Cambridge
O Level

Cambridge Assessment International Education
Cambridge Ordinary Level

ADDITIONAL MATHEMATICS **4037/12**

Paper 1 **October/November 2018**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2018 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.
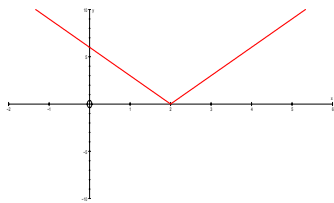
Types of mark

M   Method marks, awarded for a valid method applied to the problem.

A   Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

B   Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Guidance
1	$\sin\left(x+50^{\text{o}}\right)=-\dfrac{1}{\sqrt{2}}$ $\left(x+50^{\text{o}}=-45^{\text{o}},\ 225^{\text{o}}\right)$	M1	For order of operations – subtraction of 1, division by $\pm\sqrt{2}$ and attempt at $\sin^{-1}$
		M1	**Dep** For obtaining a solution by subtracting 50°
	$x=-95^{\text{o}},\ 175^{\text{o}}$	A2	**A1** for one correct solution **A1** for a second correct solution and no others within the range
2	$\dfrac{\text{d}y}{\text{d}x}=5x+\dfrac{1}{2}\text{e}^{2x}\ \ (+c)$	M1	For attempt to integrate to get $\dfrac{\text{d}y}{\text{d}x}$ in the form $5x+p\text{e}^{2x}$. Condone omission of $+c$
	When $x=0,\ \dfrac{\text{d}y}{\text{d}x}=4$ so $c=\dfrac{7}{2}$	M1	**Dep** For attempt to get value of c
	$y=\dfrac{5x^{2}}{2}+\dfrac{1}{4}\text{e}^{2x}+\dfrac{7}{2}x\ \ (+d)$	M1	**Dep** on first M1 only For attempt to get y in the form including $\dfrac{5x^{2}}{2}+p\text{e}^{2x}$. Condone omission of $+d$.
	When $x=0,\ y=-3$ so $d=-\dfrac{13}{4}$	M1	**Dep** on previous **DepM1** For attempt to obtain d, allow if c not found
	$y=\dfrac{5x^{2}}{2}+\dfrac{1}{4}\text{e}^{2x}+\dfrac{7}{2}x-\dfrac{13}{4}$	A1	Must have an equation
3(i)		B2	**B1** for correct shape with vertex at $\left(2,0\right)$ **Dep B1** for passing through or starting at $\left(0,6\right)$

Question	Answer	Marks	Guidance
3(ii)	**Either** $6 - 3x = 2$ $x = \dfrac{4}{3}$	**B1**	For $x = \dfrac{4}{3}$
	$6 - 3x = -2$	**M1**	For considering -2
	$x = \dfrac{8}{3}$	**A1**	
	Or $9x^2 - 36x + 32 = 0$	**M1**	For squaring each side and attempt to solve a 3 term quadratic $= 0$
	$x = \dfrac{4}{3}$	**A1**	
	$x = \dfrac{8}{3}$	**A1**	
3(iii)	$x < \dfrac{4}{3}, \ x > \dfrac{8}{3}$	**B1**	**FT** on *their* solutions in part **(ii)**, must both be positive and written as 2 separate statements
4(i)		**B1**	For $\dfrac{2}{2x+1}$
		M1	For attempt to differentiate a product
	$\dfrac{dy}{dx} = x^3 \dfrac{2}{2x+1} + 3x^2 \ln(2x+1)$	**A1**	For all other terms correct
	When $x = 0.3$, $\dfrac{dy}{dx} = 0.161$	**A1**	For awrt 0.161
4(ii)	$0.161h$	**B1**	**FT** on *their* numerical answer to part **(i)**
5(i)	7th term: $924a^6b^6x^6 = 924x^6$ $924a^6b^6 = 924$ $924a^6(bx)^6 = 924x^6$	**B1**	For any correct statement
	$(ab)^6 = 1$ or $ab = 1$ so $b = \dfrac{1}{a}$	**B1**	**Dep** on first **B1** Must be convinced, nfww

Question	Answer	Marks	Guidance
5(ii)	6th term: $792a^7b^5x^5 = 198x^5$ $792a^7b^5 = 198$ $792a^7(bx)^5 = 198x^5$	**B1**	For any correct statement
	use of $ab = 1$ to obtain $a^2 = ...$ or $b^2 = ...$	**M1**	For attempt to solve *their* equations simultaneously to obtain an equation in a^2 or b^5
	$a = \dfrac{1}{2}$	**A1**	
	$b = 2$	**A1**	
6(i)		**M1**	For $kx(5x - 125)^{-\frac{1}{3}}$
	$\dfrac{2}{3} \times 10x\left(5x^2 - 125\right)^{-\frac{1}{3}}$ $\left(\dfrac{20}{3}x\left(5x^2 - 125\right)^{-\frac{1}{3}}\right)$	**A1**	Allow unsimplified
6(ii)		**M1**	For $m\left(5x^2 - 125\right)^{\frac{2}{3}} \ (+c)$
	$\dfrac{3}{20}\left(5x^2 - 125\right)^{\frac{2}{3}} \ (+c)$	**A1**	**FT** on *their k* from part **(i)**
6(iii)	$\dfrac{3}{20}\left((375)^{\frac{2}{3}} - (55)^{\frac{2}{3}}\right)$	**M1**	**Dep** on previous **M1** For use of limits in *their* answer to part **(ii)**, must be in the form $m\left(5x^2 - 125\right)^{\frac{2}{3}} \ (+c),$
	$= 5.63$	**A1**	Allow greater accuracy

Question	Answer	Marks	Guidance
7(a)	$\left\|\begin{pmatrix}-12\\5\end{pmatrix}\right\|=13$	**B1**	For magnitude, may be implied by a correct **v**
	$\mathbf{v}=\begin{pmatrix}-36\\15\end{pmatrix}$ or $3\begin{pmatrix}-12\\5\end{pmatrix}$	**B1**	Must be a vector
7(a) Alternative	If $t\left\|\begin{pmatrix}-12\\5\end{pmatrix}\right\|=39$, $t=3$	**B1**	For value of t, may be implied by a correct **v**
	$\mathbf{v}=\begin{pmatrix}-36\\15\end{pmatrix}$ or $3\begin{pmatrix}-12\\5\end{pmatrix}$	**B1**	
7(b)		**M1**	For equating like vectors at least once
	$17r+2s+3=0$ $2r+6s+9=0$	**M1**	**Dep** For solution of resulting equations to obtain 2 solutions
	$r=0$	**A1**	
	$s=-\dfrac{3}{2}$ oe	**A1**	
8(i)	$a(a+4)-12=0$	**M1**	For correct use of det = 0
	$a^2+4a-12=0$	**M1**	**Dep** For solution of resulting quadratic equation
	leading to $a=-6$, $a=2$	**A1**	For both
8(ii)	$\mathbf{A}^{-1}=\dfrac{1}{20}\begin{pmatrix}8&-3\\-4&4\end{pmatrix}$ oe	**B2**	**B1** for $\dfrac{1}{20}$ **B1** for $\begin{pmatrix}8&-3\\-4&4\end{pmatrix}$
8(iii)	$\mathbf{B}=\mathbf{A}^{-1}\begin{pmatrix}2&3\\4&-5\end{pmatrix}$	**M1**	For pre-multiplication by their $\mathbf{A}^{-1}$
		M1	**Dep** For multiplication of 2 matrices – need to see at least 2 correct elements – may be unsimplified
	$=\dfrac{1}{20}\begin{pmatrix}4&39\\8&-32\end{pmatrix}$	**A1**	For final matrix oe

Question	Answer	Marks	Guidance
9(i)	$\text{p}(-3)=0$ leading to $-27a+9b-3c-9=0$	**M1**	For substitution of $x=-3$ and equating to zero
	$\text{p}'(x)=3ax^2+2bx+c$ $\text{p}'(0)=36$	**M1**	For differentiation in the form rx^2+sx+t and substitution of $x=0$
	$c=36$	**A1**	nfww
	$\text{p}''(x)=6ax+2b$ $\text{p}''(0)=2b$	**M1**	For further differentiation in the form $vx+w$ of *their* $\text{p}'(x)$ and substitution of $x=0$
	$b=43$	**A1**	nfww
	$a=10$	**A1**	nfww
9(ii)	$\text{p}\left(\dfrac{1}{2}\right)$	**M1**	For use of $x=\dfrac{1}{2}$ in *their* $\text{p}(x)$ from part **(i)**
	21	**A1**	
10(i)	$a=2$	**B1**	
	$\cos bx=-\dfrac{1}{2}$	**M1**	For a correct attempt to solve $\cos b\dfrac{\pi}{6}=\pm\dfrac{a}{4}$ provided $0<a\leqslant 4$ to get $b=...$
	leading to $b=4$	**A1**	
10(ii)	$\cos 4x=-\dfrac{1}{2}$	**M1**	**Dep** For attempt to solve *their* $\cos bx=\pm\dfrac{a}{4}$ provided $0<a\leqslant 4$ or use of symmetry to get $x=...$
	$x=\dfrac{\pi}{3}$ so $\left(\dfrac{\pi}{3},0\right)$	**A1**	
10(iii)	At M, $y=-2$	**B1**	
	$x=\dfrac{\pi}{4}$	**B1**	

Question	Answer	Marks	Guidance
11(i)	$2r + r\theta = 10$	**M1**	For use of arc length and attempt to get perimeter, must have 2 terms involving r
		M1	**Dep** For attempt to get r in terms of θ
	$r = \dfrac{10}{2+\theta}$	**A1**	
	$A = \dfrac{1}{2}\left(\dfrac{10}{2+\theta}\right)^2 \theta$	**M1**	For attempt to obtain the area of the sector in terms of θ only, using *their r*
	$A = \dfrac{50\theta}{(2+\theta)^2}$	**A1**	For manipulation to get the required answer nfww **AG**
11(ii)		**M1**	For attempt to differentiate a quotient or an equivalent product
	$\dfrac{\mathrm{d}A}{\mathrm{d}\theta} = \dfrac{50(2+\theta)^2 - 100\theta(2+\theta)}{(2+\theta)^4}$ or $\dfrac{\mathrm{d}A}{\mathrm{d}\theta} = 50(2+\theta)^{-2} - 100\theta(2+\theta)^{-3}$	**A1**	All correct, allow unsimplified
	When $\dfrac{\mathrm{d}A}{\mathrm{d}\theta} = 0$	**M1**	For equating *their* $\dfrac{\mathrm{d}A}{\mathrm{d}\theta}$ to 0 and attempt to solve – need to see at least one line of working
	$\theta = 2$	**A1**	Condone inclusion of -2
	$A = \dfrac{25}{4}$	**A1**	

Question	Answer	Marks	Guidance
11(ii) Alternative	Starting again using $\theta = \dfrac{10-2r}{2}$ so $A = 5r - r^2$	M1	A complete method to obtain $\dfrac{\mathrm{d}A}{\mathrm{d}r}$
	$\dfrac{\mathrm{d}A}{\mathrm{d}r} = 5 - 2r$	A1	
	When $\dfrac{\mathrm{d}A}{\mathrm{d}r} = 0$	M1	For equating to zero and attempt to solve
	$r = 2.5$	A1	
	$A = \dfrac{25}{4}$	A1	
12	$2x^2 + 7x = 0$ or $y^2 - 3y - 10 = 0$	M1	For attempt to obtain a simplified quadratic equation in one variable equated to 0
		M1	**Dep** For solution of quadratic
	$(0, 5)$	A1	
	$\left(-\dfrac{7}{2}, -2\right)$	A1	
	Midpoint $\left(-\dfrac{7}{4}, \dfrac{3}{2}\right)$	B1	
	Gradient of $AB = 2$ $\therefore \perp$ gradient $= -\dfrac{1}{2}$	M1	For attempt to obtain gradient of line perpendicular to AB using *their* coordinates
	$\perp$ bisector: $y - \dfrac{3}{2} = -\dfrac{1}{2}\left(x + \dfrac{7}{4}\right)$	M1	For a correct attempt to obtain equation of perpendicular bisector using their midpoint and *their* perpendicular gradient
	Consideration of when $y = x$	M1	**Dep** on previous **M1** For attempt to find intersection with the line $y = x$
	$x = y = \dfrac{5}{12}$	A1	For both