UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2012**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **4** printed pages.

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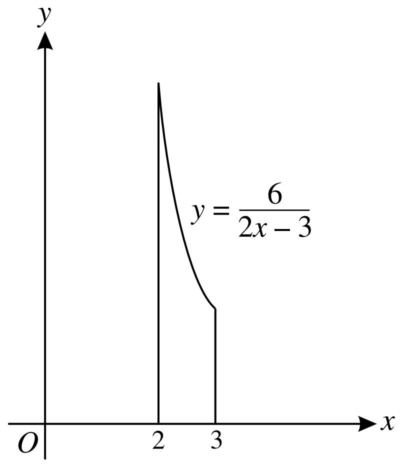
International Examinations

1



The diagram shows the region enclosed by the curve $y = \frac{6}{2x-3}$, the x-axis and the lines $x = 2$ and $x = 3$. Find, in terms of π, the volume obtained when this region is rotated through 360° about the x-axis. [4]

2 The equation of a curve is $y = 4\sqrt{x} + \frac{2}{\sqrt{x}}$.

(i) Obtain an expression for $\frac{dy}{dx}$. [3]

(ii) A point is moving along the curve in such a way that the x-coordinate is increasing at a constant rate of 0.12 units per second. Find the rate of change of the y-coordinate when $x = 4$. [2]

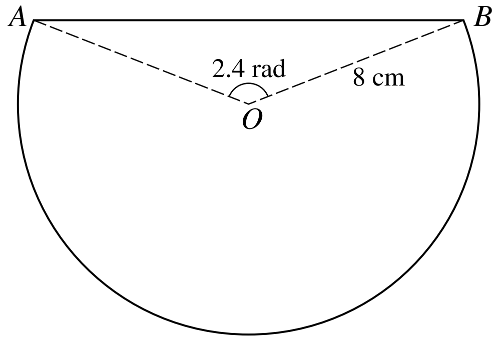
3 The coefficient of x^3 in the expansion of $(a + x)^5 + (2 - x)^6$ is 90. Find the value of the positive constant a. [5]

4 The point A has coordinates $(-1, -5)$ and the point B has coordinates $(7, 1)$. The perpendicular bisector of AB meets the x-axis at C and the y-axis at D. Calculate the length of CD. [6]

5 **(i)** Prove the identity $\tan x + \frac{1}{\tan x} \equiv \frac{1}{\sin x \cos x}$. [2]

(ii) Solve the equation $\frac{2}{\sin x \cos x} = 1 + 3\tan x$, for $0° \leqslant x \leqslant 180°$. [4]

6



The diagram shows a metal plate made by removing a segment from a circle with centre O and radius 8 cm. The line AB is a chord of the circle and angle $AOB = 2.4$ radians. Find

(i) the length of AB, [2]

(ii) the perimeter of the plate, [3]

(iii) the area of the plate. [3]

7 **(a)** In an arithmetic progression, the sum of the first n terms, denoted by S_n, is given by

$$S_n = n^2 + 8n.$$

Find the first term and the common difference. [3]

(b) In a geometric progression, the second term is 9 less than the first term. The sum of the second and third terms is 30. Given that all the terms of the progression are positive, find the first term. [5]

8 **(i)** Find the angle between the vectors $3\mathbf{i} - 4\mathbf{k}$ and $2\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}$. [4]

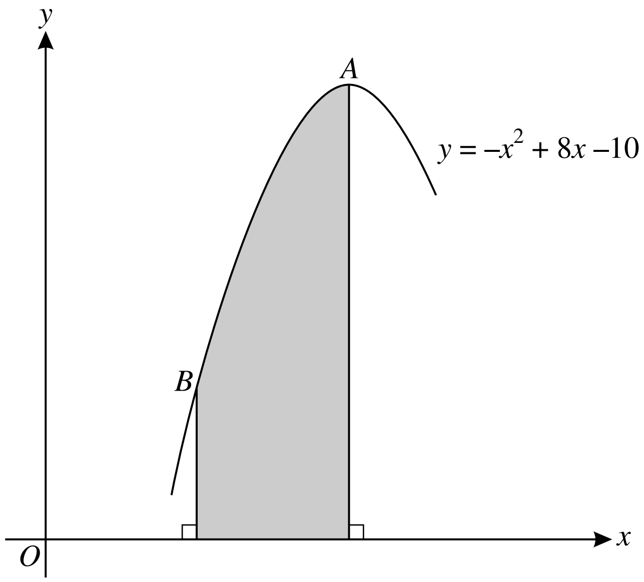
The vector $\overrightarrow{OA}$ has a magnitude of 15 units and is in the same direction as the vector $3\mathbf{i} - 4\mathbf{k}$. The vector $\overrightarrow{OB}$ has a magnitude of 14 units and is in the same direction as the vector $2\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}$.

(ii) Express $\overrightarrow{OA}$ and $\overrightarrow{OB}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. [3]

(iii) Find the unit vector in the direction of $\overrightarrow{AB}$. [3]

[Questions 9 and 10 are printed on the next page.]

9



The diagram shows part of the curve $y = -x^2 + 8x - 10$ which passes through the points A and B. The curve has a maximum point at A and the gradient of the line BA is 2.

(i) Find the coordinates of A and B. [7]

(ii) Find $\int y\,dx$ and hence evaluate the area of the shaded region. [4]

10 Functions f and g are defined by

$$f : x \mapsto 2x + 5 \quad \text{for } x \in \mathbb{R},$$
$$g : x \mapsto \frac{8}{x-3} \quad \text{for } x \in \mathbb{R},\ x \neq 3.$$

(i) Obtain expressions, in terms of x, for $f^{-1}(x)$ and $g^{-1}(x)$, stating the value of x for which $g^{-1}(x)$ is not defined. [4]

(ii) Sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ on the same diagram, making clear the relationship between the two graphs. [3]

(iii) Given that the equation $fg(x) = 5 - kx$, where k is a constant, has no solutions, find the set of possible values of k. [5]