Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME									
CENTRE NUMBER						CANDIDATE NUMBER			

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **May/June 2020**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Blank pages are indicated.

[Turn over

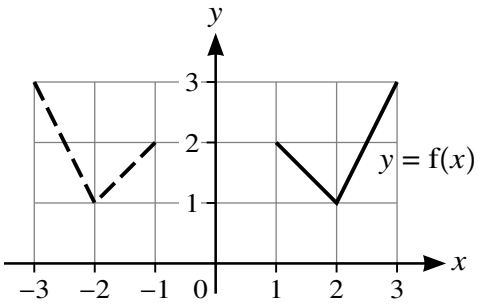
1 Find the set of values of m for which the line with equation $y = mx + 1$ and the curve with equation $y = 3x^2 + 2x + 4$ intersect at two distinct points. [4]

2 The equation of a curve is such that $\frac{dy}{dx} = 3x^{\frac{1}{2}} - 3x^{-\frac{1}{2}}$. It is given that the point (4, 7) lies on the curve.

Find the equation of the curve. [4]

3 In each of parts **(a)**, **(b)** and **(c)**, the graph shown with solid lines has equation $y = \mathrm{f}(x)$. The graph shown with broken lines is a transformation of $y = \mathrm{f}(x)$.

(a)



State, in terms of f, the equation of the graph shown with broken lines. [1]

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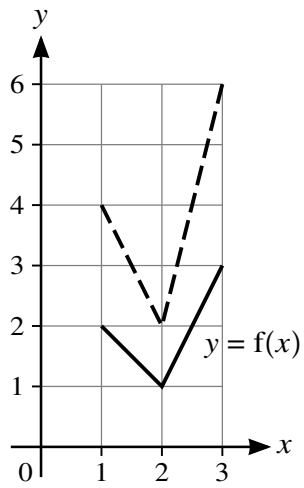
(b)



State, in terms of f, the equation of the graph shown with broken lines. [1]

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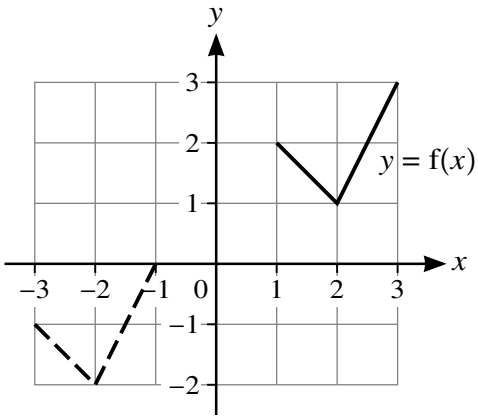
(c)



State, in terms of f, the equation of the graph shown with broken lines. [2]

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4 **(a)** Expand $(1 + a)^5$ in ascending powers of a up to and including the term in a^3. [1]

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(b) Hence expand $[1 + (x + x^2)]^5$ in ascending powers of x up to and including the term in x^3, simplifying your answer. [3]

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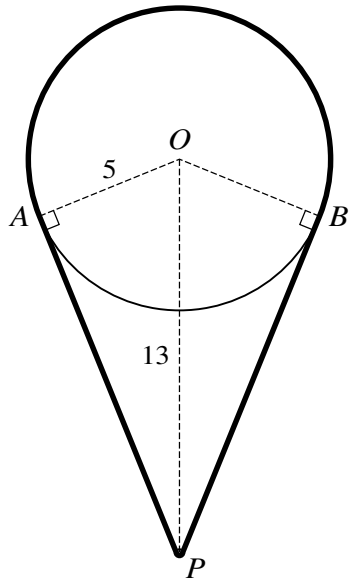

The diagram shows a cord going around a pulley and a pin. The pulley is modelled as a circle with centre O and radius 5 cm. The thickness of the cord and the size of the pin P can be neglected. The pin is situated 13 cm vertically below O. Points A and B are on the circumference of the circle such that AP and BP are tangents to the circle. The cord passes over the major arc AB of the circle and under the pin such that the cord is taut.

Calculate the length of the cord. [6]

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6 A point P is moving along a curve in such a way that the x-coordinate of P is increasing at a constant rate of 2 units per minute. The equation of the curve is $y = (5x - 1)^{\frac{1}{2}}$.

(a) Find the rate at which the y-coordinate is increasing when $x = 1$. [4]

(b) Find the value of x when the y-coordinate is increasing at $\frac{5}{8}$ units per minute. [3]

7 **(a)** Show that $\dfrac{\tan\theta}{1+\cos\theta} + \dfrac{\tan\theta}{1-\cos\theta} \equiv \dfrac{2}{\sin\theta\cos\theta}$. [4]

(b) Hence solve the equation $\frac{\tan\theta}{1+\cos\theta} + \frac{\tan\theta}{1-\cos\theta} = \frac{6}{\tan\theta}$ for $0° < \theta < 180°$. [4]

8 The first term of a progression is $\sin^2\theta$, where $0 < \theta < \frac{1}{2}\pi$. The second term of the progression is $\sin^2\theta\cos^2\theta$.

(a) Given that the progression is geometric, find the sum to infinity. [3]

It is now given instead that the progression is arithmetic.

(b) **(i)** Find the common difference of the progression in terms of $\sin\theta$. [3]

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(ii) Find the sum of the first 16 terms when $\theta = \frac{1}{3}\pi$. [3]

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9 The functions f and g are defined by

$$\mathrm{f}(x) = x^2 - 4x + 3 \quad \text{for } x > c, \text{ where } c \text{ is a constant,}$$

$$\mathrm{g}(x) = \frac{1}{x+1} \quad \text{for } x > -1.$$

(a) Express $\mathrm{f}(x)$ in the form $(x - a)^2 + b$. [2]

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It is given that f is a one-one function.

(b) State the smallest possible value of c. [1]

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It is now given that $c = 5$.

(c) Find an expression for $\mathrm{f}^{-1}(x)$ and state the domain of f^{-1}. [3]

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(d) Find an expression for $\mathrm{gf}(x)$ and state the range of gf. [3]

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10 (a) The coordinates of two points A and B are $(-7, 3)$ and $(5, 11)$ respectively.

Show that the equation of the perpendicular bisector of AB is $3x + 2y = 11$. [4]

(b) A circle passes through A and B and its centre lies on the line $12x - 5y = 70$.

Find an equation of the circle. [5]

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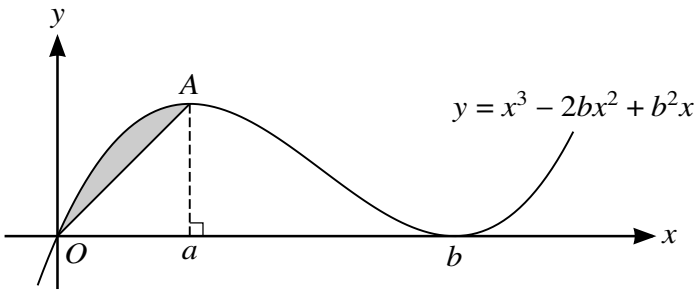

The diagram shows part of the curve with equation $y = x^3 - 2bx^2 + b^2x$ and the line OA, where A is the maximum point on the curve. The x-coordinate of A is a and the curve has a minimum point at $(b, 0)$, where a and b are positive constants.

(a) Show that $b = 3a$. [4]

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(b) Show that the area of the shaded region between the line and the curve is ka^4, where k is a fraction to be found. [7]

Additional Page

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