CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Level

MARK SCHEME for the May/June 2014 series

9709 MATHEMATICS

9709/32 Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2014 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- **Note:** B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a "fortuitous" answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR – 1 A penalty of MR – 1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR – 2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA – 1 This is deducted from A or B marks in the case of premature approximation. The PA – 1 penalty is usually discussed at the meeting.

1	*EITHER:* State or imply non-modular inequality $(x+2a)^2 > (3(x-a))^2$, or corresponding quadratic equation, or pair of linear equations $(x+2a) = \pm 3(x-a)$	B1	
	Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations for x	M1	
	Obtain critical values $x = \frac{1}{4}a$ and $x = \frac{5}{2}a$	A1	
	State answer $\frac{1}{4}a < x < \frac{5}{2}a$	A1	
	OR: Obtain critical value $x = \frac{5}{2}a$ from a graphical method, or by inspection, or by solving a linear equation or inequality	B1	
	Obtain critical value $x = \frac{1}{4}a$ similarly	B2	
	State answer $\frac{1}{4}a < x < \frac{5}{2}a$	B1	4
	[Do not condone $\leqslant$ for $<$.]		
2	Remove logarithms and obtain $5 - e^{-2x} = e^{\frac{1}{2}}$, or equivalent	B1	
	Obtain a correct value for e^{-2x}, e^{2x}, e^{-x} or e^{x}, e.g. $e^{2x} = 1/(5 - e^{\frac{1}{2}})$	B1	
	Use correct method to solve an equation of the form $e^{2x} = a$, $e^{-2x} = a$, $e^{x} = a$ or $e^{-x} = a$ where $a > 0$. [The M1 is dependent on the correct removal of logarithms.]	M1	
	Obtain answer $x = -0.605$ only.	A1	4
3	Use $\cos(A + B)$ formula to obtain an equation in $\cos x$ and $\sin x$	M1	
	Use trig formula to obtain an equation in $\tan x$ (or $\cos x$ or $\sin x$)	M1	
	Obtain $\tan x = \sqrt{3} - 4$, or equivalent (or find $\cos x$ or $\sin x$)	A1	
	Obtain answer $x = -66.2°$	A1	
	Obtain answer $x = 113.8°$ and no others in the given interval	A1	5
	[Ignore answers outside the given interval. Treat answers in radians as a misread $(-1.16, 1.99)$.]		
	[The other solution methods are *via* $\cos x = \pm 1/\sqrt{(1 + (\sqrt{3} - 4)^2)}$ and $\sin x = \pm(\sqrt{3} - 4)/\sqrt{(1 + (\sqrt{3} - 4)^2)}$.]		
4	**(i)** State $\frac{dx}{dt} = 1 - \sec^2 t$, or equivalent	B1	
	Use chain rule	M1	
	Obtain $\frac{dy}{dt} = -\frac{\sin t}{\cos t}$, or equivalent	A1	
	Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	M1	
	Obtain the given answer correctly.	A1	5
	(ii) State or imply $t = \tan^{-1}(\frac{1}{2})$	B1	
	Obtain answer $x = -0.0364$	B1	2

5	(i)	Differentiate f(x) and obtain $f'(x) = (x-2)^2 g'(x) + 2(x-2)g(x)$	B1	
		Conclude that $(x-2)$ is a factor of $f'(x)$	B1	2
	(ii)	*EITHER:* Substitute $x = 2$, equate to zero and state a correct equation, e.g. $32 + 16a + 24 + 4b + a = 0$	B1	
		Differentiate polynomial, substitute $x = 2$ and equate to zero or divide by $(x-2)$ and equate constant remainder to zero	M1*	
		Obtain a correct equation, e.g. $80 + 32a + 36 + 4b = 0$	A1	
		OR1: Identify given polynomial with $(x-2)^2(x^3 + Ax^2 + Bx + C)$ and obtain an equation in a and/or b	M1*	
		Obtain a correct equation, e.g. $\frac{1}{4}a - 4(4+a) + 4 = 3$	A1	
		Obtain a second correct equation, e.g. $-\frac{3}{4}a + 4(4+a) = b$	A1	
		OR2: Divide given polynomial by $(x-2)^2$ and obtain an equation in a and b	M1*	
		Obtain a correct equation, e.g. $29 + 8a + b + 0$	A1	
		Obtain a second correct equation, e.g. $176 + 47a + 4b = 0$	A1	
		Solve for a or for b	M1(dep*)	
		Obtain $a = -4$ and $b = 3$	A1	5
6	(i)	Use correct arc formula and form an equation in r and x	M1	
		Obtain a correct equation in any form	A1	
		Rearrange in the given form	A1	3
	(ii)	Consider sign of a relevant expression at $x = 1$ and $x = 1.5$, or compare values of relevant expressions at $x = 1$ and $x = 1.5$	M1	
		Complete the argument correctly with correct calculated values	A1	2
	(iii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 1.21	A1	
		Show sufficient iterations to 4 d.p. to justify 1.21 to 2 d.p., or show there is a sign change in the interval (1.205,1.215)	A1	3
7	(a)	*EITHER*: Substitute and expand $(-1+\sqrt{5}\,\mathrm{i})^3$ completely	M1	
		Use $\mathrm{i}^2 = -1$ correctly at least once	M1	
		Obtain $a = -12$	A1	
		State that the other complex root is $-1-\sqrt{5}\,\mathrm{i}$	B1	
		OR1: State that the other complex root is $-1-\sqrt{5}\,\mathrm{i}$	B1	
		State the quadratic factor $z^2 + 2z + 6$	B1	
		Divide the cubic by a 3-term quadratic, equate remainder to zero and solve for a or, using a 3-term quadratic, factorise the cubic and determine a	M1	
		Obtain $a = -12$	A1	
		OR2: State that the other complex root is $-1-\sqrt{5}\mathrm{i}$	B1	
		State or show the third root is 2	B1	
		Use a valid method to determine a	M1	
		Obtain $a = -12$	A1	
		OR3: Substitute and use De Moivre to cube $\sqrt{6}\mathrm{cis}(114.1^\circ)$, or equivalent	M1	
		Find the real and imaginary parts of the expression	M1	
		Obtain $a = -12$	A1	
		State that the other complex root is $-1-\sqrt{5}\mathrm{i}$	B1	4

(**b**)	*EITHER:*	Substitute $w = \cos 2\theta + \mathrm{i}\sin 2\theta$ in the given expression	B1	
		Use double angle formulae throughout	M1	
		Express numerator and denominator in terms of $\cos\theta$ and $\sin\theta$ only	A1	
		Obtain given answer correctly	A1	
	OR:	Substitute $w = \mathrm{e}^{2\mathrm{i}\theta}$ in the given expression	B1	
		Divide numerator and denominator by $\mathrm{e}^{\mathrm{i}\theta}$, or equivalent	M1	
		Express numerator and denominator in terms of $\cos\theta$ and $\sin\theta$ only	A1	
		Obtain the given answer correctly	A1	4

8	(**i**)	Use product rule	M1	
		Obtain derivative in any correct form	A1	
		Differentiate first derivative using the product rule	M1	
		Obtain second derivative in any correct form, e.g. $-\frac{1}{2}\sin\frac{1}{2}x - \frac{1}{4}x\cos\frac{1}{2}x - \frac{1}{2}\sin\frac{1}{2}x$	A1	
		Verify the given statement	A1	5
	(**ii**)	Integrate and reach $kx\sin\frac{1}{2}x + l\int \sin\frac{1}{2}x\,\mathrm{d}x$	M1*	
		Obtain $2x\sin\frac{1}{2}x - 2\int \sin\frac{1}{2}x\,\mathrm{d}x$, or equivalent	A1	
		Obtain indefinite integral $2x\sin\frac{1}{2}x + 4\cos\frac{1}{2}x$	A1	
		Use correct limits $x = 0$, $x = \pi$ correctly	M1(dep*)	
		Obtain answer $2\pi - 4$, or exact equivalent	A1	5

9	(**i**)	State or imply $\frac{\mathrm{d}N}{\mathrm{d}t} = kN(1 - 0.01N)$ and obtain the given answer $k = 0.02$	B1	1
	(**ii**)	Separate variables and attempt integration of at least one side	M1	
		Integrate and obtain term $0.02t$, or equivalent	A1	
		Carry out a relevant method to obtain A or B such that $\frac{1}{N(1-0.01N)} \equiv \frac{A}{N} + \frac{B}{1-0.01N}$, or equivalent	M1*	
		Obtain $A = 1$ and $B = 0.01$, or equivalent	A1	
		Integrate and obtain terms $\ln N - \ln(1 - 0.01N)$, or equivalent	A1√	
		Evaluate a constant or use limits $t = 0$, $N = 20$ in a solution with terms $a\ln N$ and $b\ln(1 - 0.01N)$, $ab \neq 0$	M1(dep*)	
		Obtain correct answer in any form, e.g. $\ln N - \ln(1 - 0.01N) = 0.02t + \ln 25$	A1	
		Rearrange and obtain $t = 50\ln(4N/(100 - N))$, or equivalent	A1	8
	(**iii**)	Substitute $N = 40$ and obtain $t = 49.0$	B1	1

10	**(i)**	*EITHER:*	State or imply $\overrightarrow{AB}$ and $\overrightarrow{AC}$ correctly in component form	B1	
			Using the correct processes evaluate the scalar product $\overrightarrow{AB}.\overrightarrow{AC}$, or equivalent	M1	
			Using the correct process for the moduli divide the scalar product by the product of the moduli	M1	
			Obtain answer $\frac{20}{21}$	A1	
		OR:	Use correct method to find lengths of all sides of triangle *ABC*	M1	
			Apply cosine rule correctly to find the cosine of angle *BAC*	M1	
			Obtain answer $\frac{20}{21}$	A1	4
	(ii)		State an exact value for the sine of angle *BAC*, e.g. $\sqrt{41}/21$	B1√	
			Use correct area formula to find the area of triangle *ABC*	M1	
			Obtain answer $\frac{1}{2}\sqrt{41}$, or exact equivalent	A1	3
			[SR: Allow use of a vector product, e.g. $\overrightarrow{AB}\times\overrightarrow{AC} = -6\mathbf{i}+2\mathbf{j}-\mathbf{k}$ B1√. Using correct process for the modulus, divide the modulus by 2 M1. Obtain answer $\frac{1}{2}\sqrt{41}$ A1.]		
	(iii)	*EITHER*:	State or obtain $b = 0$	B1	
			Equate scalar product of normal vector and $\overrightarrow{BC}$ (or $\overrightarrow{CB}$) to zero	M1	
			Obtain $a + b - 4c = 0$ (or $a - 4c = 0$)	A1	
			Substitute a relevant point in $4x + z = d$ and evaluate d	M1	
			Obtain answer $4x + z = 9$, or equivalent	A1	
		*OR*1:	Attempt to calculate vector product of relevant vectors, e.g. $(\mathbf{j})\times(\mathbf{i} + \mathbf{j} - 4\mathbf{k})$	M1	
			Obtain two correct components of the product	A1	
			Obtain correct product, e.g. $-4\mathbf{i} - \mathbf{k}$	A1	
			Substitute a relevant point in $4x + z = d$ and evaluate d	M1	
			Obtain $4x + z = 9$, or equivalent	A1	
		*OR*2:	Attempt to form 2-parameter equation for the plane with relevant vectors	M1	
			State a correct equation, e.g. $\mathbf{r} = 2\mathbf{i} + 4\mathbf{j} + \mathbf{k} + \lambda(\mathbf{j}) + \mu(\mathbf{i} + \mathbf{j} - 4\mathbf{k})$	A1	
			State 3 equations in x, y, z, λ and μ	A1	
			Eliminate μ	M1	
			Obtain answer $4x + z = 9$, or equivalent	A1	
		*OR*3:	State or obtain $b = 0$	B1	
			Substitute for *B* and *C* in the plane equation and obtain $2a + c = d$ and $3a - 3c = d$ (or $2a + 4b + c = d$ and $3a + 5b - 3c = d$)	B1	
			Solve for one ratio, e.g. $a : d$	M1	
			Obtain $a : c : d$, or equivalent	M1	
			Obtain answer $4x + z = 9$, or equivalent	A1	
		OR4:	Attempt to form a determinant equation for the plane with relevant vectors	M1	
			State a correct equation, e.g. $\begin{vmatrix} x-2 & y-4 & z-1 \\ 0 & 1 & 0 \\ 1 & 1 & -4 \end{vmatrix} = 0$	A1	
			Attempt to use a correct method to expand the determinant	M1	
			Obtain two correct terms of a 3-term expansion, or equivalent	A1	
			Obtain answer $4x + z = 9$, or equivalent	A1	5