Cambridge International Examinations
Cambridge Ordinary Level

Cambridge
O Level

CANDIDATE NAME	

CENTRE NUMBER		CANDIDATE NUMBER	

ADDITIONAL MATHEMATICS **4037/13**

Paper 1 **October/November 2018**

2 hours

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of **15** printed pages and **1** blank page.

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
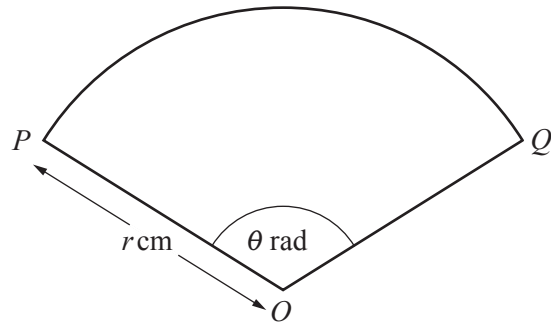
$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}bc \sin A$$

1   **(a)**   In the expansion of $(2 + px)^5$ the coefficient of x^3 is equal to $-\dfrac{8}{25}$. Find the value of the constant p.

           [3]

     (b)   Find the term independent of x in the expansion of $\left(2x^2 + \dfrac{1}{4x^2}\right)^8$.

           [3]

1   **(a)**   In the expansion of $(2 + px)^5$ the coefficient of x^3 is equal to $-\dfrac{8}{25}$. Find the value of the constant p.

           [3]

2

The diagram shows a sector POQ of a circle, centre O, radius r cm, where angle $POQ = \theta$ radians. The perimeter of the sector is 20 cm.

(i) Show that the area, A cm^2, of the sector is given by $A = 10r - r^2$. [3]

It is given that r can vary and that A has a maximum value.

(ii) Find the value of θ for which A has a maximum value. [3]

3 **Do not use a calculator in this question.**

In this question, all lengths are in centimetres.

A triangle ABC is such that angle $B = 90°$, $AB = 5\sqrt{3} + 5$ and $BC = 5\sqrt{3} - 5$.

(i) Find, in its simplest surd form, the length of AC. [3]

(ii) Find $\tan BCA$, giving your answer in the form $a + b\sqrt{3}$, where a and b are integers. [3]

4   In this question, the units of x are radians and the units of y are centimetres.

It is given that $y = (1 + \cos 3x)^{10}$.

(i)   Find the value of $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ when $x = \dfrac{\pi}{2}$. [4]

Given also that y is increasing at a rate of $6\,\mathrm{cm\,s}^{-1}$ when $x = \dfrac{\pi}{2}$,

(ii)   find the corresponding rate of change of x. [2]

5 **(i)** Show that $\log_9 4 = \log_3 2$. [2]

(ii) Hence solve $\log_9 4 + \log_3 x = 3$. [3]

6 A particle P is moving in a straight line such that its displacement, s m, from a fixed point O at time t s, is given by $s = 12e^{-0.5t} + 4t - 12$.

(i) Find the value of t when P is instantaneously at rest. [3]

(ii) Find an expression for the acceleration of P at time t s. [2]

(iii) Find the value of s when the acceleration of P is $0.3\,\text{ms}^{-2}$. [3]

(iv) Explain why the acceleration of the particle will always be positive. [1]

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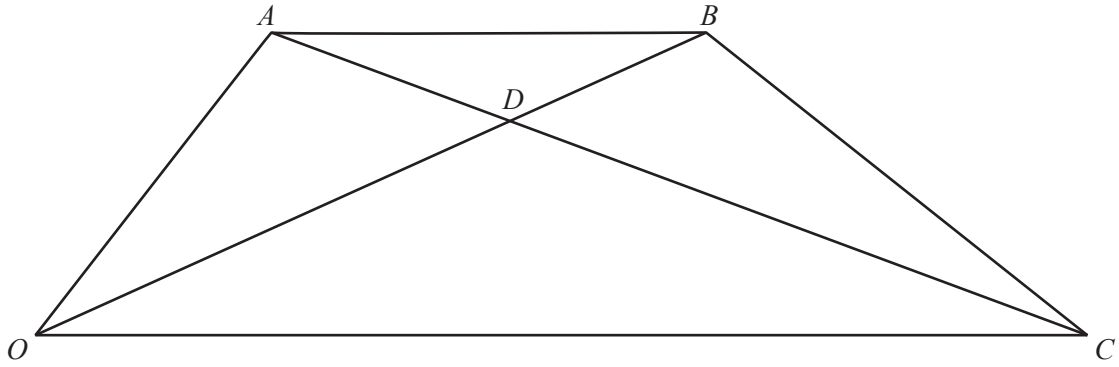
The diagram shows a quadrilateral $OABC$. The point D lies on OB such that $\overrightarrow{OD} = 2\overrightarrow{DB}$ and $\overrightarrow{AD} = m\overrightarrow{AC}$, where m is a scalar quantity.

$$\overrightarrow{OA} = \mathbf{a} \qquad \overrightarrow{OB} = \mathbf{b} \qquad \overrightarrow{OC} = \mathbf{c}$$

(i) Find $\overrightarrow{AD}$ in terms of m, $\mathbf{a}$ and $\mathbf{c}$. [1]

(ii) Find $\overrightarrow{AD}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. [2]

(iii) Given that $15\mathbf{a} = 16\mathbf{b} - 9\mathbf{c}$, find the value of m. [3]

8 $f(x) = 5 + \sin\dfrac{x}{4}$ for $0 \leqslant x \leqslant 2\pi$ radians

     $g(x) = x - \dfrac{\pi}{3}$ for $x \in \mathbb{R}$

(i) Write down the range of $f(x)$. [2]

(ii) Find $f^{-1}(x)$ and write down its range. [3]

(iii) Solve $2fg(x) = 11$. [4]

9 Find the equation of the normal to the curve $y = \dfrac{\ln(3x^2 + 1)}{x^2}$ at the point where $x = 2$, giving your answer

in the form $y = mx + c$, where m and c are correct to 2 decimal places. You must show all your working. [8]

10
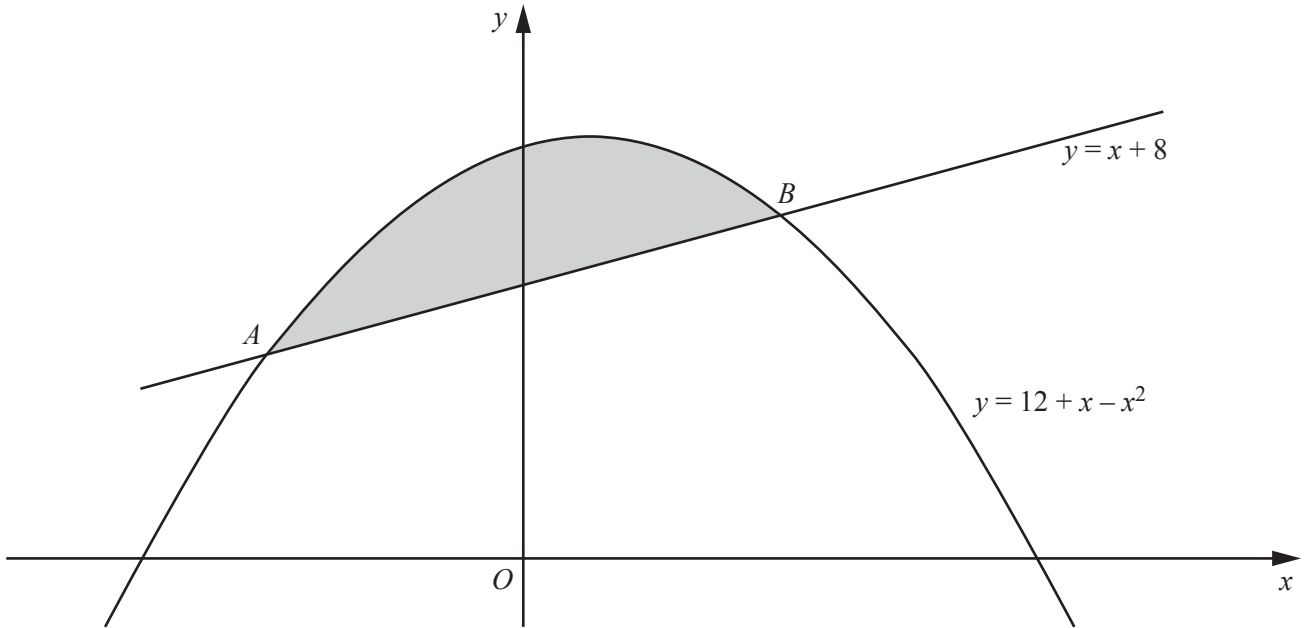
The diagram shows the curve $y = 12 + x - x^2$ intersecting the line $y = x + 8$ at the points A and B.

(i) Find the coordinates of the points A and B. [3]

(ii) Find $\int (12 + x - x^2) \, dx$. [2]

(iii) Showing all your working, find the area of the shaded region. [4]

11 The polynomial $\mathrm{p}(x) = ax^3 + 17x^2 + bx - 8$ is divisible by $2x - 1$ and has a remainder of -35 when divided by $x + 3$.

(i) By finding the value of each of the constants a and b, verify that $a = b$. [4]

Using your values of a and b,

(ii) find $\mathrm{p}(x)$ in the form $(2x - 1)\mathrm{q}(x)$, where $\mathrm{q}(x)$ is a quadratic expression, [2]

(iii) factorise p(x) completely, [1]

(iv) solve $a\sin^3\theta + 17\sin^2\theta + b\sin\theta - 8 = 0$ for $0° < \theta < 180°$. [3]

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