Cambridge O Level

Cambridge International Examinations
Cambridge Ordinary Level

CANDIDATE NAME			
CENTRE NUMBER		CANDIDATE NUMBER	

4551701028

ADDITIONAL MATHEMATICS **4037/23**

Paper 1 **October/November 2016**

2 hours

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of **16** printed pages.

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

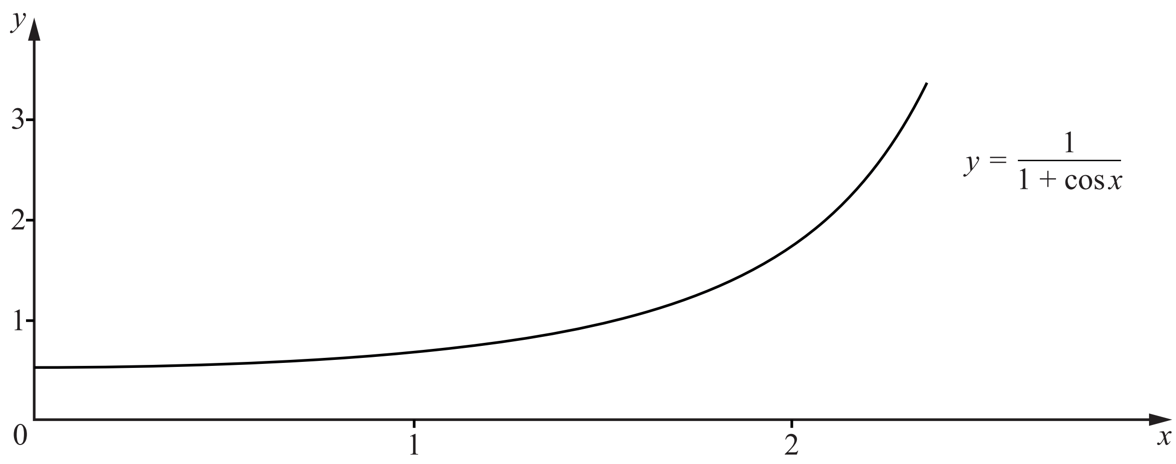
$$\Delta = \frac{1}{2} bc \sin A$$

1 **Without using a calculator**, show that $\dfrac{\sqrt{5}+3\sqrt{3}}{\sqrt{5}+\sqrt{3}} = \sqrt{k}-2$ where k is an integer to be found. [3]

2 Solve the equation $e^{3x} = 6e^{x}$. [3]

3 **(i)** Show that $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(\dfrac{\sin x}{1+\cos x}\right) = \dfrac{1}{1+\cos x}$. [4]

(ii)



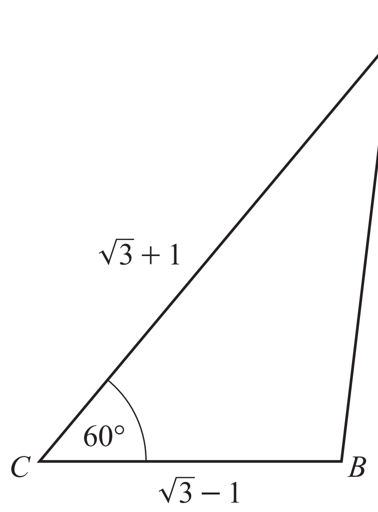
The diagram shows part of the graph of $y = \dfrac{1}{1+\cos x}$. Use the result from part **(i)** to find the area enclosed by the graph and the lines $x = 0$, $x = 2$ and $y = 0$. [2]

4 The cubic given by $p(x) = x^3 + ax^2 + bx - 24$ is divisible by $x - 2$. When $p(x)$ is divided by $x - 1$ the remainder is -20.

(i) Form a pair of equations in a and b and solve them to find the value of a and of b. [4]

(ii) Factorise $p(x)$ completely and hence solve $p(x) = 0$. [4]

5 In this question all lengths are in centimetres.



In the triangle ABC shown above, $AC = \sqrt{3}+1$, $BC = \sqrt{3}-1$ and angle $ACB = 60°$.

(i) **Without using a calculator**, show that the length of $AB = \sqrt{6}$. [3]

(ii) Show that angle $CAB = 15°$. [2]

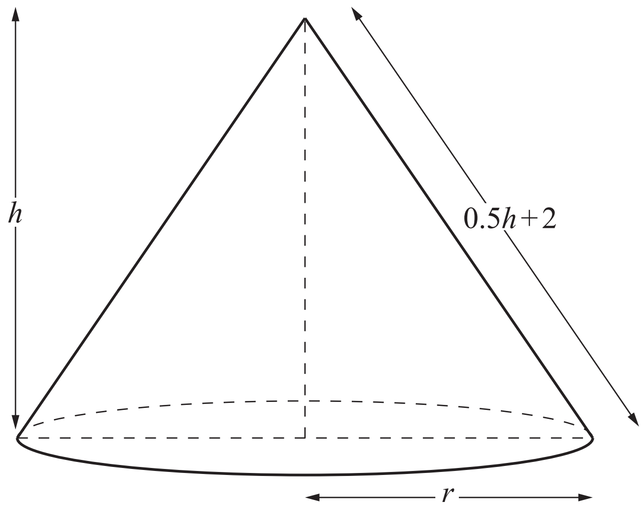
(iii) **Without using a calculator**, find the area of triangle ABC. [2]

6 A curve has equation $y = 7 + \tan x$. Find

(i) the equation of the tangent to the curve at the point where $x = \frac{\pi}{4}$, [4]

(ii) the values of x between 0 and π radians for which $\frac{\mathrm{d}y}{\mathrm{d}x} = y$. [4]

7 In this question all lengths are in metres.



A conical tent is to be made with height h, base radius r and slant height $0.5h + 2$, as shown in the diagram.

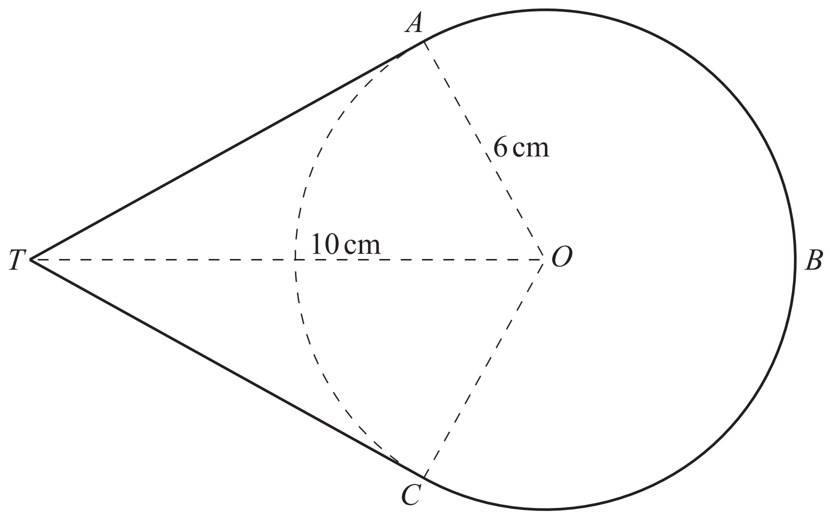
(i) Show that $r^2 = 2h + 4 - 0.75h^2$. [2]

The volume of the tent, V, is given by $\frac{1}{3}\pi r^2 h$.

(ii) Given that h can vary find, correct to 2 decimal places, the value of h which gives a stationary value of V. [5]

(iii) Determine the nature of this stationary value. [2]

8



The points A, B and C lie on a circle centre O, radius 6 cm. The tangents to the circle at A and C meet at the point T. The length of OT is 10 cm. Find

(i) the angle TOA in radians, [2]

(ii) the area of the region *TABCT*, [6]

(iii) the perimeter of the region *TABCT*. [2]

9 In this question $\mathbf{i}$ is a unit vector due east and $\mathbf{j}$ is a unit vector due north. Units of length and velocity are metres and metres per second respectively.

The initial position vectors of particles A and B, relative to a fixed point O, are $\mathbf{i} + 5\mathbf{j}$ and $q\mathbf{i} - 15\mathbf{j}$ respectively. A and B start moving at the same time. A moves with velocity $p\mathbf{i} - 3\mathbf{j}$ and B moves with velocity $3\mathbf{i} - \mathbf{j}$.

(i) Given that A travels with a speed of $5\,\mathrm{ms}^{-1}$, find the value of the positive constant p. [1]

(ii) Find the direction of motion of B as a bearing correct to the nearest degree. [2]

(iii) State the position vector of A after t seconds . [1]

(iv) State the position vector of B after t seconds. [1]

(v) Find the time taken until A and B meet. [2]

(vi) Find the position vector of the point where A and B meet. [1]

(vii) Find the value of the constant q. [1]

10 The functions f and g are defined for $x > 1$ by

$$f(x) = 2 + \ln x,$$

$$g(x) = 2e^x + 3.$$

(i) Find $fg(x)$. [1]

(ii) Find $ff(x)$. [1]

(iii) Find $g^{-1}(x)$. [2]

(iv) Solve the equation $f(x) = 4$. [1]

(v) Solve the equation $gf(x) = 20$. [4]

Question 11 is printed on the next page.

11 It is given that $\mathbf{A} = \begin{pmatrix} 2 & q \\ p & 3 \end{pmatrix}$ and that $\mathbf{A}^2 - 5\mathbf{A} = 2\mathbf{I}$, where $\mathbf{I}$ is the identity matrix.

(i) Find a relationship connecting the constants p and q. [4]

(ii) Given that p and q are positive and that $\det \mathbf{A} = -3p$, find the value of p and of q. [4]