Cambridge International Examinations
Cambridge Ordinary Level

CANDIDATE NAME	

CENTRE NUMBER			CANDIDATE NUMBER	

ADDITIONAL MATHEMATICS   **4037/12**

Paper 1   **May/June 2016**

                                                                                               2 hours

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

This document consists of **16** printed pages.

[Turn over

2

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \dfrac{n!}{(n-r)!r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}bc \sin A$$

1 **(a)** The universal set $\mathscr{E}$ is the set of real numbers and sets X, Y and Z are such that

$X = \{$integer multiples of $5\}$,

$Y = \{$integer multiples of $10\}$,

$Z = \{\pi, \sqrt{2}, e\}$.

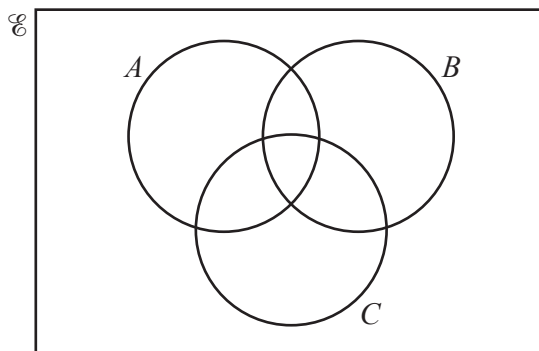Use set notation to complete the two statements below.

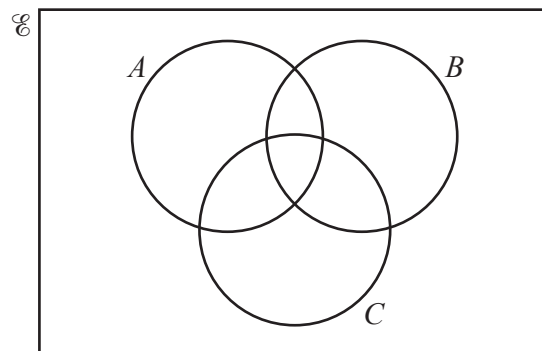Y .... X $\qquad\qquad\qquad$ $Y \cap Z = $ .... $\qquad\qquad$ [2]

(b) On each of the Venn diagrams below, shade the region indicated.

$(A' \cap B) \cup C$ $\qquad\qquad\qquad\qquad\qquad\qquad$ $A' \cap (B \cup C)$

[2]

2 **(i)** The first 3 terms in the expansion of $\left(2 - \dfrac{1}{4x}\right)^5$ are $\quad a + \dfrac{b}{x} + \dfrac{c}{x^2}$. Find the value of each of the integers a, b and c. [3]

(ii) Hence find the term independent of x in the expansion of $\left(2 - \dfrac{1}{4x}\right)^5 (3 + 4x)$. [2]

3 Vectors **a**, **b** and **c** are such that $\mathbf{a} = \begin{pmatrix} 2 \\ y \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} -5 \\ 5 \end{pmatrix}$.

  (i) Given that $|\mathbf{a}| = |\mathbf{b} - \mathbf{c}|$, find the possible values of y. [3]

  (ii) Given that $\mu(\mathbf{b} + \mathbf{c}) + 4(\mathbf{b} - \mathbf{c}) = \lambda(2\mathbf{b} - \mathbf{c})$, find the value of μ and of λ. [3]

4   **Do not use a calculator in this question.**

Find the positive value of x for which $(4 + \sqrt{5})x^2 + (2 - \sqrt{5})x - 1 = 0$,   giving your answer in the

form $\dfrac{a + \sqrt{5}}{b}$, where a and b are integers. [6]

5 **(i)** Show that $(1 - \cos\theta)(1 + \sec\theta) = \sin\theta\tan\theta$. [4]

(ii) Hence solve the equation $(1 - \cos\theta)(1 + \sec\theta) = \sin\theta$ for $0 \leqslant \theta \leqslant \pi$ radians. [3]

6 Show that $\dfrac{\mathrm{d}}{\mathrm{d}x}(\mathrm{e}^{3x}\sqrt{4x+1})$ can be written in the form $\dfrac{\mathrm{e}^{3x}(px+q)}{\sqrt{4x+1}}$, where p and q are integers to be found. [5]

7
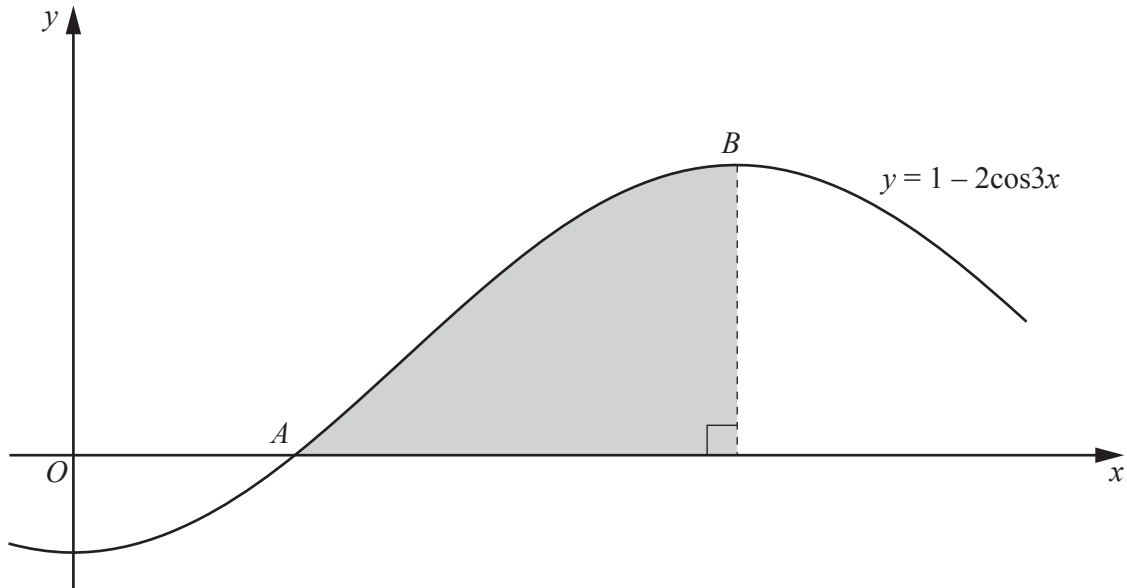
The diagram shows part of the graph of $y = 1 - 2\cos 3x$, which crosses the x-axis at the point A and has a maximum at the point B.

(i) Find the coordinates of A. [2]

(ii) Find the coordinates of B. [2]

(iii) Showing all your working, find the area of the shaded region bounded by the curve, the x-axis and the perpendicular from B to the x-axis. [4]

8
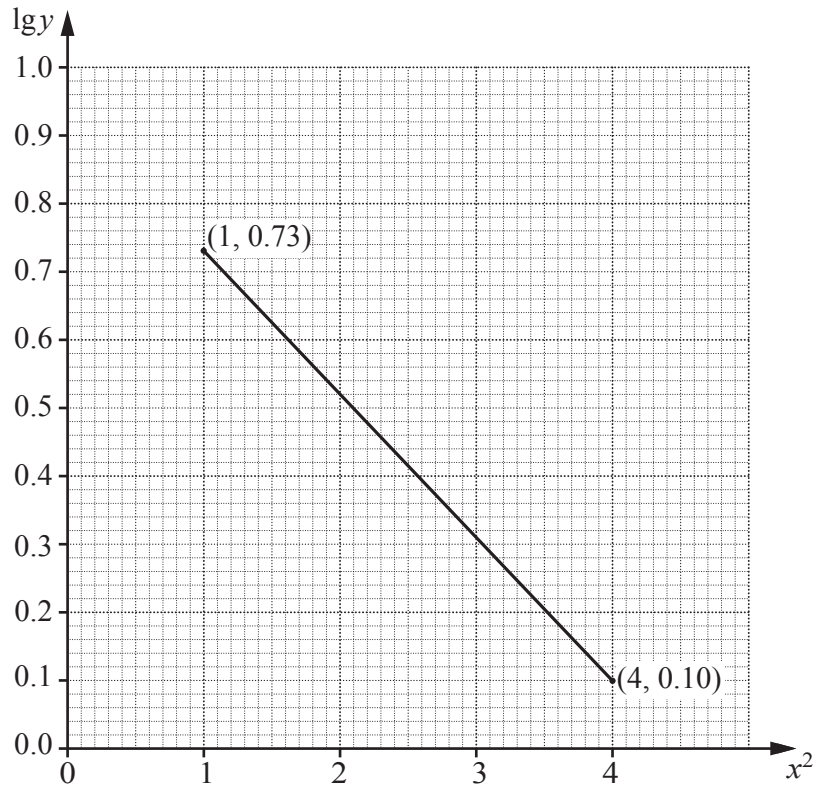
Variables x and y are such that when $\lg y$ is plotted against x^2, the straight line graph shown above is obtained.

(i) Given that $y = Ab^{x^2}$, find the value of A and of b. [4]

(ii) Find the value of y when $x = 1.5$. [2]

(iii) Find the positive value of x when $y = 2$. [2]

9 A curve passes through the point $\left(2, -\frac{4}{3}\right)$ and is such that $\dfrac{dy}{dx} = (3x + 10)^{-\frac{1}{2}}$.

   (i) Find the equation of the curve. [4]

The normal to the curve, at the point where $x = 5$, meets the line $y = -\dfrac{5}{3}$ at the point P.

   (ii) Find the x-coordinate of P. [6]

10
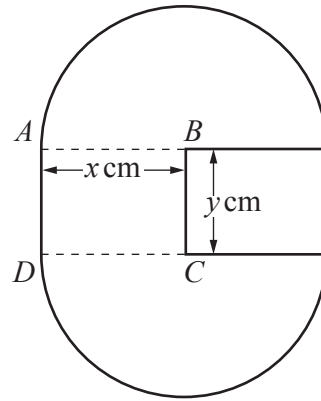
The diagram shows a badge, made of thin sheet metal, consisting of two semi-circular pieces, centres B and C, each of radius x cm. They are attached to each other by a rectangular piece of thin sheet metal, $ABCD$, such that AB and CD are the radii of the semi-circular pieces and $AD = BC = y$ cm.

(i) Given that the area of the badge is $20\,\text{cm}^2$, show that the perimeter, P cm, of the badge is given

by $P = 2x + \dfrac{40}{x}$. [4]

(ii) Given that x can vary, find the minimum value of P, justifying that this value is a minimum. [5]

11 (a)
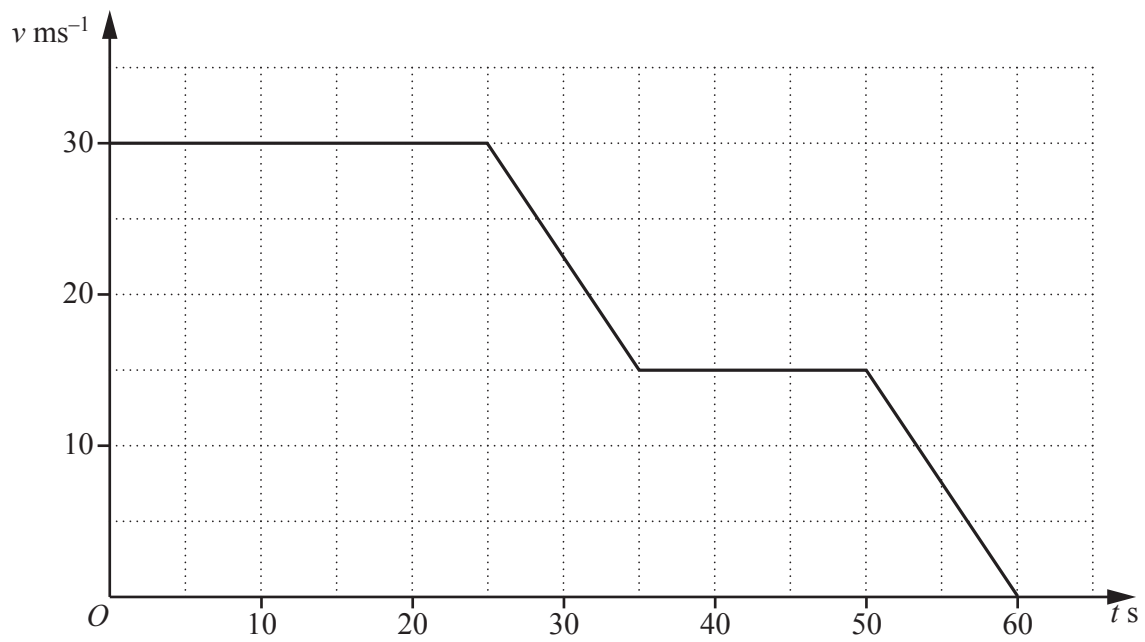
The diagram shows the velocity-time graph of a particle P moving in a straight line with velocity $v\ \text{ms}^{-1}$ at time t s after leaving a fixed point.

(i) Find the distance travelled by the particle P. [2]

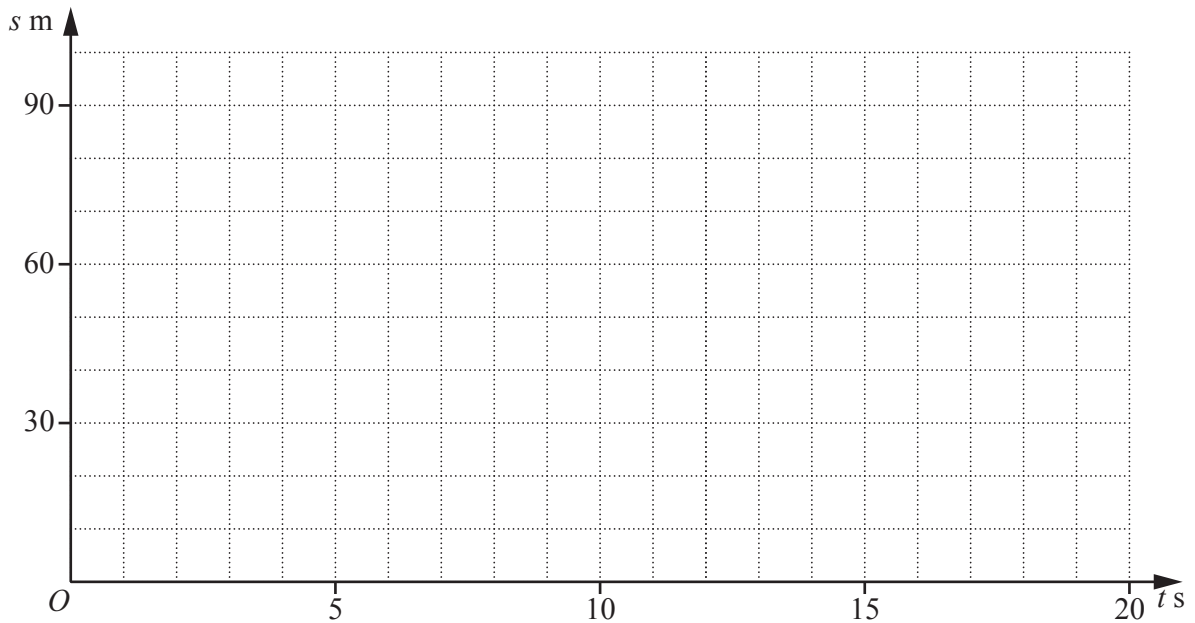**(ii)** Write down the deceleration of the particle when $t = 30$. [1]

(b) The diagram shows a velocity-time graph of a particle Q moving in a straight line with velocity $v\,\text{ms}^{-1}$, at time t s after leaving a fixed point.
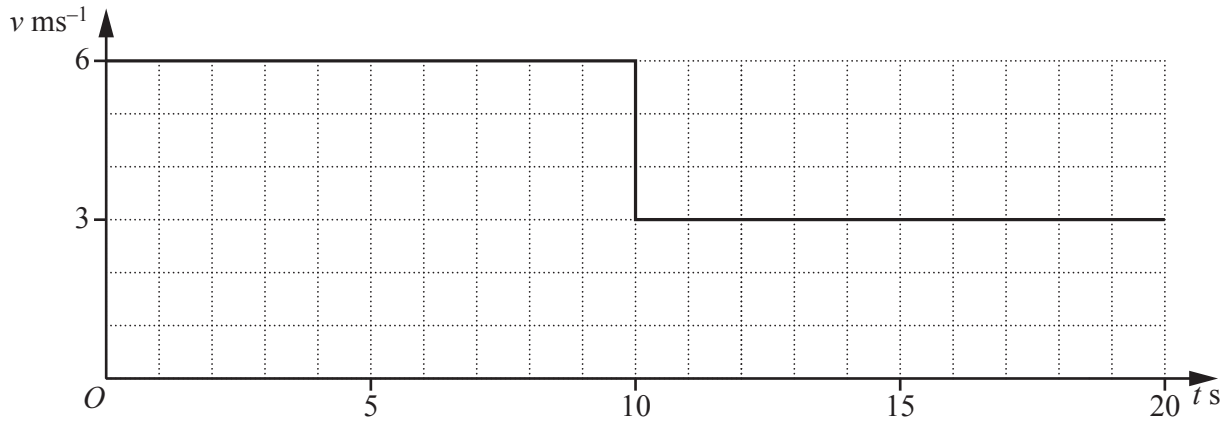
The displacement of Q at time t s is s m. On the axes below, draw the corresponding displacement-time graph for Q. [2]

Question 11(c) is printed on the next page.

(c) The velocity, $v\,\text{ms}^{-1}$, of a particle R moving in a straight line, $t\,$s after passing through a fixed point O, is given by $v = 4\text{e}^{2t} + 6$.

(i) Explain why the particle is never at rest. [1]

(ii) Find the exact value of t for which the acceleration of R is $12\,\text{ms}^{-2}$. [2]

(iii) Showing all your working, find the distance travelled by R in the interval between $t = 0.4$ and $t = 0.5$. [4]