CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge Ordinary Level

MARK SCHEME for the May/June 2015 series

4037 ADDITIONAL MATHEMATICS

4037/11 Paper 1, maximum raw mark 80

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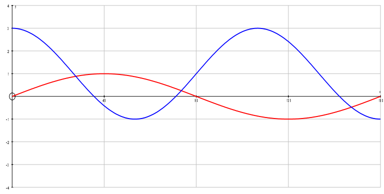
Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied
www	without wrong working

1 (i)	180° or π radians or 3.14 radians (or better)	**B1**	
(ii)	2	**B1**	
(iii) (a)		**B1**	$y = \sin 2x$ all correct
(b)		**B1**	for either ↑↓↑starting at their highest value and ending at their lowest value Or a curve with highest value at $y = 3$ and lowest value at $y = -1$
		B1	completely correct graph
(iv)	3	**B1**	
2 (i)	$\tan\theta = \frac{(8+5\sqrt{2})(4-3\sqrt{2})}{(4+3\sqrt{2})(4-3\sqrt{2})}$ $= \frac{32-24\sqrt{2}+20\sqrt{2}-30}{16-18}$	**M1**	attempt to obtain $\tan\theta$ and rationalise. Must be convinced that no calculators are being used
	$=1+2\sqrt{2}$ cao	**A1**	

(ii)	$\sec^2\theta = 1+\tan^2\theta$ $= 1+\left(-1+2\sqrt{2}\right)^2$ $= 1+1-4\sqrt{2}+8$ $= 10-4\sqrt{2}$ **Alternative solution:** $AC^2 = \left(4+3\sqrt{2}\right)^2+\left(8+5\sqrt{2}\right)^2$ $= 148+104\sqrt{2}$ $\sec^2\theta = \dfrac{148+104\sqrt{2}}{\left(4+3\sqrt{2}\right)^2}$ $= \dfrac{148+104\sqrt{2}}{\left(4+3\sqrt{2}\right)^2}\times\dfrac{34-24\sqrt{2}}{34-24\sqrt{2}}$ $= 10-4\sqrt{2}$	**M1** **DM1** **A1** **M1** **DM1** **A1**	attempt to use $\sec^2\theta = 1+\tan^2\theta$, with *their* answer to **(i)** attempt to simplify, must be convinced no calculators are being used. Need to expand $\left(-1+2\sqrt{2}\right)^2$ as 3 terms
3 (i)	$64+192x^2+240x^4+160x^6$	**B3,2,1,0**	–1 each error
(ii)	$\left(64+192x^2+240x^4\right)\left(1-\dfrac{6}{x^2}+\dfrac{9}{x^4}\right)$ Terms needed $64-(192\times 6)+(240\times 9)$ $= 1072$	**B1** **M1** **A1**	expansion of $\left(1-\dfrac{3}{x^2}\right)^2$ attempt to obtain 2 or 3 terms using *their* **(i)**

4 **(a)**	$\mathbf{X}^2=\begin{pmatrix}4-4k & -8\\ 2k & -4k\end{pmatrix}$	**B2,1,0**	–1 each incorrect element
(b)	Use of $\mathbf{AA}^{-1}=\mathbf{I}$ $\begin{pmatrix}a & 1\\ b & 5\end{pmatrix}\begin{pmatrix}\frac{5}{6} & -\frac{1}{6}\\ -\frac{2}{3} & \frac{1}{3}\end{pmatrix}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$	**M1**	use of $\mathbf{AA}^{-1}=\mathbf{I}$ and an attempt to obtain at least one equation.
	Any 2 equations will give $a=2$, $b=4$	**A1,A1**	
	Alternative method 1: $\frac{1}{5a-b}\begin{pmatrix}5 & -1\\ b & a\end{pmatrix}=\begin{pmatrix}\frac{5}{6} & -\frac{1}{6}\\ -\frac{2}{3} & \frac{1}{3}\end{pmatrix}$	**M1**	correct attempt to obtain $\mathbf{A}^{-1}$ and comparison of at least one term.
	Compare any 2 terms to give $a=2$, $b=4$	**A1,A1**	
	Alternative method 2: Inverse of $\frac{1}{6}\begin{pmatrix}5 & -1\\ -4 & 2\end{pmatrix}=\begin{pmatrix}2 & 1\\ 4 & 5\end{pmatrix}$	**M1** **A1,A1**	reasoning and attempt at inverse
5	$3x-1=x(3x-1)+x^2-4$ or $y=\left(\frac{y+1}{3}\right)y+\left(\frac{y+1}{3}\right)^2-4$		
	$4x^2-4x-3=0$ or $4y^2-4y-35=0$	**M1**	equate and attempt to obtain an equation in 1 variable
	$(2x-3)(2x+1)=0$ or $(2y-7)(2y+5)=0$	**DM1**	forming a 3 term quadratic equation and attempt to solve
	leading to $x=\frac{3}{2}, x=-\frac{1}{2}$ and	**A1**	x values
	$y=\frac{7}{2}, y=-\frac{5}{2}$	**A1**	y values
	Midpoint $\left(\frac{1}{2},\frac{1}{2}\right)$	**B1**	for midpoint, allow anywhere
	Perpendicular gradient = $-\frac{1}{3}$	**M1**	correct attempt to obtain the gradient of the perpendicular, using AB
	Perp bisector: $y-\frac{1}{2}=-\frac{1}{3}\left(x-\frac{1}{2}\right)$	**M1**	straight line equation through the midpoint; must be convinced it is a perpendicular gradient.
	$(3y+x-2=0)$	**A1**	allow unsimplified

6 (i)	$f\left(\frac{1}{2}\right)=\frac{a}{8}-\frac{15}{4}+\frac{b}{2}-2=0$ leading to $a+4b=46$ $f(1)=a-15+b-2=5$ leading to $a+b=22$	**M1** **A1**	correct use of either $f\left(\frac{1}{2}\right)$ or f(1) paired correctly both equations correct (allow unsimplified)
	giving $b=8$ (AG), $a=14$	**M1,A1**	M1 for solution of equations A1 for both *a* and *b*. **AG** for *b*.
(ii)	$(2x-1)(7x^2-4x+2)$	**M1,A1**	M1 for valid attempt to obtain g(*x*), by either observation or by algebraic long division.
(iii)	$7x^2-4x+2=0$ has no real solutions as $b^2<4ac$ $16<56$	**M1** **A1**	use of b^2-4ac correct conclusion; must be from a correct g(*x*) or 2g(*x*) www
7 (i)	$\frac{dy}{dx}=\frac{(x-1)\frac{8x}{(4x^2+2)}-\ln(4x^2+3)}{(x-1)^2}$	**M1** **B1** **A1**	differentiation of a quotient (or product) correct differentiation of $\ln(4x^2+3)$ all else correct
	When $x=0$, $y=-\ln 3$ oe	**B1**	for *y* value
	$\frac{dy}{dx}=-\ln 3$ so gradient of normal is $\frac{1}{\ln 3}$ (allow numerical equivalent)	**M1**	valid attempt to obtain gradient of the normal
	normal equation $y+\ln 3=\frac{1}{\ln 3}x$ or $y=0.910x-1.10$, or $y=\frac{10}{11}x-\frac{11}{10}$ cao (Allow $y=0.91x-1.1$)	**M1** **A1**	attempt at normal equation must be using a perpendicular
(ii)	when $x=0$, $y=-\ln 3$ when $y=0$, $x=(\ln 3)^2$ Area = ±0.66 or ±0.67 or awrt these or $\frac{1}{2}(\ln 3)^3$	**M1** **A1**	valid attempt at area

8 **(i)**	Range for f: $y \geq 3$ Range for g: $y \geq 9$	**B1** **B1**	
(ii)	$x = -2 + \sqrt{y-5}$	**M1**	attempt to obtain the inverse function
	$g^{-1}(x) = -2 + \sqrt{x-5}$ Domain of g^{-1}: $x \geq 9$	**A1** **B1**	Must be correct form for domain
	Alternative method: $y^2 + 4y + 9 - x = 0$	**M1**	attempt to use quadratic formula and find inverse
	$y = \dfrac{-4 + \sqrt{16 - 4(9-x)}}{2}$	**A1**	must have + not ±
(iii)	Need $g\left(3e^{2x}\right)$	**M1**	correct order
	$\left(3e^{2x}+2\right)^2 + 5 = 41$ or $9e^{4x} + 12e^{2x} - 32 = 0$ $\left(3e^{2x}-4\right)\left(3e^{2x}+8\right) = 0$	**DM1**	correct attempt to solve the equation
	leading to $3e^{2x} + 2 = \pm 6$ so $x = \dfrac{1}{2}\ln\dfrac{4}{3}$	**M1**	dealing with the exponential correctly in order to reach a solution for x
	or $e^{2x} = \dfrac{4}{3}$ so $x = \dfrac{1}{2}\ln\dfrac{4}{3}$	**A1**	Allow equivalent logarithmic forms
	Alternative method: Using $f(x) = g^{-1}(41)$, $g^{-1}(41) = 4$	**M1**	correct use of g^{-1}
	leading to $3e^{2x} = 4$, so $x = \dfrac{1}{2}\ln\dfrac{4}{3}$	**DM1** **M1** **A1**	dealing with $g^{-1}(41)$ to obtain an equation in terms of e^{2x} dealing with the exponential correctly in order to reach a solution for x Allow equivalent logarithmic forms
(iv)	$g'(x) = 6e^{2x}$ $g'(\ln 4) = 96$	**B1** **B1**	B1 for each

9 **(i)**	$\frac{dy}{dx} = 3x^2 - 10x + 3$	**M1**	for differentiation
	When $x = 0$, for curve $\frac{dy}{dx} = 3$,		
	gradient of line also 3 so line is a tangent.	**A1**	comparing both gradients
	Alternate method:		
	$3x + 10 = x^3 - 5x^2 + 3x + 10$	**M1**	attempt to deal with simultaneous equations
	leading to $x^2 = 0$, so tangent at $x = 0$	**A1**	obtaining $x = 0$
(ii)	When $\frac{dy}{dx} = 0$, $(3x-1)(x-3) = 0$	**M1**	equating gradient to zero and valid attempt to solve
	$x = \frac{1}{3}$, $x = 3$	**A1,A1**	A1 for each
(iii)	Area = $\frac{1}{2}(10+19)3 - \int_0^3 x^3 - 5x^2 + 3x + 10\,dx$	**B1**	area of the trapezium
	$= \frac{87}{2} - \left[\frac{x^4}{4} - \frac{5x^3}{3} + \frac{3x^2}{2} + 10x\right]_0^3$	**M1**	attempt to obtain the area enclosed by the curve and the coordinate axes, by integration
	$= \frac{87}{2} - \left(\frac{81}{4} - 45 + \frac{27}{2} + 30\right)$	**A1** **DM1**	integration all correct correct application of limits (must be using *their* 3 from **(ii)** and 0)
	$= 24.7$ or 24.8	**A1**	
	Alternative method:		
	Area $= \int_0^3 (3x+10) - \left(x^3 - 5x^2 + 3x + 10\right)dx$	**B1**	correct use of '*Y*–*y*'
	$= \int_0^3 -x^3 + 5x^2\,dx$	**M1** **A1**	attempt to integrate integration all correct
	$= \left[-\frac{x^4}{4} + \frac{5x^3}{3}\right]_0^3 = \frac{99}{4}$	**DM1** **A1**	correct application of limits
10 **(a)**	$\sin^2 x = \frac{1}{4}$		
	$\sin x = (\pm)\frac{1}{2}$	**M1**	using $\text{cosec}x = \frac{1}{\sin x}$ and obtaining $\sin x = \ldots$
	$x = 30°, 150°, 210°, 330°$	**A1,A1**	A1 for one correct pair, A1 for another correct pair with no extra solutions

(b)	$\left(\sec^2 3y - 1\right) - 2\sec 3y - 2 = 0$	M1	use of the correct identity
	$\sec^2 3y - 2\sec 3y - 3 = 0$ $(\sec 3y + 1)(\sec 3y - 3) = 0$	M1	attempt to obtain a 3 term quadratic equation in sec $3y$ and attempt to solve
	leading to $\cos 3y = -1$, $\cos 3y = \frac{1}{3}$	M1	dealing with sec and $3y$ correctly
	$3y = 180°, 540° \quad 3y = 70.5°, 289.5°, 430.5°$ $y = 60°, 180°, 23.5°, 96.5°, 143.5°$	A1,A1 A1	A1 for a correct pair, A1 for a second correct pair, A1 for correct 5th solution and no other within the range
	Alternative 1:		
	$\sec^2 3y - 2\sec 3y - 3 = 0$	M1	use of the correct identity
	leading to $3\cos^2 3y + 2\cos 3y - 1$	M1	attempt to obtain a quadratic equation in cos $3y$ and attempt to solve
	$(3\cos y - 1)(\cos y + 1) = 0$	M1	dealing with $3y$ correctly **A** marks as above
	Alternative 2:		
	$\frac{\sin^2 y}{\cos^2 y} - \frac{2}{\cos y} - 2 = 0$ $\left(1 - \cos^2 x\right) - 2\cos x - 2\cos^2 x = 0$	M1	use of the correct identity, $\tan y = \frac{\sin y}{\cos y}$ and $\sec y = \frac{1}{\cos y}$, then as before
(c)	$z - \frac{\pi}{3} = \frac{\pi}{3}, \frac{4\pi}{3}$	M1	correct order of operations
	$z = \frac{2\pi}{3}, \frac{5\pi}{3}$ or 2.09 or 2.1, 5.24	A1,A1	A1 for a correct solution A1 for a second correct solution and no other within the range