UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/03**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2009**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
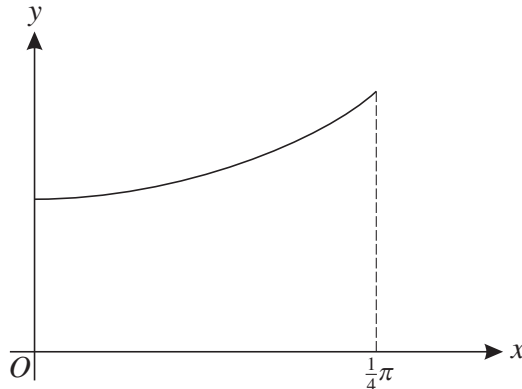
This document consists of **4** printed pages.

UNIVERSITY *of* CAMBRIDGE
International Examinations

[Turn over

1 Solve the equation $\ln(2 + e^{-x}) = 2$, giving your answer correct to 2 decimal places. [4]

2

The diagram shows the curve $y = \sqrt{(1 + 2\tan^2 x)}$ for $0 \leqslant x \leqslant \frac{1}{4}\pi$.

  (i) Use the trapezium rule with three intervals to estimate the value of

$$\int_0^{\frac{1}{4}\pi} \sqrt{(1 + 2\tan^2 x)}\,\mathrm{d}x,$$

giving your answer correct to 2 decimal places. [3]

  (ii) The estimate found in part **(i)** is denoted by E. Explain, without further calculation, whether another estimate found using the trapezium rule with six intervals would be greater than E or less than E. [1]

3   **(i)** Prove the identity $\operatorname{cosec} 2\theta + \cot 2\theta \equiv \cot \theta$. [3]

  (ii) Hence solve the equation $\operatorname{cosec} 2\theta + \cot 2\theta = 2$, for $0° \leqslant \theta \leqslant 360°$. [2]

4 The equation $x^3 - 2x - 2 = 0$ has one real root.

  (i) Show by calculation that this root lies between $x = 1$ and $x = 2$. [2]

  (ii) Prove that, if a sequence of values given by the iterative formula

$$x_{n+1} = \frac{2x_n^3 + 2}{3x_n^2 - 2}$$

converges, then it converges to this root. [2]

  (iii) Use this iterative formula to calculate the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

5 When $(1 + 2x)(1 + ax)^{\frac{2}{3}}$, where a is a constant, is expanded in ascending powers of x, the coefficient of the term in x is zero.

    (i) Find the value of a.   [3]

    (ii) When a has this value, find the term in x^3 in the expansion of $(1 + 2x)(1 + ax)^{\frac{2}{3}}$, simplifying the coefficient.   [4]

6 The parametric equations of a curve are

$$x = a\cos^3 t, \quad y = a\sin^3 t,$$

where a is a positive constant and $0 < t < \frac{1}{2}\pi$.

    (i) Express $\dfrac{dy}{dx}$ in terms of t.   [3]

    (ii) Show that the equation of the tangent to the curve at the point with parameter t is

$$x\sin t + y\cos t = a\sin t\cos t. \qquad [3]$$

    (iii) Hence show that, if this tangent meets the x-axis at X and the y-axis at Y, then the length of XY is always equal to a.   [2]

7   **(i)** Solve the equation $z^2 + (2\sqrt{3})iz - 4 = 0$, giving your answers in the form $x + iy$, where x and y are real.   [3]

    (ii) Sketch an Argand diagram showing the points representing the roots.   [1]

    (iii) Find the modulus and argument of each root.   [3]

    (iv) Show that the origin and the points representing the roots are the vertices of an equilateral triangle.   [1]

8   **(i)** Express $\dfrac{100}{x^2(10 - x)}$ in partial fractions.   [4]

    (ii) Given that $x = 1$ when $t = 0$, solve the differential equation

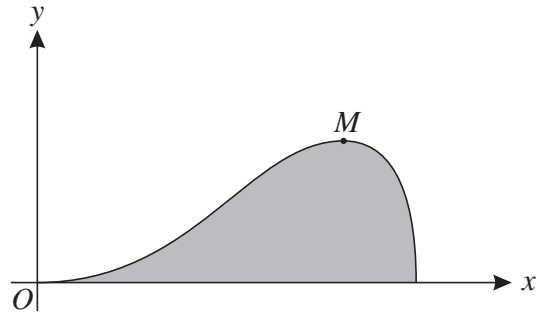$$\frac{dx}{dt} = \tfrac{1}{100}x^2(10 - x),$$

    obtaining an expression for t in terms of x.   [6]

9 The line l has equation $\mathbf{r} = 4\mathbf{i} + 2\mathbf{j} - \mathbf{k} + t(2\mathbf{i} - \mathbf{j} - 2\mathbf{k})$. It is given that l lies in the plane with equation $2x + by + cz = 1$, where b and c are constants.

    (i) Find the values of b and c.   [6]

    (ii) The point P has position vector $2\mathbf{j} + 4\mathbf{k}$. Show that the perpendicular distance from P to l is $\sqrt{5}$.   [5]

10

The diagram shows the curve $y = x^2 \sqrt{(1 - x^2)}$ for $x \geqslant 0$ and its maximum point M.

   (i) Find the exact value of the x-coordinate of M.   [4]

 (ii) Show, by means of the substitution $x = \sin\theta$, that the area A of the shaded region between the curve and the x-axis is given by

$$A = \tfrac{1}{4} \int_0^{\frac{1}{2}\pi} \sin^2 2\theta \, d\theta.$$ [3]

(iii) Hence obtain the exact value of A.   [4]