Cambridge International Examinations
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MATHEMATICS **9709/11**

Paper 1 **May/June 2017**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2017 series for most Cambridge IGCSE®, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
SOI	Seen or implied
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

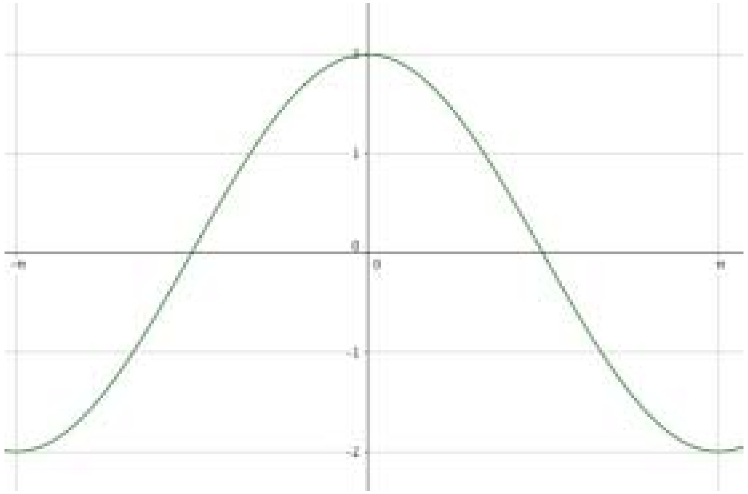
Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1	$(3-2x)^6$		
	Coeff of $x^2 = 3^4 \times (-2)^2 \times {}_6C_2 = a$ Coeff of $x^3 = 3^3 \times (-2)^3 \times {}_6C_3 = b$	**B3,2,1**	Mark unsimplified forms. –1 each independent error but powers must be correct. Ignore any 'x' present.
	$\frac{a}{b} = -\frac{9}{8}$	**B1**	OE. Negative sign must appear before or in the numerator
	Total:	**4**	
2	$\overrightarrow{OA} = \begin{pmatrix} 3 \\ -6 \\ p \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} 2 \\ -6 \\ -7 \end{pmatrix}$		
2(i)	Angle $AOB = 90° \rightarrow 6 + 36 - 7p = 0$	**M1**	Use of $x_1x_2 + y_1y_2 + z_1z_2 = 0$ or Pythagoras
	$\rightarrow p = 6$	**A1**	
	Total:	**2**	

Question	Answer	Marks	Guidance
2(ii)	$\overrightarrow{OC} = \frac{2}{3}\begin{pmatrix}3\\-6\\p\end{pmatrix} = \begin{pmatrix}2\\-4\\4\end{pmatrix}$	**B1 FT**	CAO FT on their value of p
	$\overrightarrow{BC} = \mathbf{c} - \mathbf{b} = \begin{pmatrix}0\\2\\11\end{pmatrix}$; magnitude = $\sqrt{125}$	**M1 M1**	Use of **c** – **b**. Allow magnitude of **b** + **c** or **b** – **c** Allow first **M1** in terms of p
	Unit vector = $\frac{1}{\sqrt{125}}\begin{pmatrix}0\\2\\11\end{pmatrix}$	**A1**	OE Allow ± and decimal equivalent
3(i)	$\frac{1+\cos\theta}{\sin\theta} + \frac{\sin\theta}{1+\cos\theta} \equiv \frac{2}{\sin\theta}$.		
	$\frac{(1+c)^2+s^2}{s(1+c)} = \frac{1+2c+c^2+s^2}{s(1+c)}$	**M1**	Correct use of fractions
	$= \frac{2+2c}{s(1+c)} = \frac{2(1+c)}{s(1+c)} \to \frac{2}{s}$	**M1 A1**	Use of trig identity, **A1** needs evidence of cancelling
	Total:	**3**	
3(ii)	$\frac{2}{\boldsymbol{s}} = \frac{3}{\boldsymbol{c}} \to t = \frac{2}{3}$	**M1**	Use part **(i)** and $t = s \div c$, may restart from given equation
	$\to \theta = 33.7°$ or $213.7°$	**A1 A1FT**	FT for 180° + 1st answer. 2nd **A1** lost for extra solns in range
	Total:	**3**	

Question	Answer	Marks	Guidance
4(a)	$a = 32$, $a + 4d = 22$, $\rightarrow d = -2.5$	**B1**	
	$a + (n - 1)d = -28 \rightarrow n = 25$	**B1**	
	$S_{25} = \frac{25}{2}\left(64 - 2.5 \times 24\right) = 50$	**M1 A1**	**M1** for correct formula with $n = 24$ or $n = 25$
	Total:	**4**	
4(b)	$a = 2000$, $r = 1.025$	**B1**	$r = 1 + 2.5\%$ ok if used correctly in S_n formula
	$S_{10} = 2000\left(\frac{1.025^{10} - 1}{1.025 - 1}\right) = 22400$ or a value which rounds to this	**M1 A1**	**M1** for correct formula with $n = 9$ or $n = 10$ and their a and r
			SR: correct answer only for $n = 10$ **B3**, for $n = 9$, **B1** (£19 900)
	Total:	**3**	

Question	Answer	Marks	Guidance
5	$y = 2\cos x$		
5(i)		**B1**	One whole cycle – starts and finishes at –ve value
		**DB1**	Smooth curve, flattens at ends and middle. Shows (0, 2).
	Total:	**2**	
5(ii)	$P(\frac{\pi}{3}, 1)$ $Q(\pi, -2)$		
	$\rightarrow PQ^2 = \left(\frac{2\pi}{3}\right)^2 + 3^2 \rightarrow PQ = 3.7$	**M1 A1**	Pythagoras (on their coordinates) must be correct, OE.
	Total:	**2**	

Question	Answer	Marks	Guidance
5(iii)	Eqn of PQ $y-1=-\frac{9}{2\pi}\left(x-\frac{\pi}{3}\right)$	M1	Correct form of line equation or sim equations from their P & Q
	If $y=0 \to h=\frac{5\pi}{9}$	A1	AG, condone $x=\frac{5\pi}{9}$
	If $x=0 \to k=\frac{5}{2}$,	A1	SR: non-exact solutions **A1** for both
	Total:	**3**	
6(i)	Volume = $\left(\frac{1}{2}\right)x^2\frac{\sqrt{3}}{2}h = 2000 \to h=\frac{8000}{\sqrt{3}x^2}$	M1	Use of (area of triangle, with attempt at ht) × h =2000, $h=\mathrm{f}(x)$
	$A = 3xh+(2)\times\left(\frac{1}{2}\right)\times x^2\times\frac{\sqrt{3}}{2}$	M1	Uses 3 rectangles and at least one triangle
	Sub for $h \to A=\frac{\sqrt{3}}{2}x^2+\frac{24000}{\sqrt{3}}x^{-1}$	A1	AG
	Total:	**3**	
6(ii)	$\frac{\mathrm{d}A}{\mathrm{d}x}=\frac{\sqrt{3}}{2}2x-\frac{24000}{\sqrt{3}}x^{-2}$	B1	CAO, allow decimal equivalent
	$= 0$ when $x^3 = 8000 \to x = 20$	M1 A1	Sets their $\frac{\mathrm{d}A}{\mathrm{d}x}$ to 0 and attempt to solve for x
	Total:	**3**	

Question	Answer	Marks	Guidance
6(iii)	$\frac{d^2A}{dx^2} = \frac{\sqrt{3}}{2}2 + \frac{48000}{\sqrt{3}}x^{-3} > 0$	M1	Any valid method, ignore value of $\frac{d^2A}{dx^2}$ providing it is positive
	→ Minimum	A1 FT	FT on their x providing it is positive
	Total:	**2**	
7	$\frac{dy}{dx} = 7 - x^2 - 6x$		
7(i)	$y = 7x - \frac{x^3}{3} - \frac{6x^2}{2}$ $(+ c)$	B1	CAO
	Uses (3, − 10) → $c = 5$	M1 A1	Uses the given point to find c
	Total:	**3**	
7(ii)	$7 - x^2 - 6x = 16 - (x+3)^2$	B1 B1	**B1** $a = 16$, **B1** $b = 3$.
	Total:	**2**	
7(iii)	$16 - (x+3)^2 > 0 \rightarrow (x+3)^2 < 16$, and solve	M1	or factors $(x + 7)(x - 1)$
	End-points $x = 1$ or -7	A1	
	$\rightarrow -7 < x < 1$	A1	needs <, not ⩽. (SR $x < 1$ only, or $x > -7$ only **B1** i.e. 1/3)
	Total:	**3**	

Question	Answer	Marks	Guidance
8(i)	Letting M be midpoint of AB		
	$OM = 8$ (Pythagoras) $\rightarrow XM = 2$	**B1**	(could find $\sqrt{40}$ and use $\sin^{-1}$or $\cos^{-1}$)
	$\tan AXM = \frac{6}{2}$ $AXB = 2\tan^{-1}3 = 2.498$	**M1 A1**	AG Needs × 2 and correct trig for **M1**
	(Alternative 1: $\sin AOM = \frac{6}{10}, AOM = 0.6435, AXB = \pi - 0.6435$)		(Alternative 1: Use of isosceles triangles, **B1** for AOM, **M1**,**A1** for completion) (Alternative 2: Use of circle theorem, **B1** for AOB, **M1**,**A1**for completion)
	Total:	**3**	
8(ii)	$AX = \sqrt{(6^2 + 2^2)} = \sqrt{40}$	**B1**	CAO, could be gained in part (i) or part (iii)
	Arc $AYB = r\theta = \sqrt{40} \times 2.498$	**M1**	Allow for incorrect $\sqrt{40}$ (not $r = 6\, or\, 12\, or\, 10$)
	Perimeter = 12 + arc = 27.8 cm	**A1**	
	Total:	**3**	
8(iii)	area of sector $AXBY = \frac{1}{2} \times (\sqrt{40})^2 \times 2.498$	**M1**	Use of $\frac{1}{2}r^2\theta$ with their r, (not $r = 6\, or\, r = 10$)
	Area of triangle $AXB = \frac{1}{2} \times 12 \times 2$, Subtract these $\rightarrow$ 38.0 cm²	**M1 A1**	Use of $\frac{1}{2}bh$ and subtraction. Could gain **M1** with $r = 10$.
	Total:	**3**	

Question	Answer	Marks	Guidance
9	$\mathrm{f}: x \mapsto \dfrac{2}{3-2x}$ $\mathrm{g}: x \mapsto 4x + a$,		
9(i)	$y = \dfrac{2}{3-2x} \rightarrow y(3-2x) = 2 \rightarrow 3-2x = \dfrac{2}{y}$	**M1**	Correct first 2 steps
	$\rightarrow 2x = 3 - \dfrac{2}{y} \rightarrow \mathrm{f}^{-1}(x) = \dfrac{3}{2} - \dfrac{1}{x}$	**M1 A1**	Correct order of operations, any correct form with $\mathrm{f}(x)$ *or* $y =$
	Total:	**3**	
9(ii)	$\mathrm{gf}(-1) = 3$ $\mathrm{f}(-1) = \dfrac{2}{5}$	**M1**	Correct first step
	$\dfrac{8}{5} + a = 3 \rightarrow a = \dfrac{7}{5}$	**M1 A1**	Forms an equation in a and finds a, OE
			(or $\dfrac{8}{3-2x} + a = 3$, **M1** Sub and solves **M1**, **A1**)
	Total:	**3**	
9(iii)	$\mathrm{g}^{-1}(x) = \dfrac{x-a}{4} = \mathrm{f}^{-1}(x)$	**M1**	Finding $\mathrm{g}^{-1}(x)$ and equating to their $\mathrm{f}^{-1}(x)$ even if $a = 7/5$
	$\rightarrow x^2 - x(a+6) + 4 (= 0)$	**M1**	Use of $b^2 - 4ac$ on a quadratic with a in a coefficient
	Solving $(a+6)^2 = 16$ *or* $a^2 + 12a + 20 (= 0)$	**M1**	Solution of a 3 term quadratic
	$\rightarrow a = -2$ or -10	**A1**	
	Total:	**4**	

Question	Answer	Marks	Guidance
10(i)	$\frac{dy}{dx} = \frac{-4}{(5-3x)^2} \times (-3)$	B1 B1	**B1** without $\times(-3)$ **B1** For $\times(-3)$
	Gradient of tangent = 3, Gradient of normal – ⅓	*M1	Use of $m_1 m_2 = -1$ after calculus
	$\rightarrow$ eqn: $y-2 = -\frac{1}{3}(x-1)$	DM1	Correct form of equation, with (1, their y), not (1,0)
	$\rightarrow$ $y = -\frac{1}{3}x + \frac{7}{3}$	A1	This mark needs to have come from $y = 2$, y must be subject
	Total:	**5**	
10(ii)	Vol = $\pi \int_0^1 \frac{16}{(5-3x)^2} dx$	M1	Use of $V = \pi \int y^2 dx$ with an attempt at integration
	$\pi\left[\frac{-16}{(5-3x)} \div -3\right]$	A1 A1	**A1** without($\div -3$), **A1** for ($\div -3$)
	= ($\pi\left(\frac{16}{6} - \frac{16}{15}\right)$) = $\frac{8\pi}{5}$ (if limits switched must show – to +)	M1 A1	Use of both correct limits **M1**
	Total:	**5**	