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## 4037/12

May/June 2020

**2 hours**

You must answer on the question paper.

No additional materials are needed.

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Blank pages are indicated.

**Mathematical Formulae****1. ALGEBRA***Quadratic Equation*

For the equation  $ax^2 + bx + c = 0$ ,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

*Binomial Theorem*

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where  $n$  is a positive integer and  $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

*Arithmetic series*

$$u_n = a + (n-1)d$$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n-1)d\}$$

*Geometric series*

$$u_n = ar^{n-1}$$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

**2. TRIGONOMETRY***Identities*

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

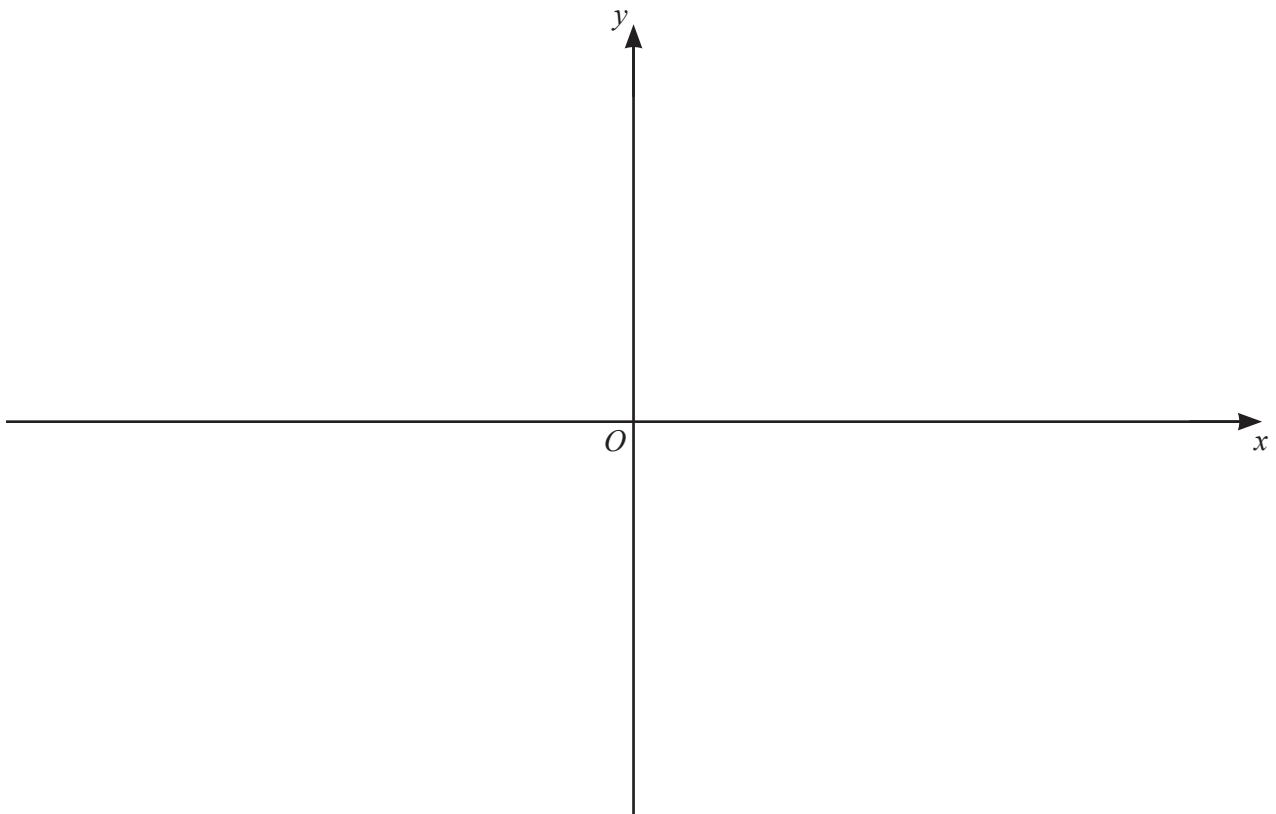
*Formulae for  $\triangle ABC$* 

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

- 1 On the axes below, sketch the graph of  $y = |(x-2)(x+1)(x+2)|$  showing the coordinates of the points where the curve meets the axes. [3]



- 2 The volume,  $V$ , of a sphere of radius  $r$  is given by  $V = \frac{4}{3}\pi r^3$ .

The radius,  $r$  cm, of a sphere is increasing at the rate of  $0.5 \text{ cm s}^{-1}$ . Find, in terms of  $\pi$ , the rate of change of the volume of the sphere when  $r = 0.25$ . [4]

- 3 (a) Find the first 3 terms in the expansion of  $\left(4 - \frac{x}{16}\right)^6$  in ascending powers of  $x$ . Give each term in its simplest form. [3]

- (b) Hence find the term independent of  $x$  in the expansion of  $\left(4 - \frac{x}{16}\right)^6 \left(x - \frac{1}{x}\right)^2$ . [3]

- 4 (a) (i) Find how many different 5-digit numbers can be formed using the digits 1, 2, 3, 5, 7 and 8, if each digit may be used only once in any number. [1]
- (ii) How many of the numbers found in **part (i)** are not divisible by 5? [1]
- (iii) How many of the numbers found in **part (i)** are even and greater than 30 000? [4]
- (b) The number of combinations of  $n$  items taken 3 at a time is 6 times the number of combinations of  $n$  items taken 2 at a time. Find the value of the constant  $n$ . [4]

5  $f : x \mapsto (2x+3)^2 \text{ for } x > 0$

(a) Find the range of  $f$ . [1]

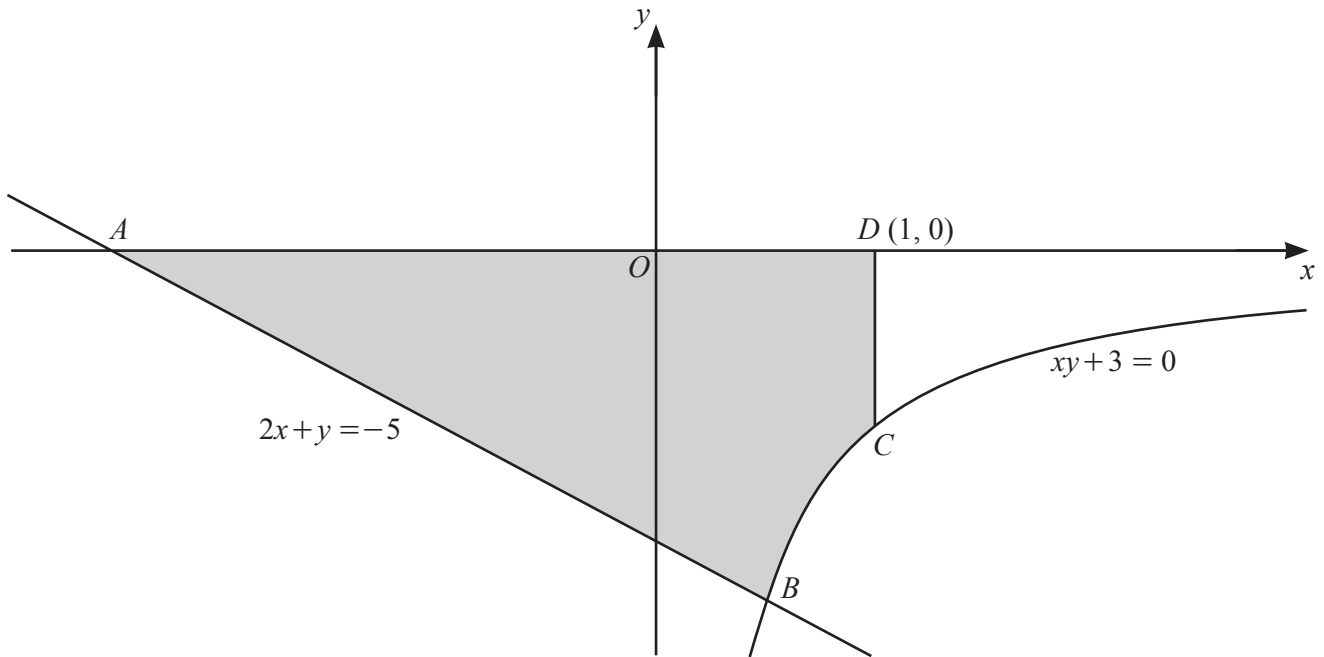
(b) Explain why  $f$  has an inverse. [1]

(c) Find  $f^{-1}$ . [3]

(d) State the domain of  $f^{-1}$ . [1]

(e) Given that  $g : x \mapsto \ln(x+4) \text{ for } x > 0$ , find the exact solution of  $fg(x) = 49$ . [3]

6



The diagram shows the straight line  $2x + y = -5$  and part of the curve  $xy + 3 = 0$ . The straight line intersects the  $x$ -axis at the point  $A$  and intersects the curve at the point  $B$ . The point  $C$  lies on the curve. The point  $D$  has coordinates  $(1, 0)$ . The line  $CD$  is parallel to the  $y$ -axis.

(a) Find the coordinates of each of the points  $A$  and  $B$ .

[3]

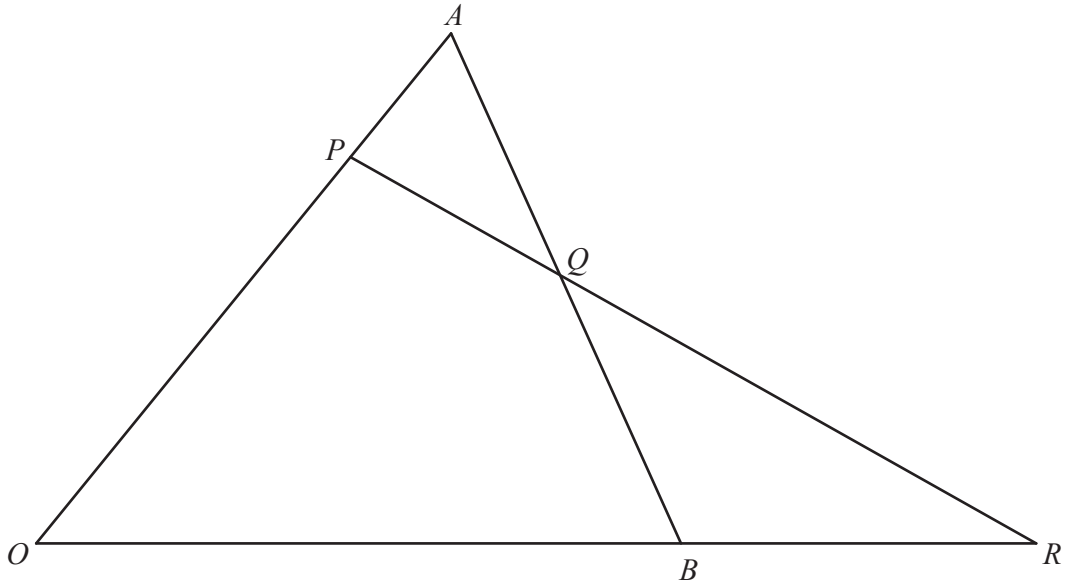


- (b) Find the area of the shaded region, giving your answer in the form  $p + \ln q$ , where  $p$  and  $q$  are positive integers. [6]

- 7 (a) Given that  $y = (x^2 - 1)\sqrt{5x + 2}$ , show that  $\frac{dy}{dx} = \frac{Ax^2 + Bx + C}{2\sqrt{5x + 2}}$ , where  $A$ ,  $B$  and  $C$  are integers. [5]

- (b) Find the coordinates of the stationary point of the curve  $y = (x^2 - 1)\sqrt{5x + 2}$ , for  $x > 0$ . Give each coordinate correct to 2 significant figures. [3]

- (c) Determine the nature of this stationary point. [2]



The diagram shows a triangle  $OAB$  such that  $\overrightarrow{OA} = \mathbf{a}$  and  $\overrightarrow{OB} = \mathbf{b}$ . The point  $P$  lies on  $OA$  such that  $OP = \frac{3}{4}OA$ . The point  $Q$  is the mid-point of  $AB$ . The lines  $OB$  and  $PQ$  are extended to meet at the point  $R$ . Find, in terms of  $\mathbf{a}$  and  $\mathbf{b}$ ,

(a)  $\overrightarrow{AB}$ , [1]

(b)  $\overrightarrow{PQ}$ . Give your answer in its simplest form. [3]

It is given that  $n\overrightarrow{PQ} = \overrightarrow{QR}$  and  $\overrightarrow{BR} = k\mathbf{b}$ , where  $n$  and  $k$  are positive constants.

(c) Find  $\overrightarrow{QR}$  in terms of  $n$ ,  $\mathbf{a}$  and  $\mathbf{b}$ . [1]

(d) Find  $\overrightarrow{QR}$  in terms of  $k$ ,  $\mathbf{a}$  and  $\mathbf{b}$ . [2]

(e) Hence find the value of  $n$  and of  $k$ . [3]

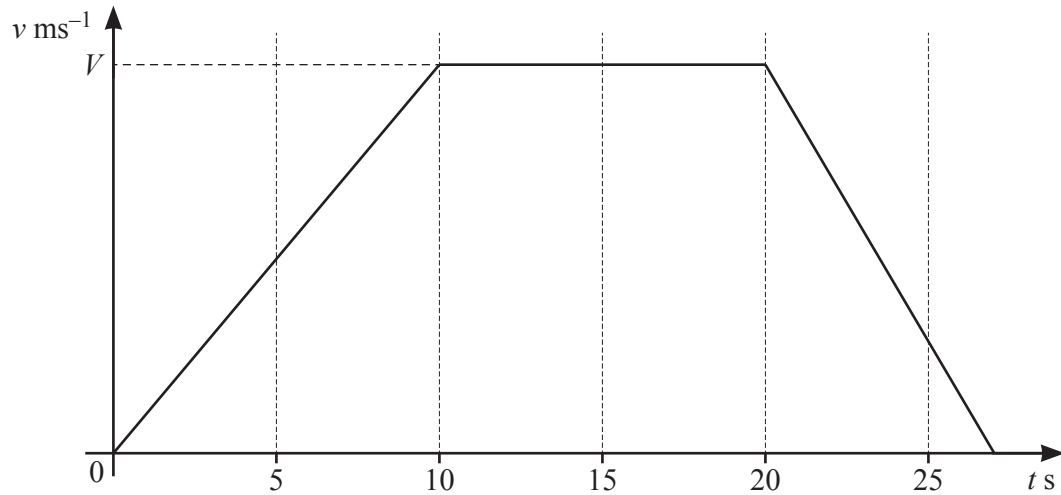
- 9 (a) A particle  $P$  moves in a straight line such that its displacement,  $x$  m, from a fixed point  $O$  at time  $t$  s is given by  $x = 10 \sin 2t - 5$ .

(i) Find the speed of  $P$  when  $t = \pi$ . [1]

(ii) Find the value of  $t$  for which  $P$  is first at rest. [2]

(iii) Find the acceleration of  $P$  when it is first at rest. [2]

(b)



The diagram shows the velocity–time graph for a particle  $Q$  travelling in a straight line with velocity  $v\text{ ms}^{-1}$  at time  $t\text{ s}$ . The particle accelerates at  $3.5\text{ ms}^{-2}$  for the first 10 s of its motion and then travels at constant velocity,  $V\text{ ms}^{-1}$ , for 10 s. The particle then decelerates at a constant rate and comes to rest. The distance travelled during the interval  $20 \leq t \leq 25$  is 112.5 m.

(i) Find the value of  $V$ . [1]

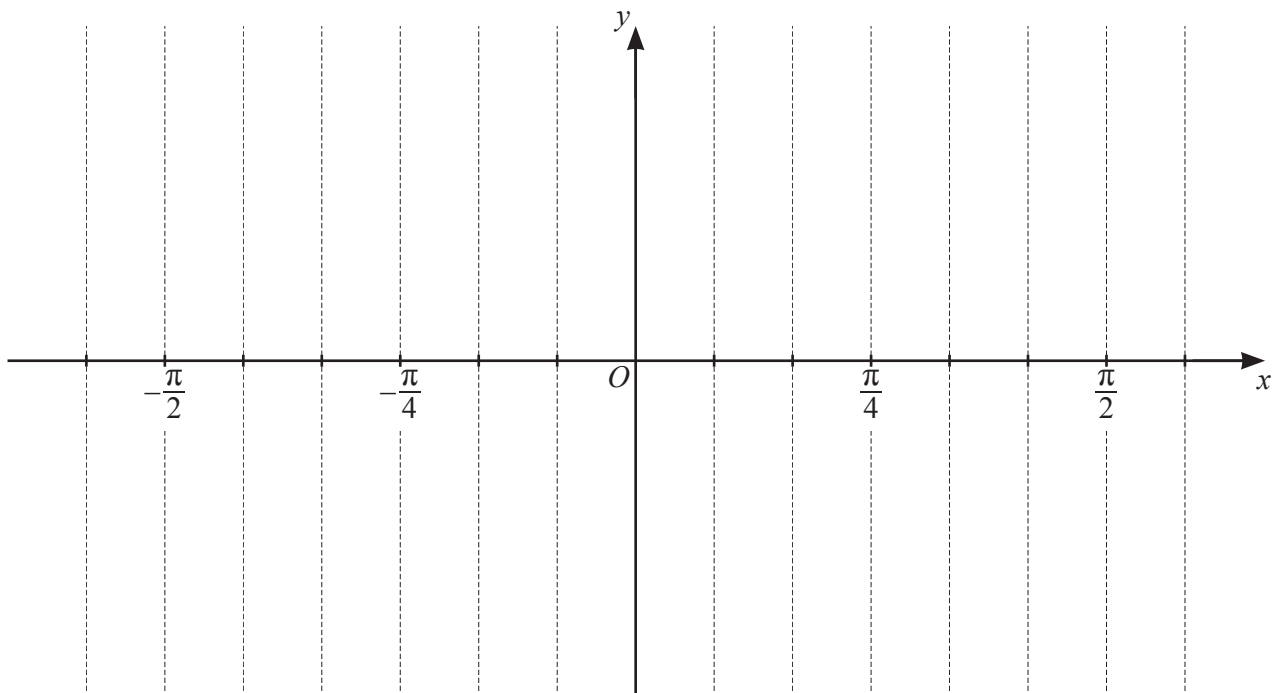
(ii) Find the velocity of  $Q$  when  $t = 25$ . [3]

(iii) Find the value of  $t$  when  $Q$  comes to rest. [3]

Question 10 is printed on the next page.

- 10 (a) Solve  $\tan 3x = -1$  for  $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$  radians, giving your answers in terms of  $\pi$ . [4]

- (b) Use your answers to **part (a)** to sketch the graph of  $y = 4 \tan 3x + 4$  for  $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$  radians on the axes below. Show the coordinates of the points where the curve meets the axes.



[3]

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