Cambridge O Level

**Cambridge International Examinations**
Cambridge Ordinary Level

CANDIDATE NAME

CENTRE NUMBER | CANDIDATE NUMBER

*4775967560*

**ADDITIONAL MATHEMATICS** **4037/12**

Paper 1 **October/November 2016**

**2 hours**

Candidates answer on the Question Paper.

No Additional Materials are required.

**READ THESE INSTRUCTIONS FIRST**

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of **16** printed pages.

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CAMBRIDGE
International Examinations

***Mathematical Formulae***

## 1. ALGEBRA

*Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} .$$

*Binomial Theorem*

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where $n$ is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$ .

## 2. TRIGONOMETRY

*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

*Formulae for $\Delta ABC$*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

**1** **(a)** Sets $\mathscr{E}$, $A$ and $B$ are such that

$$\text{n}(\mathscr{E}) = 26,\ \text{n}(A \cap B') = 7,\ \text{n}(A \cap B) = 3 \text{ and } \text{n}(B) = 15\ .$$

Using a Venn diagram, or otherwise, find

**(i)** $\text{n}(A)$, [1]

**(ii)** $\text{n}(A \cup B)$, [1]

**(iii)** $\text{n}(A \cup B)'$. [1]

**(b)** It is given that $\mathscr{E} = \{x : 0 < x < 30\}$ , $P = \{\text{multiples of } 5\}$, $Q = \{\text{multiples of } 6\}$ and $R = \{\text{multiples of } 2\}$. Use set notation to complete the following statements.

**(i)** $Q$ ........... $R$, [1]

**(ii)** $P \cap Q =$ ........... [1]

**2** Given that $\dfrac{p^{\frac{1}{3}}q^{-\frac{1}{2}}r^{\frac{3}{2}}}{p^{-\frac{2}{3}}\sqrt{(qr)^5}} = p^a q^b r^c$, find the value of each of the integers $a$, $b$ and $c$. [3]

**3** By using the substitution $y = \log_3 x$, or otherwise, find the values of $x$ for which

$$3(\log_3 x)^2 + \log_3 x^5 - \log_3 9 = 0\ .$$ [6]

**4** **(i)** Find the first 3 terms in the expansion of $\left(2x^2 - \frac{1}{3x}\right)^5$, in descending powers of $x$. [3]

**(ii)** Hence find the coefficient of $x^7$ in the expansion of $\left(3 + \frac{1}{x^3}\right)\left(2x^2 - \frac{1}{3x}\right)^5$. [2]

**5** **(i)** Find the equation of the normal to the curve $y = \frac{1}{2}\ln(3x+2)$ at the point $P$ where $x = -\frac{1}{3}$. [4]

The normal to the curve at the point $P$ intersects the $y$-axis at the point $Q$. The curve $y = \frac{1}{2}\ln(3x+2)$ intersects the $y$-axis at the point $R$ .

**(ii)** Find the area of the triangle $PQR$. [3]

**6** **(a)** Matrices **X**, **Y** and **Z** are such that

$$\mathbf{X} = \begin{pmatrix} 2 & 3 \\ 4 & -1 \\ 6 & 5 \end{pmatrix}, \quad \mathbf{Y} = \begin{pmatrix} 1 & -1 & 0 \end{pmatrix} \quad \text{and } \mathbf{Z} = \begin{pmatrix} 0 & -1 \\ 5 & 3 \end{pmatrix}.$$

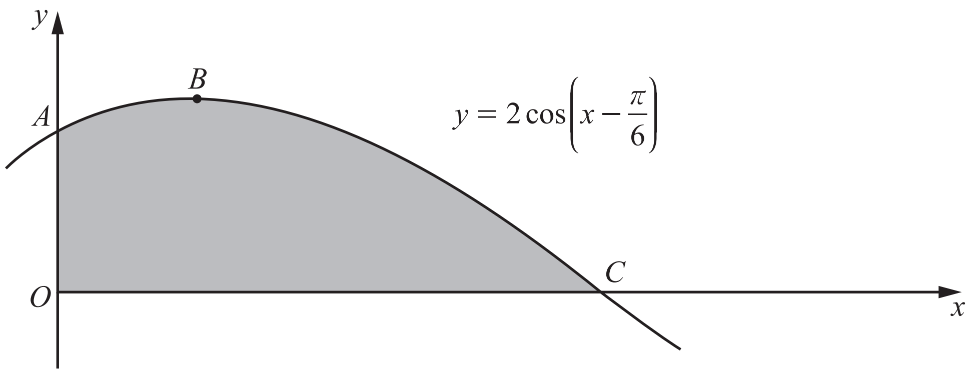
Write down all the matrix products which are possible using any two of these matrices. Do not evaluate these products. [2]

**(b)** Matrices **A**, **B** and **C** are such that $\mathbf{A} = \begin{pmatrix} 2 & -1 \\ 4 & 7 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} -4 & 2 \\ 10 & 4 \end{pmatrix}$ and $\mathbf{AC} = \mathbf{B}$.

**(i)** Find $\mathbf{A}^{-1}$. [2]

**(ii)** Hence find **C**. [3]

**7**




The diagram shows part of the graph of $y = 2\cos\left(x - \frac{\pi}{6}\right)$. The graph intersects the $y$-axis at the point $A$, has a maximum point at $B$ and intersects the $x$-axis at the point $C$.

**(i)** Find the coordinates of $A$. [1]

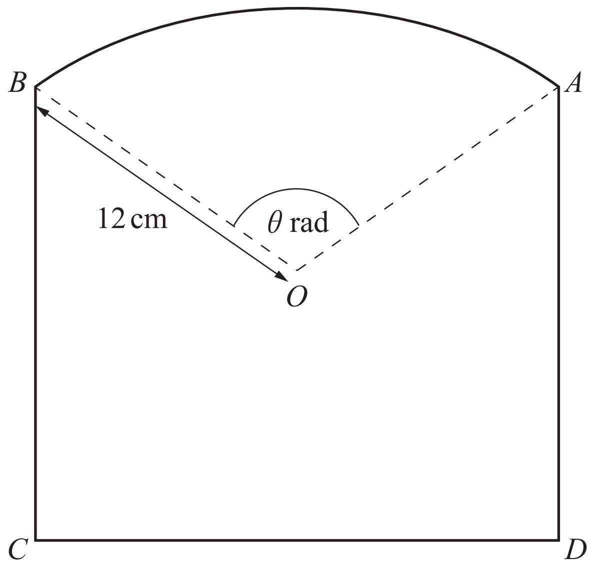
**(ii)** Find the coordinates of $B$. [2]

**(iii)** Find the coordinates of $C$. [2]

**(iv)** Find $\int 2\cos\left(x-\frac{\pi}{6}\right)\mathrm{d}x$ . [1]

**(v)** Hence find the area of the shaded region. [2]

8




The diagram shows a sector $AOB$ of the circle, centre $O$, radius 12 cm, together with points $C$ and $D$ such that $ABCD$ is a rectangle. The angle $AOB$ is $\theta$ radians and the perimeter of the sector $AOB$ is 47 cm.

**(i)** Show that $\theta = 1.92$ radians correct to 2 decimal places. [2]

**(ii)** Find the length of $CD$. [2]

**(iii)** Given that the total area of the shape is $425\,\text{cm}^2$, find the length of $AD$. [5]

**9 Do not use a calculator in this question.**

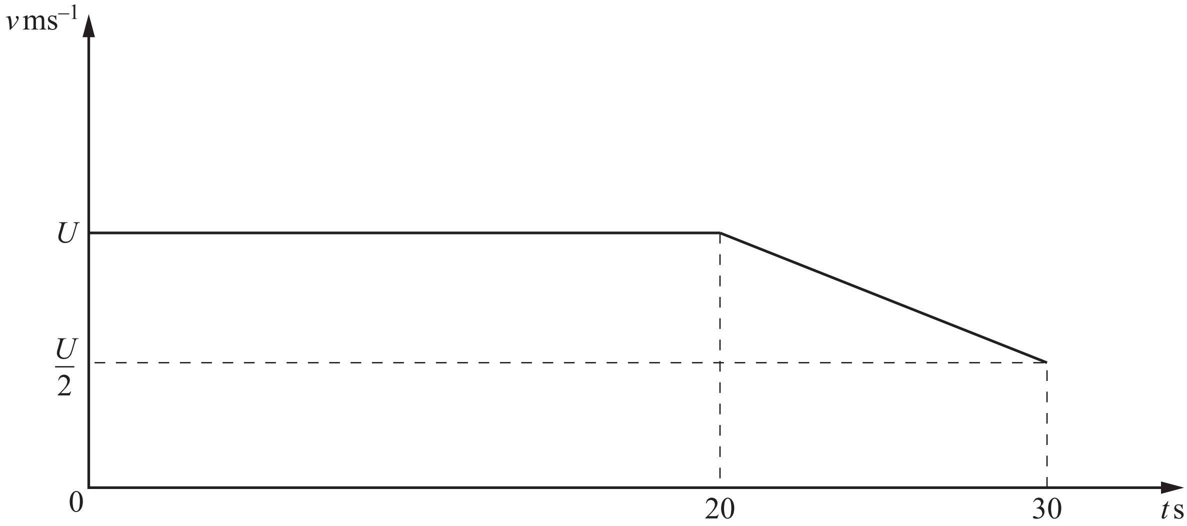
The polynomial $\mathrm{p}(x)$ is $ax^3 - 4x^2 + bx + 18$. It is given that $\mathrm{p}(x)$ and $\mathrm{p}'(x)$ are both divisible by $2x - 3$.

**(i)** Show that $a = 4$ and find the value of $b$. [4]

**(ii)** Using the values of $a$ and $b$ from part **(i)**, factorise $\mathrm{p}(x)$ completely. [2]

**(iii)** Hence find the values of $x$ for which $\mathrm{p}(x) = x + 2$ . [3]

**10 (a)**




The diagram shows part of the velocity-time graph for a particle, moving at $v\,\text{ms}^{-1}$ in a straight line, $t$ s after passing through a fixed point. The particle travels at $U\,\text{ms}^{-1}$ for 20 s and then decelerates uniformly for 10 s to a velocity of $\frac{U}{2}\,\text{ms}^{-1}$. In this 30 s interval, the particle travels 165 m.

**(i)** Find the value of $U$. [3]

**(ii)** Find the acceleration of the particle between $t = 20$ and $t = 30$. [2]

**(b)** A particle $P$ travels in a straight line such that, $t$ s after passing through a fixed point $O$, its velocity, $v\,\text{ms}^{-1}$, is given by $v = \left(e^{\frac{t^2}{8}} - 4\right)^3$.

**(i)** Find the speed of $P$ at $O$. [1]

**(ii)** Find the value of $t$ for which $P$ is instantaneously at rest. [2]

**(iii)** Find the acceleration of $P$ when $t = 1$. [4]

**Question 11 is printed on the next page.**

**11** The variables $x$ and $y$ are such that when $\ln y$ is plotted against $x$, a straight line graph is obtained. This line passes through the points $x = 4$, $\ln y = 0.20$ and $x = 12$, $\ln y = 0.08$.

**(i)** Given that $y = Ab^x$, find the value of $A$ and of $b$. [5]

**(ii)** Find the value of $y$ when $x = 6$ . [2]

**(iii)** Find the value of $x$ when $y = 1.1$ . [2]