**UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS**

**GCE Advanced Subsidiary Level and GCE Advanced Level**

# MARK SCHEME for the May/June 2010 question paper

# for the guidance of teachers

## 9709 MATHEMATICS

**9709/31** Paper 31, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

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**Mark Scheme Notes**

Marks are of the following three types:

M	Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A	Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B	Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol $\sqrt{}$ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note:	B2 or A2 means that the candidate can earn 2 or 0.
	B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking $g$ equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

**Penalties**

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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| **1** | *EITHER:* | State or imply non-modular inequality $(x+3a)^2 > (2(x-2a))^2$, or corresponding quadratic equation, or pair of linear equations $(x+3a) = \pm 2(x-2a)$ | B1 | |
| | | Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations | M1 | |
| | | Obtain critical values $x = \frac{1}{3}a$ and $x = 7a$ | A1 | |
| | | State answer $\frac{1}{3}a < x < 7a$ | A1 | |
| | *OR:* | Obtain the critical value $x = 7a$ from a graphical method, or by inspection, or by solving a linear equation or inequality | B1 | |
| | | Obtain the critical value $x = \frac{1}{3}a$ similarly | B2 | |
| | | State answer $\frac{1}{3}a < x < 7a$ | B1 | [4] |
| | | [Do not condone $\leqslant$ for $<$; accept 0.33 for $\frac{1}{3}$.] | | |

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| **2** | Use correct cos $2A$ formula and obtain an equation in sin $\theta$ | M1 | |
| | Obtain $4\sin^2\theta + \sin\theta - 3 = 0$, or equivalent | A1 | |
| | Make reasonable attempt to solve a 3-term quadratic in sin $\theta$ | M1 | |
| | Obtain answer 48.6° | A1 | |
| | Obtain answer 131.4° and no others in the given range | A1 √ | |
| | Obtain answer 270° and no others in the given range | A1 | [6] |
| | [Treat the giving of answers in radians as a misread. Ignore answers outside the given range.] | | |

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| **3** | **(i)** | *EITHER:* | State or imply $n\ln x + \ln y = \ln C$ | B1 | |
| | | | Substitute $x$- and $y$-values and solve for $n$ | M1 | |
| | | | Obtain $n = 1.50$ | A1 | |
| | | | Solve for $C$ | M1 | |
| | | | Obtain $C = 6.00$ | A1 | |
| | | *OR:* | Obtain two correct equations by substituting $x$- and $y$-values in $x^n y = C$ | B1 | |
| | | | Solve for $n$ | M1 | |
| | | | Obtain $n = 1.50$ | A1 | |
| | | | Solve for $C$ | M1 | |
| | | | Obtain $C = 6.00$ | A1 | [5] |
| | **(ii)** | | State that the graph of ln $y$ against ln $x$ has equation $n$ln $x$ + ln $y$ = ln $C$ which is *linear* in ln $y$ and ln $x$, or has equation of the form $nX + Y = \ln C$, where $X = \ln x$ and $Y = \ln y$, and is thus a straight line | B1 | [1] |

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| **4** | **(i)** | State correct expansion of $\cos(3x - x)$ or $\cos(3x + x)$ | B1 | |
| | | Substitute expansions in $\frac{1}{2}(\cos 2x - \cos 4x)$, or equivalent | M1 | |
| | | Simplify and obtain the given identity correctly | A1 | [3] |
| | **(ii)** | Obtain integral $\frac{1}{4}\sin 2x - \frac{1}{8}\sin 4x$ | B1 | |
| | | Substitute limits correctly in an integral of the form $a\sin 2x + b\sin 4x$ | M1 | |
| | | Obtain given answer following full, correct and exact working | A1 | [3] |

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| **5** | | Separate variables correctly | B1 | |
| | | Integrate and obtain term $\ln x$ | B1 | |
| | | Integrate and obtain term $\frac{1}{2}\ln(y^2+4)$ | B1 | |
| | | Evaluate a constant or use limits $y = 0$, $x = 1$ in a solution containing $a\ln x$ and $b\ln(y^2 + 4)$ | M1 | |
| | | Obtain correct solution in any form, e.g. $\frac{1}{2}\ln(y^2+4) = \ln x + \frac{1}{2}\ln 4$ | A1 | |
| | | Rearrange as $y^2 = 4(x^2 - 1)$, or equivalent | A1 | [6] |
| **6** | **(i)** | Using the formulae $\frac{1}{2}r^2\theta$ and $\frac{1}{2}r^2\sin\theta$, or equivalent, form an equation | M1 | |
| | | Obtain a correct equation in $r$ and $x$ and/or $x/2$ in any form | A1 | |
| | | Obtain the given equation correctly | A1 | [3] |
| | **(ii)** | Consider the sign of $x - (\frac{3}{4}\pi - \sin x)$ at $x = 1.3$ and $x = 1.5$, or equivalent | M1 | |
| | | Complete the argument with correct calculations | A1 | [2] |
| | **(iii)** | Use the iterative formula correctly at least once | M1 | |
| | | Obtain final answer 1.38 | A1 | |
| | | Show sufficient iterations to at least 4 d.p. to justify its accuracy to 2 d.p., or show there is a sign change in the interval (1.375, 1.385) | A1 | [3] |
| **7** | **(i)** | Obtain modulus $\sqrt{8}$ | B1 | |
| | | Obtain argument $\frac{1}{4}\pi$ or 45° | B1 | [2] |
| | **(ii)** | Show 1, i and $u$ in relatively correct positions on an Argand diagram | B1 | |
| | | Show the perpendicular bisector of the line joining 1 and i | B1 | |
| | | Show a circle with centre $u$ and radius 1 | B1 | |
| | | Shade the correct region | B1 | [4] |
| | **(iii)** | State or imply relevance of the appropriate tangent from $O$ to the circle | B1 √ | |
| | | Carry out complete strategy for finding $\lvert z \rvert$ for the critical point | M1 | |
| | | Obtain answer $\sqrt{7}$ | A1 | [3] |
| **8** | **(i)** | State or imply the form $\frac{A}{x+1} + \frac{B}{x+3}$ and use a relevant method to find $A$ or $B$ | M1 | |
| | | Obtain $A = 1$, $B = -1$ | A1 | [2] |
| | **(ii)** | Square the result of part **(i)** and substitute the fractions of part **(i)** | M1 | |
| | | Obtain the given answer correctly | A1 | [2] |
| | **(iii)** | Integrate and obtain $-\frac{1}{x+1} - \ln(x+1) + \ln(x+3) - \frac{1}{x+3}$ | B3 | |
| | | Substitute limits correctly in an integral containing at least two terms of the correct form | M1 | |
| | | Obtain given answer following full and exact working | A1 | [5] |

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| 9 | (i) | Use quotient or product rule to differentiate $(1 - x)/(1 + x)$ | M1 | |
| | | Obtain correct derivative in any form | A1 | |
| | | Use chain rule to find $\frac{dy}{dx}$ | M1 | |
| | | Obtain a correct expression in any form | A1 | |
| | | Obtain the gradient of the normal in the given form correctly | A1 | [5] |
| | (ii) | Use product rule | M1 | |
| | | Obtain correct derivative in any form | A1 | |
| | | Equate derivative to zero and solve for $x$ | M1 | |
| | | Obtain $x = \frac{1}{2}$ | A1 | [4] |
| 10 | (i) | Express general point of $l$ or $m$ in component form, e.g. $(1 + s, 1 - s, 1 + 2s)$ or $(4 + 2t, 6 + 2t, 1 + t)$ | B1 | |
| | | Equate at least two corresponding pairs of components and solve for $s$ or $t$ | M1 | |
| | | Obtain $s = -1$ or $t = -2$ | A1 | |
| | | Verify that all three component equations are satisfied | A1 | [4] |
| | (ii) | Carry out correct process for evaluating the scalar product of the direction vectors of $l$ and $m$ | M1 | |
| | | Using the correct process for the moduli, divide the scalar product by the product of the moduli and evaluate the inverse cosine of the result | M1 | |
| | | Obtain answer 74.2° (or 1.30 radians) | A1 | [3] |
| | (iii) | *EITHER:* Use scalar product to obtain $a - b + 2c = 0$ and $2a + 2b + c = 0$ | B1 | |
| | | Solve and obtain one ratio, e.g. $a : b$ | M1 | |
| | | Obtain $a : b : c = 5 : -3 : -4$, or equivalent | A1 | |
| | | Substitute coordinates of a relevant point and values for $a$, $b$ and $c$ in general equation of plane and evaluate $d$ | M1 | |
| | | Obtain answer $5x - 3y - 4z = -2$, or equivalent | A1 | |
| | | *OR* 1: Using two points on $l$ and one on $m$, or *vice versa*, state three equations in $a$, $b$, $c$ and $d$ | B1 | |
| | | Solve and obtain one ratio, e.g. $a : b$ | M1 | |
| | | Obtain a ratio of three of the unknowns, e.g. $a : b : c = -5 : 3 : 4$ | A1 | |
| | | Use coordinates of a relevant point and found ratio to find the fourth unknown, e.g. $d$ | M1 | |
| | | Obtain answer $-5x + 3y + 4z = 2$, or equivalent | A1 | |
| | | *OR* 2: Form a correct 2-parameter equation for the plane, e.g. $\mathbf{r} = \mathbf{i} + \mathbf{j} + \mathbf{k} + \lambda(\mathbf{i} - \mathbf{j} + 2\mathbf{k}) + \mu(2\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ | B1 | |
| | | State three equations in $x$, $y$, $z$, $\lambda$ and $\mu$ | M1 | |
| | | State three correct equations | A1 | |
| | | Eliminate $\lambda$ and $\mu$ | M1 | |
| | | Obtain answer $5x - 3y - 4z = -2$, or equivalent | A1 | |
| | | *OR* 3: Attempt to calculate vector product of direction vectors of $l$ and $m$ | M1 | |
| | | Obtain two correct components of the product | A1 | |
| | | Obtain correct product, e.g. $-5\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$ | A1 | |
| | | Form a plane equation and use coordinates of a relevant point to calculate $d$ | M1 | |
| | | Obtain answer $-5x + 3y + 4z = 2$, or equivalent | A1 | [5] |