UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the May/June 2011 question paper

for the guidance of teachers

9709 MATHEMATICS

9709/31 Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- Cambridge will not enter into discussions or correspondence in connection with these mark schemes.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1	Either:	Obtain $1+\frac{1}{3}kx$, where $k=\pm 6$ or ± 1	M1	
		Obtain $1-2x$	A1	
		Obtain $-4x^2$	A1	
		Obtain $-\frac{40}{3}x^3$ or equivalent	A1	
	Or:	Differentiate expression to obtain form $k(1-6x)^{-\frac{2}{3}}$ and evaluate f(0) and f′(0)	M1	
		Obtain $f'(x) = -2(1-6x)^{-\frac{2}{3}}$ and hence the correct first two terms $1-2x$	A1	
		Obtain $f''(x) = -8(1-6x)^{-\frac{5}{3}}$ and hence $-4x^2$	A1	
		Obtain $f'''(x) = -80(1-6x)^{-\frac{8}{3}}$ and hence $-\frac{40}{3}x^3$ or equivalent	A1	[4]

2	**(i)**	Obtain $\frac{k\cos 2x}{1+\sin 2x}$ for any non-zero constant k	M1	
		Obtain $\frac{2\cos 2x}{1+\sin 2x}$	A1	[2]
	(ii)	Use correct quotient or product rule	M1	
		Obtain $\frac{x\sec^2 x-\tan x}{x^2}$ or equivalent	A1	[2]

3	**(i)**	Obtain $\pm\begin{pmatrix}3\\-4\\6\end{pmatrix}$ as normal to plane	B1	
		Form equation of p as $3x-4y+6z=k$ or $-3x+4y-6z=k$ and use relevant point to find k	M1	
		Obtain $3x-4y+6z=80$ or $-3x+4y-6z=-80$	A1	[3]
	(ii)	State the direction vector $\begin{pmatrix}0\\1\\0\end{pmatrix}$ or equivalent	B1	
		Carry out correct process for finding scalar product of two relevant vectors	M1	
		Use correct complete process with moduli and scalar product and evaluate $\sin^{-1}$ or $\cos^{-1}$ of result	M1	
		Obtain 30.8° or 0.538 radians	A1	[4]

4	**(i)**	Verify that $-96 + 100 + 8 - 12 = 0$	B1	
		Attempt to find quadratic factor by division by $(x + 2)$, reaching a partial quotient $12x^2 + kx$, inspection or use of an identity	M1	
		Obtain $12x^2 + x - 6$	A1	
		State $(x + 2)(4x + 3)(3x - 2)$	A1	[4]
		[The M1 can be earned if inspection has unknown factor $Ax^2 + Bx - 6$ and an equation in A and/or B or equation $12x^2 + Bx + C$ and an equation in B and/or C.]		
	(ii)	State $3^y = \frac{2}{3}$ and no other value	B1	
		Use correct method for finding y from equation of form $3^y = k$, where $k > 0$	M1	
		Obtain -0.369 and no other value	A1	[3]
5	**(i)**	Use at least one of $e^{2x} = 9$, $e^y = 2$ and $e^{2y} = 4$	B1	
		Obtain given result $58 + 2k = c$ **AG**	B1	[2]
	(ii)	Differentiate left-hand side term by term, reaching $ae^{2x} + be^y\frac{dy}{dx} + ce^{2y}\frac{dy}{dx}$	M1	
		Obtain $12e^{2x} + ke^y\frac{dy}{dx} + 2e^{2y}\frac{dy}{dx}$	A1	
		Substitute (ln 3, ln 2) in an attempt involving implicit differentiation at least once, where RHS = 0	M1	
		Obtain $108 - 12k - 48 = 0$ or equivalent	A1	
		Obtain $k = 5$ and $c = 68$	A1	[5]
6	**(i)**	State or imply area of segment is $\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$ or $50\theta - 50\sin\theta$	B1	
		Attempt to form equation from area of segment = $\frac{1}{5}$ of area of circle, or equivalent	M1	
		Confirm given result $\theta = \frac{2}{5}\pi + \sin\theta$	A1	[3]
	(ii)	Use iterative formula correctly at least once	M1	
		Obtain value for θ of 2.11	A1	
		Show sufficient iterations to justify value of θ or show sign change in interval (2.105, 2.115)	A1	
		Use correct trigonometry to find an expression for the length of AB	M1	
		e.g. $20\sin 1.055$ or $\sqrt{200 - 200\cos 2.11}$		
		Hence 17.4	A1	[5]
		$[2.1 \to 2.1198 \to 2.1097 \to 2.1149 \to 2.1122]$		

7	**(i)**	State or imply $\mathrm{d}x = 2t\,\mathrm{d}t$ or equivalent	B1	
		Express the integral in terms of x and $\mathrm{d}x$	M1	
		Obtain given answer $\int_1^5 (2x-2)\ln x\,\mathrm{d}x$, including change of limits **AG**	A1	[3]
	(ii)	Attempt integration by parts obtaining $(ax^2+bx)\ln x \pm \int (ax^2+bx)\frac{1}{x}\mathrm{d}x$ or equivalent	M1	
		Obtain $(x^2-2x)\ln x - \int (x^2-2x)\frac{1}{x}\mathrm{d}x$ or equivalent	A1	
		Obtain $(x^2-2x)\ln x - \frac{1}{2}x^2 + 2x$	A1	
		Use limits correctly having integrated twice	M1	
		Obtain $15\ln 5 - 4$ or exact equivalent	A1	[5]
		[Equivalent for M1 is $(2x-2)(ax\ln x + bx) - \int (ax\ln x + bx)\,2\mathrm{d}x$]		
8	**(i)**	Either: Multiply numerator and denominator by $(1-2\mathrm{i})$, or equivalent	M1	
		Obtain $-3\mathrm{i}$	A1	
		State modulus is 3	A1	
		Refer to u being on negative imaginary axis or equivalent and confirm argument as $-\frac{1}{2}\pi$	A1	
		Or: Using correct processes, divide moduli of numerator and denominator	M1	
		Obtain 3	A1	
		Subtract argument of denominator from argument of numerator	M1	
		Obtain $-\tan^{-1}\frac{1}{2} - \tan^{-1}2$ or $-0.464 - 1.107$ and hence $-\frac{1}{2}\pi$ or -1.57	A1	[4]
	(ii)	Show correct half-line from u at angle $\frac{1}{4}\pi$ to real direction	B1	
		Use correct trigonometry to find required value	M1	
		Obtain $\frac{3}{2}\sqrt{2}$ or equivalent	A1	[3]
	(iii)	Show, or imply, locus is a circle with centre $(1+\mathrm{i})u$ and radius 1	M1	
		Use correct method to find distance from origin to furthest point of circle	M1	
		Obtain $3\sqrt{2}+1$ or equivalent	A1	[3]

9	**(i)**	Express $\cos 4\theta$ as $2\cos^2 2\theta - 1$ or $\cos^2 2\theta - \sin^2 2\theta$ or $1 - 2\sin^2 2\theta$	B1	
		Express $\cos 4\theta$ in terms of $\cos\theta$	M1	
		Obtain $8\cos^4\theta - 8\cos^2\theta + 1$	A1	
		Use $\cos 2\theta = 2\cos^2\theta - 1$ to obtain given answer $8\cos^4\theta - 3$ **AG**	A1	[4]
	(ii)	**(a)** State or imply $\cos^4\theta = \frac{1}{2}$	B1	
		Obtain 0.572	B1	
		Obtain –0.572	B1	[3]
		(b) Integrate and obtain form $k_1\theta + k_2 \sin 4\theta + k_3 \sin 2\theta$	M1	
		Obtain $\frac{3}{8}\theta + \frac{1}{32}\sin 4\theta + \frac{1}{4}\sin 2\theta$	A1	
		Obtain $\frac{3}{32}\pi + \frac{1}{4}$ following completely correct work	A1	[3]
10	**(i)**	Separate variables correctly and integrate of at least one side	M1	
		Carry out an attempt to find A and B such that $\dfrac{1}{N(1800-N)} \equiv \dfrac{A}{N} + \dfrac{B}{1800-N}$, or equivalent	M1	
		Obtain $\dfrac{2}{N} + \dfrac{2}{1800-N}$ or equivalent	A1	
		Integrates to produce two terms involving natural logarithms	M1	
		Obtain $2\ln N - 2\ln(1800 - N) = t$ or equivalent	A1	
		Evaluate a constant, or use $N = 300$ and $t = 0$ in a solution involving $a\ln N$, $b\ln(1800)$ and ct	M1	
		Obtain $2\ln N - 2\ln(1800 - N) = t - 2\ln 5$ or equivalent	A1	
		Use laws of logarithms to remove logarithms	M1	
		Obtain $N = \dfrac{1800e^{\frac{1}{2}t}}{5 + e^{\frac{1}{2}t}}$ or equivalent	A1	[9]
	(ii)	State or imply that N approaches 1800	B1	[1]