$	**B1**	CWO. Do not award final B mark if more than 2 accelerations seen and not discarded, 2/3 maximum Ignore any comments, correct or incorrect
		3	

Question	Answer	Marks	Guidance
7(b)	Get distance in first 10 seconds as 25	**B1**	From suvat or from $\int_0^{10} 0.5t\,\mathrm{d}t$
	$v = 0$ when $t = 12$ and $t = 20$	**B1**	SOI
	Attempt to integrate v $\left[s = \int \left(0.25t^2 - 8t + 60\right)\mathrm{d}t\right]$	***M1**	For integration, the power of t must increase by 1 in at least 1 term with a change of coefficient in the same term. $s = vt$ is M0
	$s = \frac{0.25}{3}t^3 - \frac{8}{2}t^2 + 60t\left(+c\right)$ $\left[= \frac{1}{12}t^3 - 4t^2 + 60t\left(+c\right)\right]$	**A1**	Allow unsimplified
	Attempt to evaluate their $\left[\frac{1}{12}t^3 - 4t^2 + 60t\right]$ for $t = 10$ to $t = 12$ and $t = 12$ to $t = 20$	**DM1**	Using the correct limits correctly
	$s = \left[25 + 288 - \frac{850}{3} - \left(\frac{800}{3} - 288\right) = 25 + \frac{14}{3} - \left(-\frac{64}{3}\right)\right] = 51$ m	**A1**	

Question	Answer	Marks	Guidance
7(b)	**Special Case for those who use a calculator to integrate. Maximum 4/6**		
	Get distance in first 10 seconds as 25	**B1**	From suvat or $\int_0^{10} 0.5t\,\mathrm{d}t$
	$v = 0$ when $t = 12$ and $t = 20$	**B1**	SOI
	Either $s = \int_{10}^{12} \left(0.25t^2 - 8t + 60\right)\mathrm{d}t = \frac{14}{3} = 4.67$ Or $s = \left\|\int_{12}^{20} \left(0.25t^2 - 8t + 60\right)\mathrm{d}t\right\| = \frac{64}{3} = 21.3$	**B1**	Allow $\int_{10}^{20} \left\|0.25t^2 - 8t + 60\right\|\mathrm{d}t = 26$
	$s = \left[25 + \frac{14}{3} + \frac{64}{3}\right] = 51$ m	**B1**	Allow if $t = 12$ and $t = 20$ not found for 3 marks
		6	