

CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge Ordinary Level

MARK SCHEME for the May/June 2015 series

4037 ADDITIONAL MATHEMATICS

4037/12

Paper 1, maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied
www	without wrong working

1	$k^2 - 4(2k + 5) < 0$ $k^2 - 8k - 20 < 0$ $(k - 10)(k + 2) < 0$ critical values of 10 and -2 $-2 < k < 10$ Alternative 1: $\frac{dy}{dx} = 2(2k + 5)x + k$ When $\frac{dy}{dx} = 0$, $x = \frac{-k}{2(2k + 5)}$, $y = \frac{8k + 20 - k^2}{4(2k + 5)}$ When $y = 0$, obtain critical values of 10 and -2 $-2 < k < 10$ Alternative 2: $y = (2k + 5) \left(\left(x + \frac{k}{2(2k + 5)} \right)^2 - \frac{k^2}{4(2k + 5)} \right) + 1$ Looking at $1 - \frac{k^2}{4(2k + 5)} = 0$ leads to critical values of 10 and -2 $-2 < k < 10$	M1 M1 A1 A1 M1 M1 A1 A1 M1 M1 A1 A1	use of $b^2 - 4ac$, (not as part of quadratic formula unless isolated at a later stage) with correct values for a , b and c Do not need to see $<$ at this point attempt to obtain critical values correct critical values correct range attempt to differentiate, equate to zero and substitute x value back in to obtain a y value consider $y = 0$ in order to obtain critical values correct critical values correct range attempt to complete the square and consider ' $1 - \frac{k^2}{4(2k + 5)}$ ', attempt to solve above = to 0, to obtain critical values correct critical values correct range
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2	$\frac{\tan \theta + \cot \theta}{\operatorname{cosec} \theta} = \frac{\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta}}{\frac{1}{\sin \theta}}$ $= \frac{\frac{\sin^2 \theta + \cos^2 \theta}{\sin \theta \cos \theta}}{\frac{1}{\sin \theta}}$ $= \frac{1}{\cos \theta}$ $= \sec \theta$ <p>Alternative:</p> $\frac{\tan \theta + \cot \theta}{\operatorname{cosec} \theta} = \frac{\frac{\tan^2 \theta + 1}{\tan \theta}}{\operatorname{cosec} \theta}$ $= \frac{\sec^2 \theta}{\tan \theta \frac{1}{\sin \theta}}$ $= \frac{\sec^2 \theta}{\sec \theta}$ $= \sec \theta$	<p>M1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p>	<p>for $\tan \theta = \frac{\sin \theta}{\cos \theta}$, $\cot \theta = \frac{\cos \theta}{\sin \theta}$ and $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$; allow when used</p> <p>dealing correctly with fractions in the numerator; allow when seen</p> <p>use of the appropriate identity; allow when seen</p> <p>must be convinced it is from completely correct work (beware missing brackets)</p> <p>for either $\tan \theta = \frac{1}{\cot \theta}$ or $\cot \theta = \frac{1}{\tan \theta}$ and $\operatorname{cosec} \theta = \frac{1}{\sin \theta}$; allow when used</p> <p>dealing correctly with fractions in numerator; allow when seen</p> <p>use of the appropriate identity; allow when seen</p> <p>must be convinced it is from completely correct work</p>
3	$\mathbf{A}^{-1} = \frac{1}{2} \begin{pmatrix} 3 & -2 \\ -5 & 4 \end{pmatrix}$ $\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 3 & -2 \\ -5 & 4 \end{pmatrix} \begin{pmatrix} 8 \\ 9 \end{pmatrix}$ $\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 6 \\ -4 \end{pmatrix}$ $x = 3, y = -2$	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1, A1</p>	<p>$\frac{1}{2}$ multiplied by a matrix</p> <p>for matrix</p> <p>attempt to use the inverse matrix, must be pre-multiplication</p>

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<p>4 (i)</p> <p>Area =</p> $\left(\frac{1}{2} \times 12^2 \times 1.7\right) + \left(\frac{1}{2} \times 12^2 \sin(2\pi - 1.7 - 2.4)\right)$ <p>= awrt 181</p> <p>(ii)</p> $BC^2 = 12^2 + 12^2 - (2 \times 12 \times 12 \cos 2.1832)$ <p>or $BC = 2 \times 12 \times \sin\left(\frac{2\pi - 4.1}{2}\right)$</p> $BC = 21.296$ <p>Perimeter = $(12 \times 1.7) + 12 + 12 + 21.296$</p> $= 65.7$		<p>B1,B1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>M1</p> <p>A1</p>	<p>B1 for sector area, allow unsimplified</p> <p>B1 for correct angle BOC, allow unsimplified</p> <p>correct attempt at area of triangle, allow unsimplified using <i>their</i> angle BOC</p> <p>(Their angle BOC must not be 1.7 or 2.4)</p> <p>correct attempt at BC, may be seen in (i), allow if used in (ii). Allow use of <i>their</i> angle BOC.</p> <p>for arc length, allow unsimplified for a correct 'plan' (an arc + 2 radii and BC)</p>
<p>5 (a) (i)</p> <p>20160</p> <p>(ii)</p> $3 \times {}^6P_4 \times 2$ $= 2160$ <p>(iii)</p> $5 \times 2 \times {}^6P_4$ $= 3600$ <p>Alternative 1:</p> ${}^6C_4 \times 5! \times 2$ $= 3600$ <p>Alternative 2:</p> $\left({}^7P_5 - {}^6P_5\right) \times 2$ $= 3600$ <p>Alternative 3:</p> $2! \left({}^6P_4 + \left({}^6P_1 \times {}^5P_3 \right) + \left({}^6P_2 \times {}^4P_2 \right) + \left({}^6P_3 \times {}^3P_1 \right) + {}^6P_4 \right)$ $= 3600$		<p>B1</p> <p>B1,B1</p> <p>B1,B1</p> <p>B1</p> <p>B2</p> <p>B1</p> <p>B2</p> <p>B1</p> <p>B2</p> <p>B1</p> <p>B2</p> <p>B1</p>	<p>B1 for 6P_4 (must be seen in a product)</p> <p>B1 for all correct, with no further working</p> <p>B1 for 6P_4 (must be seen in a product)</p> <p>B1 for 5 (must be in a product)</p> <p>B1 for all correct, with no further working</p> <p>for ${}^6C_4 \times 5!$</p> <p>for ${}^6C_4 \times 5! \times 2$</p> <p>for $\left({}^7P_5 - {}^6P_5\right)$</p> <p>for $\left({}^7P_5 - {}^6P_5\right) \times 2$</p> <p>4 terms correct or omission of 2! in each term</p> <p>all correct</p>

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(b)	(i)	${}^{14}C_4 \times {}^{10}C_4$ or ${}^{14}C_8 \times {}^8C_4$ (or numerical or factorial equivalent) $= 210210$	B1,B1	B1 for either ${}^{14}C_4$ or ${}^{14}C_8$ as part of a product B1 for correct answer, with no further working
	(ii)	${}^8C_4 \times {}^6C_4$ $= 1050$	B1,B1	B1 for either 8C_4 or 6C_4 as part of a product B1 for correct answer with no further working
6	(i)	$10\ln 4$ or 13.9 or better	B1	
	(ii)	$\left(\frac{dx}{dt}\right) \frac{20t}{t^2 + 4} - 4$ When $\frac{dx}{dt} = 0$, $\frac{20t}{t^2 + 4} = 4$ leading to $t^2 - 5t + 4 = 0$ $t = 1, t = 4$	M1 B1 DM1 A1	attempt to differentiate and equate to zero $\frac{20t}{t^2 + 4}$ or equivalent seen attempt to solve <i>their</i> $\frac{dx}{dt} = 0$, must be a 2 or 3 term quadratic equation with real roots for both

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(iii)	<p>If $(v =) \frac{20t}{t^2 + 4} - 4$</p> <p>$(a =) \frac{20(t^2 + 4) - 20t(2t)}{(t^2 + 4)^2}$</p> <p>$20(4 - t^2)$ or $80 - 20t^2$ or $4 - t^2$ or equivalent expression involving $-t^2$</p> <p>When acceleration is 0, $t = 2$ only</p> <p>Alternative 1 for first 3 marks:</p> <p>If $(v =) \frac{20t - 4t^2 - 16}{t^2 + 4}$</p> <p>$(a =) \frac{(t^2 + 4)(20 - 8t) - (20t - 4t^2 - 16)(2t)}{(t^2 + 4)^2}$</p> <p>Alternative 2 for M1 mark:</p> <p>If $(v =) 20t(t^2 + 4)^{-1} - 4$</p> <p>$(a =) 20t(-2t(t^2 + 4)^{-2}) + 20(t^2 + 4)^{-1}$</p> <p>Alternative 3 for the first 3 marks</p> <p>If $(v =) (20t - 4t^2 - 16)(t^2 + 4)^{-1}$</p> <p>$(a =) (20t - 4t^2 - 16)(-2t(t^2 + 4)^{-2}) + (20 - 8t)(t^2 + 4)^{-1}$</p> <p>Numerator = $-2t(20t - 4t^2 - 16) + (20 - 8t)(t^2 + 4)$</p>	<p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>A1</p>	<p>attempt to differentiate <i>their</i> $\frac{dx}{dt}$</p> <p>$20(t^2 + 4)$</p> <p>$20t(2t)$</p> <p>$20(4 - t^2)$ or $80 - 20t^2$ or $4 - t^2$</p> <p>$t = 2$, dependent on obtaining first and second A marks</p> <p>attempt to differentiate <i>their</i> $\frac{dx}{dt}$</p> <p>for $(t^2 + 4)(20 - 8t)$</p> <p>for $(20t - 4t^2 - 16)(2t)$</p> <p>attempt to differentiate <i>their</i> $\frac{dx}{dt}$</p> <p>attempt to differentiate <i>their</i> $\frac{dx}{dt}$</p> <p>attempt to differentiate <i>their</i> $\frac{dx}{dt}$</p> <p>for $2t(20t - 4t^2 - 16)$</p> <p>for $(20 - 8t)(t^2 + 4)$</p>
7	<p>(i) $\overrightarrow{DA} = 3\mathbf{a} - \mathbf{b}$</p> <p>(ii) $\overrightarrow{DB} = 7\mathbf{a} - \mathbf{b}$</p> <p>(iii) $\overrightarrow{AX} = \lambda(4\mathbf{a} + \mathbf{b})$</p> <p>(iv) $\overrightarrow{DX} = 3\mathbf{a} - \mathbf{b} + \lambda(4\mathbf{a} + \mathbf{b})$</p>	<p>B1</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p>	<p>mark final answer, allow unsimplified</p> <p>mark final answer, allow unsimplified</p> <p>mark final answer, allow unsimplified</p> <p><i>their</i> (i) + <i>their</i> (iii) or equivalent valid method or $3\mathbf{a} - \mathbf{b} + \textit{their} (iii)$</p> <p>Allow unsimplified</p>

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(v)	$3\mathbf{a} - \mathbf{b} + \lambda(4\mathbf{a} + \mathbf{b}) = \mu(7\mathbf{a} - \mathbf{b})$ Equate like vectors: $3 + 4\lambda = 7\mu$ $-1 + \lambda = -\mu$ leads to $\lambda = \frac{4}{11}$, $\mu = \frac{7}{11}$	M1 DM1 A1,A1	equating <i>their</i> (iv) and $\mu \times$ <i>their</i> (ii) for an attempt to equate like vectors and attempt to solve 2 linear equations for λ and μ A1 for each
8 (i)	$5e^{2x} - \frac{1}{2}e^{-2k} \quad (+c)$	B1, B1	B1 for each term, allow unsimplified
(ii)	$\left(5e^{2k} - \frac{1}{2}e^{-2k}\right) - \left(5e^{-2k} - \frac{1}{2}e^{2k}\right)$	M1 A1	use of limits provided integration has taken place. Signs must be correct if brackets are not included. allow any correct form
(iii)	$\left(5e^{2k} - \frac{1}{2}e^{-2k}\right) \left(5e^{-2k} - \frac{1}{2}e^{2k}\right) = -60$ or $\frac{11}{2}e^{2k} - \frac{11}{2}e^{-2k} = -60$ or equivalent leading to $11e^{2k} - 11e^{-2k} + 120 = 0$	B1 DB1	correct expression from (ii) either simplified or unsimplified equated to -60 , must be first line seen. must be convinced as AG
(iv)	$11y^2 + 120y - 11 = 0$ $(11y - 1)(y + 11) = 0$ leading to $k = \frac{1}{2}\ln\frac{1}{11}, \ln\frac{1}{\sqrt{11}}, -\ln\sqrt{11}, -\frac{1}{2}\ln 11$	M1 DM1 A1	attempt to obtain a quadratic equation in y or e^{2k} and solve to get y or e^{2k} (only need positive solution) attempt to deal with e to get $k =$. any of given answers only.

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<p>9</p>	$\frac{dy}{dx} = 4 - 6\sin 2x$ <p>When $x = \frac{\pi}{4}$, $y = \pi$</p> $\frac{dy}{dx} = -2 \text{ so gradient of normal} = \frac{1}{2}$ <p>Normal equation $y - \pi = \frac{1}{2}\left(x - \frac{\pi}{4}\right)$</p> <p>When $x = 0$, $y = \frac{7\pi}{8}$</p> <p>When $y = 0$, $x = -\frac{7\pi}{4}$</p> $\text{Area} = \frac{1}{2} \times \frac{7\pi}{4} \times \frac{7\pi}{8} = \frac{49\pi^2}{64}$	<p>M1,A1</p> <p>B1</p> <p>DM1</p> <p>DM1</p> <p>A1</p> <p>A1</p> <p>B1ft</p>	<p>M1 for attempt to differentiate A1 for all correct</p> <p>for y</p> <p>for substitution of $x = \frac{\pi}{4}$ into <i>their</i> $\frac{dy}{dx}$ and use of '$m_1 m_2 = -1$', dependent on first M1</p> <p>correct attempt to obtain the equation of the normal, dependent on previous DM mark</p> <p>must be terms of π</p> <p>must be terms of π</p> <p>Follow through on <i>their</i> x and y intercepts; must be exact values</p>
<p>10 (a)</p> <p>(b)</p> <p>(c)</p>	<p>$\cos^2 3x = \frac{1}{2}$, $\cos 3x = (\pm)\frac{1}{\sqrt{2}}$ $3x = 45^\circ, 135^\circ, 225^\circ, 315^\circ$ $x = 15^\circ, 45^\circ, 75^\circ, 105^\circ$</p> <p>$3(\cot^2 y + 1) + 5\cot y - 5 = 0$ Leading to $3\cot^2 y + 5\cot y - 2 = 0$ or $2\tan^2 y - 5\tan y - 3 = 0$ $(3\cot y - 1)(\cot y + 2) = 0$ or $(\tan y - 3)(2\tan y + 1) = 0$</p> <p>$\tan y = 3$, $\tan y = \frac{1}{2}$</p> <p>$y = 71.6^\circ, 251.6^\circ$ $153.4^\circ, 333.4^\circ$</p> <p>$\sin\left(z + \frac{\pi}{3}\right) = \frac{1}{2}$ $z + \frac{\pi}{3} = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}$ $z = \frac{\pi}{2}, \frac{11\pi}{6}$ (allow 1.57, 5.76)</p>	<p>M1</p> <p>A1,A1</p> <p>M1</p> <p>M1</p> <p>M1</p> <p>A1,A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p>	<p>complete correct method, dealing with sec and 3, correctly A1 for each correct pair</p> <p>use of a correct identity to get an equation in terms of one trig ratio only</p> <p>for $\cot y = \frac{1}{\tan y}$ to obtain either a quadratic equation in $\tan y$ or solutions in terms of $\tan y$; allow where appropriate</p> <p>for solution of a quadratic equation in terms of either $\tan y$ or $\cot y$</p> <p>A1 for each correct 'pair'</p> <p>completely correct method of solution</p> <p>one correct solution in range</p> <p>correct attempt to obtain a second solution within the range second correct solution (and no other)</p>