Cambridge
O Level

**Cambridge International Examinations**
Cambridge Ordinary Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

*0061241896*

**ADDITIONAL MATHEMATICS** **4037/21**

Paper 2 **May/June 2016**

**2 hours**

Candidates answer on the Question Paper.

No Additional Materials are required.

**READ THESE INSTRUCTIONS FIRST**

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of **15** printed pages and **1** blank page.



CAMBRIDGE
International Examinations

***Mathematical Formulae***

## 1. ALGEBRA

*Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

*Binomial Theorem*

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1}\, b + \binom{n}{2} a^{n-2}\, b^2 + \ldots + \binom{n}{r} a^{n-r}\, b^r + \ldots + b^n,$$

where $n$ is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

## 2. TRIGONOMETRY

*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

*Formulae for $\Delta ABC$*

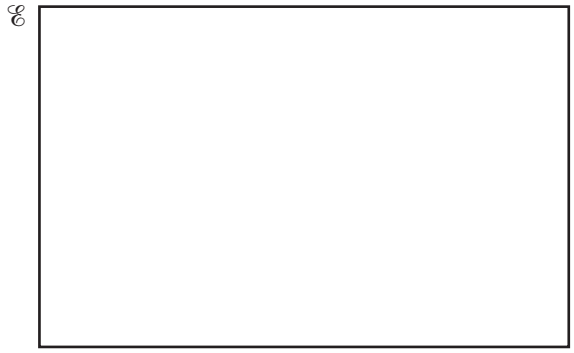
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

**1** Find the values of $x$ for which $(x-4)(x+2) > 7$. [3]

**2** **(a)** Illustrate the statements $A \subset B$ and $B \subset C$ using the Venn diagram below. [1]




**(b)** It is given that
the elements of set $\mathscr{E}$ are the letters of the alphabet,
the elements of set $P$ are the letters in the word *maths*,
the elements of set $Q$ are the letters in the word *exam*.

**(i)** Write the following using set notation.

The letter $h$ is in the word *maths*. [1]

**(ii)** Write the following using set notation.

The number of letters occurring in both of the words *maths* and *exam* is two. [1]

**(iii)** List the elements of the set $P \cap Q'$. [1]

**3 Do not use a calculator in this question.**

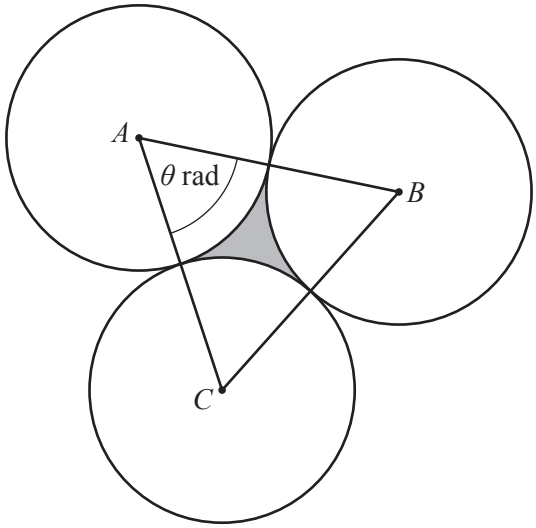
**(i)** Find the value of $-\log_p p^2$. [1]

**(ii)** Find $\lg\left(\frac{1}{10^n}\right)$. [1]

**(iii)** Show that $\frac{\lg 20 - \lg 4}{\log_5 10} = (\lg y)^2$, where $y$ is a constant to be found. [2]

**(iv)** Solve $\log_r 2x + \log_r 3x = \log_r 600$. [2]

**4**




The diagram shows 3 circles with centres $A$, $B$ and $C$, each of radius 5 cm. Each circle touches the other two circles. Angle $BAC$ is $\theta$ radians.

**(i)** Write down the value of $\theta$. [1]

**(ii)** Find the area of the shaded region between the circles. [4]





**[Turn over**

**5 Do not use a calculator in this question.**

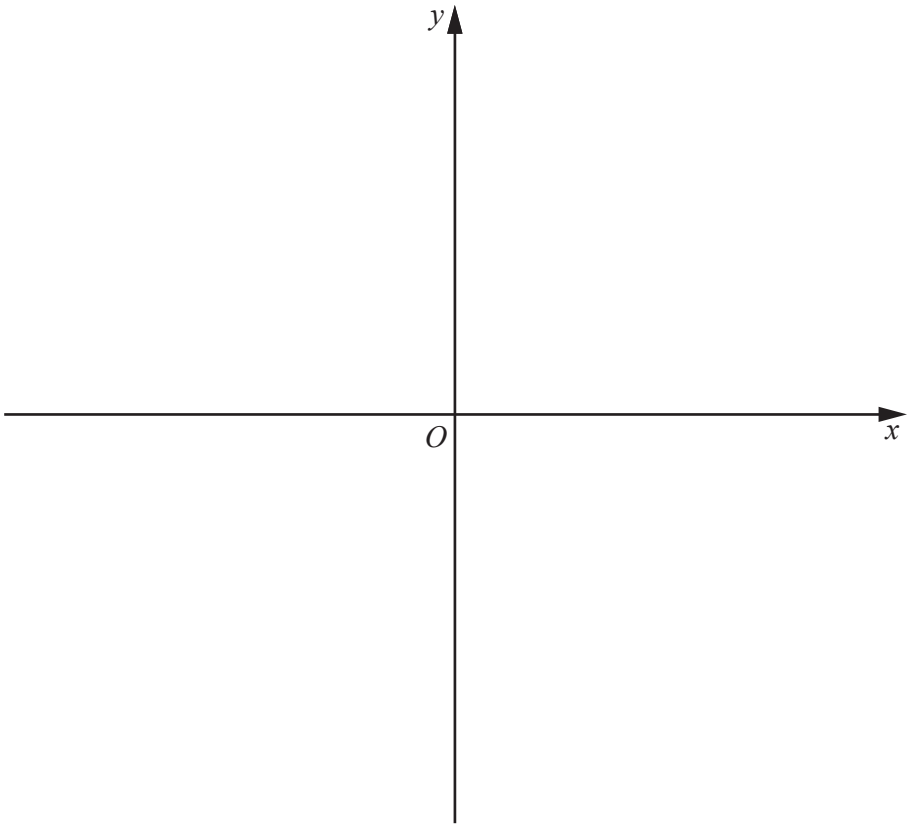
**(a)** Express $\dfrac{\sqrt{8}}{\sqrt{7}-\sqrt{5}}$ in the form $\sqrt{a}+\sqrt{b}$, where $a$ and $b$ are integers. [3]

**(b)** Given that $28+p\sqrt{3}=(q+2\sqrt{3})^2$, where $p$ and $q$ are integers, find the values of $p$ and of $q$. [3]

**6** **(i)** Express $4x^2 + 8x - 5$ in the form $p(x + q)^2 + r$, where $p$, $q$ and $r$ are constants to be found. [3]

**(ii)** State the coordinates of the vertex of $y = |4x^2 + 8x - 5|$. [2]

**(iii)** On the axes below, sketch the graph of $y = |4x^2 + 8x - 5|$, showing the coordinates of the points where the curve meets the axes. [3]



**7** $O$, $P$, $Q$ and $R$ are four points such that $\overrightarrow{OP} = \mathbf{p}$, $\overrightarrow{OQ} = \mathbf{q}$ and $\overrightarrow{OR} = 3\mathbf{q} - 2\mathbf{p}$.

**(i)** Find, in terms of **p** and **q**,

**(a)** $\overrightarrow{PQ}$, [1]

**(b)** $\overrightarrow{QR}$. [1]

**(ii)** Justifying your answer, what can be said about the positions of the points $P$, $Q$ and $R$? [2]

**(iii)** Given that $\overrightarrow{OP} = \mathbf{i} + 3\mathbf{j}$ and that $\overrightarrow{OQ} = 2\mathbf{i} + \mathbf{j}$, find the unit vector in the direction $\overrightarrow{OR}$. [3]

**8** **(a)** **(i)** Use the Binomial Theorem to expand $(a+b)^4$, giving each term in its simplest form. [2]

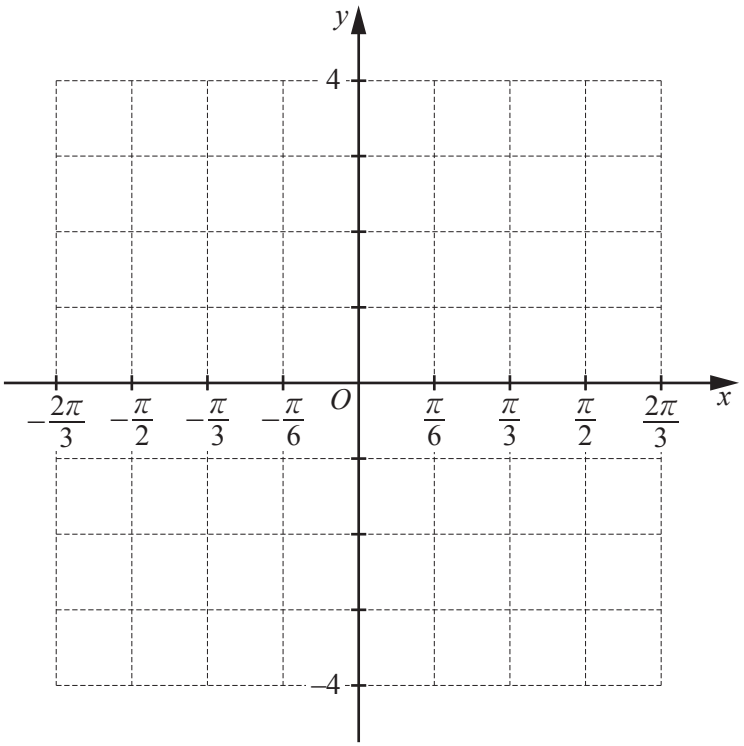
**(ii)** Hence find the term independent of $x$ in the expansion of $\left(2x+\frac{1}{5x}\right)^4$. [2]

**(b)** The coefficient of $x^3$ in the expansion of $\left(1+\frac{x}{2}\right)^n$ equals $\frac{5n}{12}$. Find the value of the positive integer $n$. [3]

**9** **(a)** Given that $y = a\tan bx + c$ has period $\frac{\pi}{4}$ radians and passes through the points $(0, -2)$ and $\left(\frac{\pi}{16}, 0\right)$, find the value of each of the constants $a$, $b$ and $c$. [3]

$a =$ ..... $b =$ ..... $c =$ .....

**(b)** **(i)** On the axes below, draw the graph of $y = 2\cos 3x + 1$ for $-\frac{2\pi}{3} \leqslant x \leqslant \frac{2\pi}{3}$ radians. [3]




**(ii)** Using your graph, or otherwise, find the exact solutions of $(2\cos 3x + 1)^2 = 1$ for $-\frac{2\pi}{3} \leqslant x \leqslant \frac{2\pi}{3}$ radians. [2]

**10 (a) (i)** Find how many 5-digit even numbers can be made using each of the digits 1, 2, 3, 4, 5 once only. [2]

**(ii)** Find how many different 3-digit numbers can be made using the digits 1, 2, 3, 4, 5 if each digit can be used once only. [2]

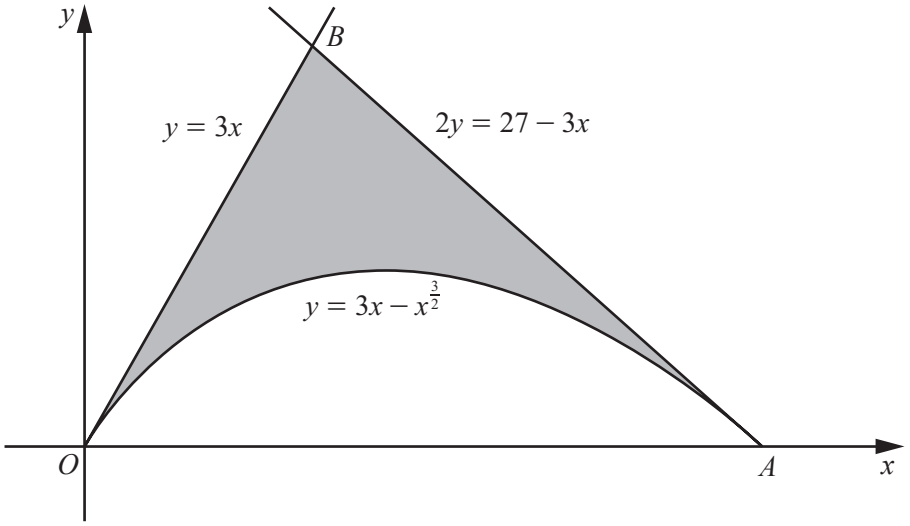
**(b)** A man and two women are to sit in a row of five empty chairs. Calculate the number of ways they can be seated if

**(i)** the two women must sit next to each other, [2]

**(ii)** all three people must sit next to each other. [2]

**11** **(i)** Find $\int(3x - x^{\frac{3}{2}})\,dx$. [2]

The diagram shows part of the curve $y = 3x - x^{\frac{3}{2}}$ and the lines $y = 3x$ and $2y = 27 - 3x$. The curve and the line $y = 3x$ meet the $x$-axis at $O$ and the curve and the line $2y = 27 - 3x$ meet the $x$-axis at $A$.




**(ii)** Find the coordinates of $A$. [1]

**(iii)** Verify that the coordinates of $B$ are $(3, 9)$. [1]

**(iv)** Find the area of the shaded region. [4]

**12** A curve has equation $y = \frac{2x-5}{x-1} - 12x$.

**(i)** Find $\frac{dy}{dx}$. [3]

**(ii)** Find $\frac{d^2y}{dx^2}$. [2]

**(iii)** Find the coordinates of the stationary points of the curve and determine their nature. [5]

**BLANK PAGE**