

# Mark Scheme (Results) Summer 2009

GCE

GCE Mathematics (6663/01)

June 2009  
6663 Core Mathematics C1  
Mark Scheme

Question Number	Scheme	Marks
Q1 (a) (b)	$(3\sqrt{7})^2 = 63$ $(8 + \sqrt{5})(2 - \sqrt{5}) = 16 - 5 + 2\sqrt{5} - 8\sqrt{5}$ $= 11, -6\sqrt{5}$	B1 (1) M1 A1, A1 (3) [4]
(a) (b)	<p>B1 for 63 only</p> <p>M1 for an attempt to expand <u>their</u> brackets with <math>\geq 3</math> terms correct.</p> <p>They may collect the <math>\sqrt{5}</math> terms to get <math>16 - 5 - 6\sqrt{5}</math></p> <p>Allow <math>-\sqrt{5} \times \sqrt{5}</math> or <math>-(\sqrt{5})^2</math> or <math>-\sqrt{25}</math> instead of the -5</p> <p>These 4 values may appear in a list or table but they should have minus signs included</p> <p><b>The next two marks should be awarded for the final answer but check that correct values follow from correct working. Do not use ISW rule</b></p> <p>1<sup>st</sup> A1 for 11 from <math>16 - 5</math> <u>or</u> <math>-6\sqrt{5}</math> from <math>-8\sqrt{5} + 2\sqrt{5}</math></p> <p>2<sup>nd</sup> A1 for <u>both</u> 11 and <math>-6\sqrt{5}</math>.</p> <p><u>S.C - Double sign error in expansion</u></p> <p>For <math>16 - 5 - 2\sqrt{5} + 8\sqrt{5}</math> leading to <math>11 + \dots</math> allow <u>one</u> mark</p>	

Question Number	Scheme	Marks
Q2	$32 = 2^5 \text{ or } 2048 = 2^{11}, \quad \sqrt{2} = 2^{1/2} \text{ or } \sqrt{2048} = (2048)^{1/2}$ $a = \frac{11}{2} \quad \left( \text{or } 5\frac{1}{2} \text{ or } 5.5 \right)$	B1, B1 B1 [3]
	<p>1<sup>st</sup> B1 for <math>32 = 2^5</math> or <math>2048 = 2^{11}</math>            This should be explicitly seen: <math>32\sqrt{2} = 2^a</math> followed by <math>2^5\sqrt{2} = 2^a</math> is OK            Even writing <math>32 \times 2 = 2^5 \times 2 (= 2^6)</math> is OK but simply writing <math>32 \times 2 = 2^6</math> is NOT</p> <p>2<sup>nd</sup> B1 for <math>2^{1/2}</math> or <math>(2048)^{1/2}</math> seen. This mark may be implied</p> <p>3<sup>rd</sup> B1 for answer as written. <b>Need</b> <math>a = \dots</math> so <math>2^{11/2}</math> is B0</p> <p style="text-align: center;"><math>a = \frac{11}{2} \left( \text{or } 5\frac{1}{2} \text{ or } 5.5 \right)</math> with no working scores full marks.</p> <p style="text-align: center;">If <math>a = 5.5</math> seen then award 3/3 unless it is clear that the value follows from totally incorrect work.</p> <p style="text-align: center;">Part solutions: e.g. <math>2^5\sqrt{2}</math> scores the first B1.</p> <p><u>Special case:</u>            If <math>\sqrt{2} = 2^{1/2}</math> is not explicitly seen, but the final answer includes <math>\frac{1}{2}</math>,            e.g. <math>a = 2\frac{1}{2}</math>, <math>a = 4\frac{1}{2}</math>, the second B1 is given by implication.</p>	

Question Number	Scheme	Marks
Q3 (a)	$\frac{dy}{dx} = 6x^2 - 6x^{-3}$	M1 A1 A1 (3)
(b)	$\frac{2x^4}{4} + \frac{3x^{-1}}{-1} (+C)$	M1 A1
	$\frac{x^4}{2} - 3x^{-1} + C$	A1
		(3) [6]
(a)	<p>M1 for an attempt to differentiate <math>x^n \rightarrow x^{n-1}</math></p> <p>1<sup>st</sup> A1 for <math>6x^2</math></p> <p>2<sup>nd</sup> A1 for <math>-6x^{-3}</math> or <math>-\frac{6}{x^3}</math> Condone <math>+ -6x^{-3}</math> here. Inclusion of <math>+c</math> scores A0 here.</p>	
(b)	<p>M1 for some attempt to integrate an <math>x</math> term of the given <math>y</math>. <math>x^n \rightarrow x^{n+1}</math></p> <p>1<sup>st</sup> A1 for <b>both</b> <math>x</math> terms correct but unsimplified- as printed or better. Ignore <math>+c</math> here</p> <p>2<sup>nd</sup> A1 for both <math>x</math> terms correct and simplified and <math>+c</math>. Accept <math>-\frac{3}{x}</math> but <u>NOT</u> <math>+ -3x^{-1}</math></p> <p>Condone the <math>+c</math> appearing on the first (unsimplified) line but missing on the final (simplified) line</p> <p>Apply ISW if a correct answer is seen</p> <p>If part (b) is attempted first and this is clearly labelled then apply the scheme and allow the marks. Otherwise assume the first solution is for part (a).</p>	

Question Number	Scheme	Marks
Q4 (a)  (b)  (c)	<p><math>5x &gt; 10, x &gt; 2</math> [Condone <math>x &gt; \frac{10}{2} = 2</math> for M1A1]</p> <p><math>(2x + 3)(x - 4) = 0</math>, ‘Critical values’ are <math>-\frac{3}{2}</math> and 4</p> <p><math>-\frac{3}{2} &lt; x &lt; 4</math></p> <p><math>2 &lt; x &lt; 4</math></p>	<p>M1, A1 (2)</p> <p>M1, A1</p> <p>M1 A1ft (4)</p> <p>B1ft (1) [7]</p>
(a)  (b)	<p>M1 for attempt to collect like terms on each side leading to <math>ax &gt; b</math>, or <math>ax &lt; b</math>, or <math>ax = b</math></p> <p>Must have <math>a</math> or <math>b</math> correct so eg <math>3x &gt; 4</math> scores M0</p> <p>1<sup>st</sup> M1 for an attempt to factorize or solve to find critical values. Method must potentially give 2 critical values</p> <p>1<sup>st</sup> A1 for <math>-\frac{3}{2}</math> and 4 seen. They may write <math>x &lt; -\frac{3}{2}, x &lt; 4</math> and still get this A1</p> <p>2<sup>nd</sup> M1 for choosing the “inside region” for their critical values</p> <p>2<sup>nd</sup> A1ft follow through their 2 distinct critical values</p> <p>Allow <math>x &gt; -\frac{3}{2}</math> with “or” “,” “<math>\cup</math>” ““ <math>x &lt; 4</math> to score M1A0 but “and” or “<math>\cap</math>” score M1A1</p> <p><math>x \in (-\frac{3}{2}, 4)</math> is M1A1 but <math>x \in [-\frac{3}{2}, 4]</math> is M1A0. Score M0A0 for a number line or graph only</p>	
(c)	<p>B1ft Allow if <b>a correct answer is seen</b> or follow through their answer to (a) and their answer to (b) but their answers to (a) and (b) <u>must be regions</u>. Do not follow through single values.</p> <p>If their follow through answer is the empty set accept <math>\emptyset</math> or <math>\{\}</math> or equivalent in words</p> <p>If (a) or (b) are not given then score this mark for cao</p> <p>NB You may see <math>x &lt; 4</math> (with anything or nothing in-between) <math>x &lt; -1.5</math> in (b) and empty set in (c) for B1ft</p> <p><b>Do not award marks for part (b) if only seen in part (c)</b></p> <p>Use of <math>\leq</math> instead of <math>&lt;</math> (or <math>\geq</math> instead of <math>&gt;</math>) loses one accuracy mark only, at first occurrence.</p>	





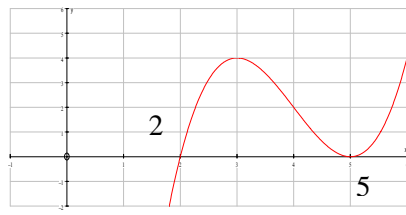
Question Number	Scheme	Marks
Q6	<p><math>b^2 - 4ac</math> attempted, in terms of <math>p</math>.</p> <p><math>(3p)^2 - 4p = 0</math> o.e.</p> <p>Attempt to solve for <math>p</math> e.g. <math>p(9p - 4) = 0</math> Must potentially lead to <math>p = k, k \neq 0</math></p> <p><math>p = \frac{4}{9}</math> (Ignore <math>p = 0</math>, if seen)</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1cso</p> <p>[4]</p>
	<p>1<sup>st</sup> M1 for an attempt to substitute into <math>b^2 - 4ac</math> or <math>b^2 = 4ac</math> with <math>b</math> or <math>c</math> correct            Condone <math>x</math>'s in one term only.            This can be inside a square root as part of the quadratic formula for example.  <b>Use of inequalities can score the M marks only</b></p> <p>1<sup>st</sup> A1 for any correct equation: <math>(3p)^2 - 4 \times 1 \times p = 0</math> or better</p> <p>2<sup>nd</sup> M1 for an attempt to factorize or solve their quadratic expression in <math>p</math>.            Method must be sufficient to lead to their <math>p = \frac{4}{9}</math>.</p> <p>Accept factors or use of quadratic formula or <math>(p \pm \frac{2}{9})^2 = k^2</math> (o.e. eg) <math>(3p \pm \frac{2}{3})^2 = k^2</math> or equivalent work on <u>their</u> eqn.</p> <p><math>9p^2 = 4p \Rightarrow \frac{9p^{\cancel{2}}}{\cancel{9}} = 4</math> which would lead to <math>9p = 4</math> is OK for this 2<sup>nd</sup> M1</p> <p>ALT <u>Comparing coefficients</u></p> <p>M1 for <math>(x + \alpha)^2 = x^2 + \alpha^2 + 2\alpha x</math> and A1 for a correct equation eg <math>3p = 2\sqrt{p}</math></p> <p>M1 for forming solving leading to <math>\sqrt{p} = \frac{2}{3}</math> or better</p> <p><u>Use of quadratic/discriminant formula (or any formula) Rule for awarding M mark</u>            If the formula is quoted accept some correct substitution leading to a partially correct expression.            If the formula is not quoted only award for a fully correct expression using their values.</p>	

Question Number	Scheme	Marks
Q7	<p>(a) <math>(a_2 =)2k - 7</math></p> <p>(b) <math>(a_3 =)2(2k - 7) - 7</math> or <math>4k - 14 - 7, = 4k - 21</math> (*)</p> <p>(c) <math>(a_4 =)2(4k - 21) - 7 (= 8k - 49)</math></p> $\sum_{r=1}^4 a_r = k + "(2k - 7)" + (4k - 21) + "(8k - 49)"$ $k + (2k - 7) + (4k - 21) + (8k - 49) = 15k - 77 = 43 \quad k = 8$	<p>B1 (1)</p> <p>M1, A1cso (2)</p> <p>M1</p> <p>M1</p> <p>M1 A1 (4)</p> <p>[7]</p>
	<p>(b) M1 must see <math>2(\text{their } a_2) - 7</math> or <math>2(2k - 7) - 7</math> or <math>4k - 14 - 7</math>. Their <math>a_2</math> must be a function of <math>k</math>. A1cso must see the <math>2(2k - 7) - 7</math> or <math>4k - 14 - 7</math> expression and the <math>4k - 21</math> with no incorrect working</p> <p>(c) 1<sup>st</sup> M1 for an attempt to find <math>a_4</math> using the given rule. Can be awarded for <math>8k - 49</math> seen. <b>Use of formulae for the sum of an arithmetic series scores M0M0A0 for the next 3 marks.</b> 2<sup>nd</sup> M1 for attempting the sum of the 1<sup>st</sup> 4 terms. Must have "+" not just , or clear attempt to sum. Follow through their <math>a_2</math> and <math>a_4</math> provided they are linear functions of <math>k</math>. Must lead to linear expression in <math>k</math>. Condone use of their linear <math>a_3 \neq 4k - 21</math> here too. 3<sup>rd</sup> M1 for forming a linear equation in <math>k</math> using their sum and the 43 and attempt to solve for <math>k</math> as far as <math>pk = q</math> A1 for <math>k = 8</math> only so <math>k = \frac{120}{15}</math> is A0</p> <p><u>Answer Only</u> (e.g. trial improvement) Accept <math>k = 8</math> <u>only if</u> <math>8 + 9 + 11 + 15 = 43</math> is seen as well</p> <p><u>Sum</u> <math>a_2 + a_3 + a_4 + a_5</math> or <math>a_2 + a_3 + a_4</math></p> <p>Allow: M1 if <math>8k - 49</math> is seen, M0 for the sum (since they are not adding the 1<sup>st</sup> 4 terms) then M1 if they use their sum along with the 43 to form a linear equation and attempt to solve but A0</p>	



Question Number	Scheme	Marks
Q8 (a)	$AB: m = \frac{2-7}{8-6}, \left( = -\frac{5}{2} \right)$ <p>Using <math>m_1 m_2 = -1</math>: <math>m_2 = \frac{2}{5}</math></p> $y - 7 = \frac{2}{5}(x - 6), \quad 2x - 5y + 23 = 0 \quad (\text{o.e. with integer coefficients})$	B1 M1 M1, A1 (4)
(b)	Using $x = 0$ in the answer to (a), $y = \frac{23}{5}$ or 4.6	M1, A1ft (2)
(c)	Area of triangle = $\frac{1}{2} \times 8 \times \frac{23}{5} = \frac{92}{5}$ (o.e.) e.g. $\left( 18\frac{2}{5}, 18.4, \frac{184}{10} \right)$	M1 A1 (2)
(a)	<p>B1 for an expression for the gradient of <math>AB</math>. Does not need the <math>= -2.5</math></p> <p>1<sup>st</sup> M1 for use of the perpendicular gradient rule. Follow through their <math>m</math></p> <p>2<sup>nd</sup> M1 for the use of (6, 7) and their changed gradient to form an equation for <math>l</math>.</p> <p>Can be awarded for <math>\frac{y-7}{x-6} = \frac{2}{5}</math> o.e.</p> <p>Alternative is to use (6, 7) in <math>y = mx + c</math> to <u>find a value</u> for <math>c</math>. Score when <math>c = \dots</math> is reached.</p> <p>A1 for a correct equation in the required form and must have “= 0” and integer coefficients</p>	
(b)	M1 for using $x = 0$ in their answer to part (a) e.g. $-5y + 23 = 0$	
A1ft	for $y = \frac{23}{5}$ provided that $x = 0$ clearly seen <u>or</u> $C(0, 4.6)$ . Follow through their equation in (a)	
(c)	M1 for $\frac{1}{2} \times 8 \times y_C$ so can follow through their $y$ coordinate of $C$ .	
A1	for 18.4 (o.e.) but their $y$ coordinate of $C$ must be positive	
<u>Use of 2 triangles or trapezium and triangle</u>	Award M1 when an expression for area of $OCB$ only is seen	
<u>Determinant approach</u>	Award M1 when an expression containing $\frac{1}{2} \times 8 \times y_C$ is seen	

Question Number	Scheme	Marks
Q9 (a)	$\left[ (3-4\sqrt{x})^2 = \right] 9-12\sqrt{x}-12\sqrt{x}+(-4)^2 x$ $9x^{-\frac{1}{2}} + 16x^{\frac{1}{2}} - 24$	M1 A1, A1 (3)
(b)	$f'(x) = -\frac{9}{2}x^{-\frac{3}{2}}, + \frac{16}{2}x^{-\frac{1}{2}}$	M1 A1, A1ft (3)
(c)	$f'(9) = -\frac{9}{2} \times \frac{1}{27} + \frac{16}{2} \times \frac{1}{3} = -\frac{1}{6} + \frac{16}{6} = \frac{5}{2}$	M1 A1 (2)
(a)	<p>M1 for an attempt to expand <math>(3-4\sqrt{x})^2</math> with at least 3 terms correct- as printed or better</p> <p><u>Or</u> <math>9-k\sqrt{x}+16x</math> (<math>k \neq 0</math>) . See also the MR rule below</p> <p>1<sup>st</sup> A1 for their coefficient of <math>\sqrt{x} = 16</math>. Condone writing <math>(\pm)9x^{(\pm)\frac{1}{2}}</math> instead of <math>9x^{-\frac{1}{2}}</math></p> <p>2<sup>nd</sup> A1 for <math>B = -24</math> or their constant term = -24</p>	
(b)	<p>M1 for an attempt to differentiate an <math>x</math> term <math>x^n \rightarrow x^{n-1}</math></p> <p>1<sup>st</sup> A1 for <math>-\frac{9}{2}x^{-\frac{3}{2}}</math> <u>and</u> their constant <math>B</math> differentiated to zero. NB <math>-\frac{1}{2} \times 9x^{-\frac{3}{2}}</math> is A0</p> <p>2<sup>nd</sup> A1ft follow through their <math>Ax^{\frac{1}{2}}</math> but can be scored without a value for <math>A</math>, i.e. for <math>\frac{A}{2}x^{\frac{1}{2}}</math></p>	
(c)	<p>M1 for some correct substitution of <math>x = 9</math> in <u>their</u> expression for <math>f'(x)</math> including an attempt at <math>(9)^{\pm\frac{k}{2}}</math> (<math>k</math> odd) somewhere that leads to some appropriate multiples of <math>\frac{1}{3}</math> or 3</p> <p>A1 accept <math>\frac{15}{6}</math> or any exact equivalent of 2.5 e.g. <math>\frac{45}{18}, \frac{135}{54}</math> or even <math>\frac{67.5}{27}</math></p> <p><u>Misread (MR)</u> Only allow MR of the form <math>\frac{(3-k\sqrt{x})^2}{\sqrt{x}}</math> N.B. Leads to answer in (c) of <math>\frac{k^2-1}{6}</math></p> <p>Score as M1A0A0, M1A1A1ft, M1A1ft</p>	

Question Number	Scheme	Marks
<p>Q10 (a)</p> <p>(b)</p> <p>(c)</p>	<p> <math display="block">x(x^2 - 6x + 9)</math> <math display="block">= x(x - 3)(x - 3)</math> </p> <p>Shape </p> <p><u>Through</u> origin (<u>not</u> touching)            Touching <math>x</math>-axis only once            Touching at (3, 0), or 3 on <math>x</math>-axis            [Must be on graph not in a table]</p> <p>Moved horizontally (either way)            (2, 0) and (5, 0), or 2 and 5 on <math>x</math>-axis</p>  	<p>B1            M1 A1 (3)            B1</p> <p>B1            B1            B1ft (4)</p> <p>M1            A1 (2)</p> <p>[9]</p>
<p>(a)</p> <p>S.C.</p> <p>(b)</p> <p>(c)</p>	<p>B1 for correctly taking out a factor of <math>x</math>            M1 for an attempt to factorize their 3TQ e.g. <math>(x + p)(x + q)</math> where <math> pq  = 9</math>.            So <math>(x - 3)(x + 3)</math> will score M1 but A0            A1 for a fully correct factorized expression - accept <math>x(x - 3)^2</math>            If they “solve” use ISW            If the only correct linear factor is <math>(x - 3)</math>, perhaps from factor theorem, award B0M1A0            Do not award marks for factorising in part (b)</p> <p><b>For the graphs</b>            “Sharp points” will lose the 1<sup>st</sup> B1 in (b) but otherwise be generous on shape            Condone (0, 3) in (b) and (0, 2), (0,5) in (c) if the points are marked in the correct places.</p> <p>2<sup>nd</sup> B1 for a curve that starts or terminates at (0, 0) score B0</p> <p>4<sup>th</sup> B1ft for a curve that touches (not crossing or terminating) at <math>(a, 0)</math> where their <math>y = x(x - a)^2</math></p> <p>M1 for their graph moved horizontally (only) <u>or</u> a fully correct graph            Condone a partial stretch if ignoring their values looks like a simple translation            A1 for their graph translated 2 to the right <u>and</u> crossing or touching the axis at 2 and 5 only  <b>Allow a fully correct graph (as shown above) to score M1A1 whatever they have in (b)</b></p>	

Question Number	Scheme	Marks
Q11 (a) (b) (c)	$x = 2: \quad y = 8 - 8 - 2 + 9 = 7 \quad (*)$ $\frac{dy}{dx} = 3x^2 - 4x - 1$ $x = 2: \quad \frac{dy}{dx} = 12 - 8 - 1 (= 3)$ $y - 7 = 3(x - 2), \quad \underline{y = 3x + 1}$ $m = -\frac{1}{3} \quad \text{(for } -\frac{1}{m} \text{ with their } m)$ $3x^2 - 4x - 1 = -\frac{1}{3}, \quad 9x^2 - 12x - 2 = 0 \quad \text{or} \quad x^2 - \frac{4}{3}x - \frac{2}{9} = 0 \quad (\text{o.e.})$ $\left( x = \frac{12 + \sqrt{144 + 72}}{18} \right) (\sqrt{216} = \sqrt{36} \cdot \sqrt{6} = 6\sqrt{6}) \quad \text{or} \quad (3x - 2)^2 = 6 \rightarrow 3x = 2 \pm \sqrt{6}$ $x = \frac{1}{3}(2 + \sqrt{6}) \quad (*)$	B1 (1) M1 A1 A1ft M1, <u>A1</u> (5) B1ft M1, A1 M1 A1cso (5) [11]
(a) (b) (c) ALT	<p>B1 there must be a clear attempt to substitute <math>x = 2</math> leading to 7            e.g. <math>2^3 - 2 \times 2^2 - 2 + 9 = 7</math></p> <p>1<sup>st</sup> M1 for an attempt to differentiate with at least one of the given terms fully correct.            1<sup>st</sup> A1 for a fully correct expression            2<sup>nd</sup> A1ft for sub. <math>x = 2</math> in <u>their</u> <math>\frac{dy}{dx}</math> (<math>\neq y</math>) accept for a correct expression e.g.  <math>3 \times (2)^2 - 4 \times 2 - 1</math></p> <p>2<sup>nd</sup> M1 for use of their “3” (provided it comes from their <math>\frac{dy}{dx}</math> (<math>\neq y</math>) and <math>x=2</math>) to find equation of tangent. Alternative is to use (2, 7) in <math>y = mx + c</math> to <u>find a value</u> for <math>c</math>.            Award when <math>c = \dots</math> is seen.</p> <p><b>No attempted use of <math>\frac{dy}{dx}</math> in (b) scores 0/5</b></p> <p>1<sup>st</sup> M1 for forming an equation from their <math>\frac{dy}{dx}</math> (<math>\neq y</math>) and their <math>-\frac{1}{m}</math> (must be changed from <math>m</math>)            1<sup>st</sup> A1 for a correct 3TQ all terms on LHS (condone missing =0)            2<sup>nd</sup> M1 for proceeding to <math>x = \dots</math> or <math>3x = \dots</math> by formula or completing the square for a 3TQ.            Not factorising. Condone <math>\pm</math></p> <p>2<sup>nd</sup> A1 for proceeding to given answer with no incorrect working seen. Can still have <math>\pm</math>.</p> <p><u>Verify (for M1A1M1A1)</u></p> <p>1<sup>st</sup> M1 for attempting to square need <math>\geq 3</math> correct values in <math>\frac{4+6+4\sqrt{6}}{9}</math>, 1<sup>st</sup> A1 for <math>\frac{10+4\sqrt{6}}{9}</math></p> <p>2<sup>nd</sup> M1 Dependent on 1<sup>st</sup> M1 in this case for substituting in all terms of their <math>\frac{dy}{dx}</math></p> <p>2<sup>nd</sup> A1cso for cso <u>with a full comment</u> e.g. “the <math>x</math> co-ord of <math>Q</math> is ...”</p>	