Cambridge International Examinations
Cambridge Ordinary Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

ADDITIONAL MATHEMATICS **4037/22**

Paper 2 **May/June 2018**

2 hours

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of **15** printed pages and **1** blank page.

DC (JM) 162868

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

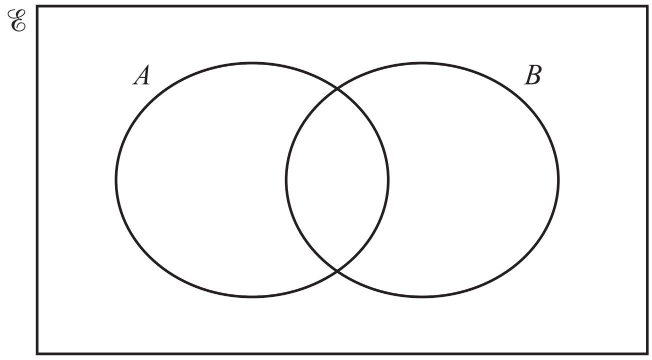
$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 **(i)** Show that $\cos\theta\cot\theta + \sin\theta = \operatorname{cosec}\theta$. [3]

(ii) Hence solve $\cos\theta\cot\theta + \sin\theta = 4$ for $0° \leqslant \theta \leqslant 90°$. [2]

2 **(a)** On the Venn diagram below, shade the region that represents $A \cap B'$.



[1]

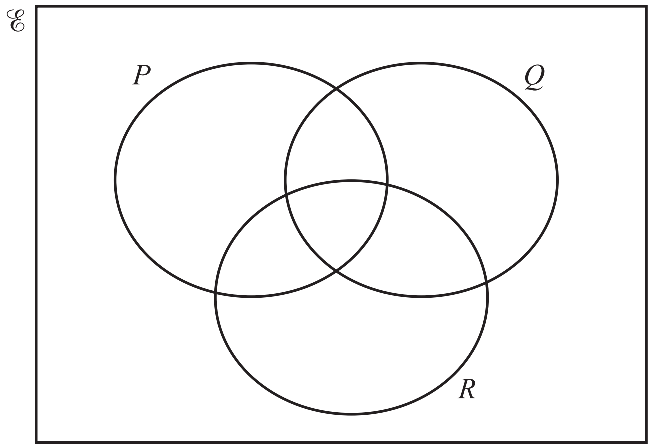
(b) The universal set $\mathscr{E}$ and sets P, Q and R are such that

$(P \cup Q \cup R)' = \varnothing$, $\quad P' \cap (Q \cap R) = \varnothing$,

$n(Q \cap R) = 8$, $\quad n(P \cap R) = 8$, $\quad n(P \cap Q) = 10$,

$n(P) = 21$, $\quad n(Q) = 15$, $\quad n(\mathscr{E}) = 30$.

Complete the Venn diagram to show this information and state the value of $n(R)$.



$n(R) =$ .... [4]

3 It is given that $x + 3$ is a factor of the polynomial $p(x) = 2x^3 + ax^2 - 24x + b$. The remainder when $p(x)$ is divided by $x - 2$ is -15. Find the remainder when $p(x)$ is divided by $x + 1$. [6]

4 Find the coordinates of the points where the line $2y - 3x = 6$ intersects the curve $\frac{x^2}{4} + \frac{y^2}{9} = 5$. [5]

5 **(a)** Four parts in a play are to be given to four of the girls chosen from the seven girls in a drama class. Find the number of different ways in which this can be done. [2]

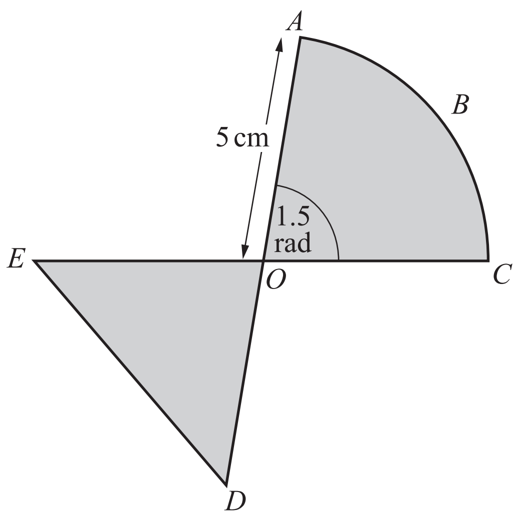
(b) Three singers are chosen at random from a group of 5 Chinese, 4 Indian and 2 British singers. Find the number of different ways in which this can be done if

(i) no Chinese singer is chosen, [1]

(ii) one singer of each nationality is chosen, [2]

(iii) the three singers chosen are all of the same nationality. [2]

6



In the diagram, ABC is an arc of the circle centre O, radius 5 cm, and angle AOC is 1.5 radians. AD and CE are diameters of the circle and DE is a straight line.

(i) Find the total perimeter of the shaded regions. [3]

(ii) Find the total area of the shaded regions. [3]

7 Vectors **i** and **j** are vectors parallel to the x-axis and y-axis respectively.

Given that $\mathbf{a} = 2\mathbf{i} + 3\mathbf{j}$, $\mathbf{b} = \mathbf{i} - 5\mathbf{j}$ and $\mathbf{c} = 3\mathbf{i} + 11\mathbf{j}$, find

(i) the exact value of $|\mathbf{a} + \mathbf{c}|$, [2]

(ii) the value of the constant m such that $\mathbf{a} + m\mathbf{b}$ is parallel to $\mathbf{j}$, [2]

(iii) the value of the constant n such that $n\mathbf{a} - \mathbf{b} = \mathbf{c}$. [2]

8 **(a)** $\mathbf{A} = \begin{pmatrix} 2 & -1 \\ 1 & -3 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 0 & -2 \\ 3 & -5 \end{pmatrix}$. Find $(\mathbf{BA})^{-1}$. [4]

(b) The matrix $\mathbf{X}$ is such that $\mathbf{XC} = \mathbf{D}$, where $\mathbf{C} = \begin{pmatrix} -2 & 5 & 3 \\ 0 & 10 & 4 \end{pmatrix}$ and $\mathbf{D} = \begin{pmatrix} -4 & 5 & 4 \end{pmatrix}$.

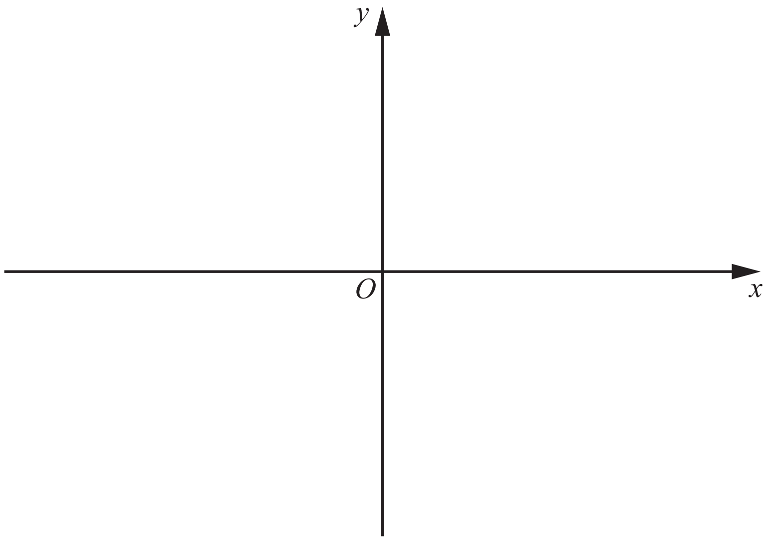
(i) State the order of the matrix $\mathbf{C}$. [1]

(ii) Find the matrix $\mathbf{X}$. [2]

9 **(i)** Differentiate $x^4(\sqrt{\sin x})$ with respect to x. [4]

(ii) Hence find $\int \left(x + \frac{x^4 \cos x}{\sqrt{\sin x}} + 8x^3(\sqrt{\sin x}) \right) \mathrm{d}x$. [3]

10 (a) (i) On the axes below, sketch the graph of $y = |(x+3)(x-5)|$ showing the coordinates of the points where the curve meets the x-axis. [2]



(ii) Write down a suitable domain for the function $f(x) = |(x+3)(x-5)|$ such that f has an inverse. [1]

(b) The functions g and h are defined by

$$g(x) = 3x - 1 \qquad \text{for } x > 1,$$
$$h(x) = \frac{4}{x} \qquad \text{for } x \neq 0.$$

(i) Find $hg(x)$. [1]

(ii) Find $(hg)^{-1}(x)$. [2]

(c) Given that $p(a) = b$ and that the function p has an inverse, write down $p^{-1}(b)$. [1]

11 **(a)** Find $\int \sqrt[3]{2x-1}\,dx$. [2]

(b) **(i)** Find $\int \sin 4x\,dx$. [2]

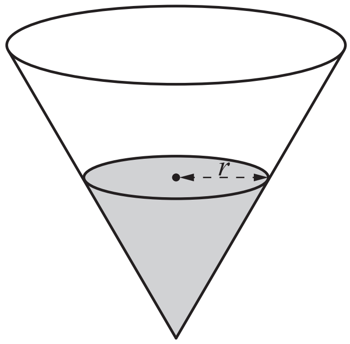
(ii) Hence evaluate $\int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \sin 4x\,dx$. [2]

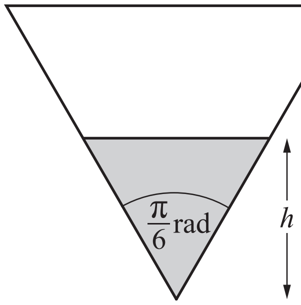
(c) Show that $\int_0^{\ln 8} e^{\frac{x}{3}}\,dx = 3$. [5]

12 In this question all lengths are in centimetres.

The volume of a cone of height h and base radius r is given by $V = \frac{1}{3}\pi r^2 h$.

It is known that $\sin\frac{\pi}{12} = \frac{\sqrt{6}-\sqrt{2}}{4}$, $\cos\frac{\pi}{12} = \frac{\sqrt{6}+\sqrt{2}}{4}$, $\tan\frac{\pi}{12} = 2-\sqrt{3}$.





A water cup is in the shape of a cone with its axis vertical. The diagrams show the cup and its cross-section. The vertical angle of the cone is $\frac{\pi}{6}$ radians. The depth of water in the cup is h. The surface of the water is a circle of radius r.

(i) Find an expression for r in terms of h and show that the volume of water in the cup is given by $V = \frac{\pi(7-4\sqrt{3})h^3}{3}$. [4]

(ii) Water is poured into the cup at a rate of $30\,\text{cm}^3\,\text{s}^{-1}$. Find, correct to 2 decimal places, the rate at which the depth of water is increasing when $h = 5$. [4]

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